Chapter 5
Gravity

School of Earth and Environmental Sciences
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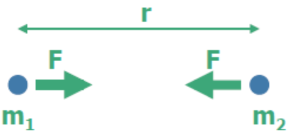
5.2 Gravitational potential and acceleration
Theory of gravity

1) Universal law of gravitation:



$$F = \frac{Gm_1 m_2}{r^2}$$

Universal gravitational constant G=6.67 x 10^{-11} Nm^2/kg^2

2) Second law of motion: $F = ma$

We can combine them to obtain the gravitational acceleration of m_2 toward m_1:

$$a = \frac{Gm_1}{r^2}$$

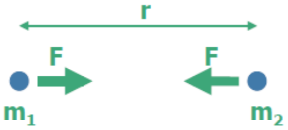
The gravitational attraction due to sphere mass M is the same as placing all the mass at the center of the sphere as long as the mass being attracted is outside the sphere.

5.2 Gravitational potential and acceleration

Theory of gravity

Gravitational acceleration:

$$a = \frac{Gm_1}{r^2}$$



Gravitational potential:

$$V = -\frac{Gm_1}{r}$$

Definition: The gravitational potential, V, due to a point mass m_1, at a distance r from m_1, is the work done by the gravitational force in moving a unit mass from infinity to to a position r from m_1.

Relationship:

$$a = -\frac{\partial V}{\partial r}$$

The gravitational acceleration is equal to the rate of change in the potential field

5.3 Gravity of the Earth

Gravitational acceleration on the Earth

If the Earth was a sphere the gravitational acceleration at the surface of the Earth would be:

$$g = \frac{GM_E}{R_E^{\ 2}}$$

Typical value: 9.81 m/s^2

Note that g is not dependent on the mass of the object being accelerated toward the center of the Earth

Gravity unit: gal **1 gal = 0.01 m/s^2** Typical value: g = 981 gal

Named after Galileo who did much of the pioneering studies into gravity

5.3 Gravity of the Earth
Gravitational acceleration on the Earth

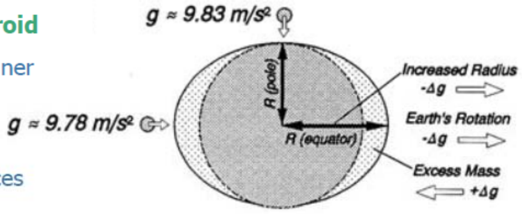
Earth is not a stationary sphere

The Earth is an oblate spheroid

- Fatter at the equator and thinner at the poles

The Earth is rotating

- Centrifugal acceleration reduces gravitational attraction. The further you are from the rotation axis the greater the centrifugal acceleration



Reference gravity formula

- The mathematically determined gravitational acceleration on a rotating oblate spheroid

$$g(\lambda) = g_e(1 + \alpha \sin^2 \lambda + \beta \sin^4 \lambda)$$

Where λ is the latitude, g_e the gravitational acceleration at the equator g_e = 9.7803185 m/s², and constants α = 0.005278895 β = 0.000023462

5.3 Gravity of the Earth
Orbits of satellites

When a mass gets caught in the gravitational field of a planet it starts to orbit the planet.

The gravitational attraction of the planet balances the outward centrifugal force:

$$\frac{GM_E m}{r^2} = m\omega^2 r$$

The distance of the satellite from the Earth is dependent on the gravitational attraction

➔ **By monitoring the orbits of man made satellites we can determine variations in g around the globe**

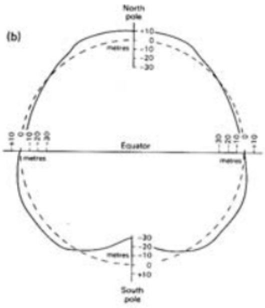
➔ **Also, using satellite altimetry we can measure the distance of the Earth surface below the satellite and determine the Earth's shape**

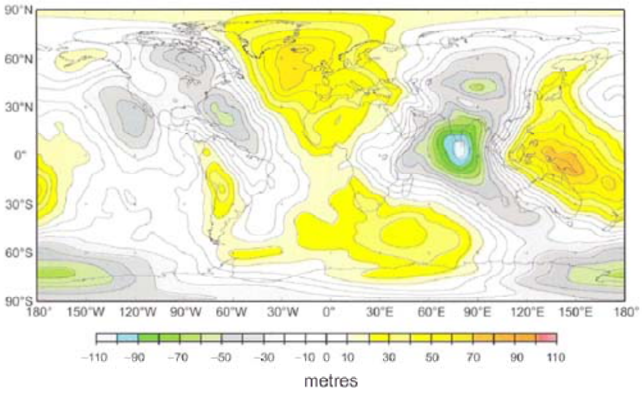
5.4 Shape of the Earth
The geoid

Mean sea level is an equipotential surface

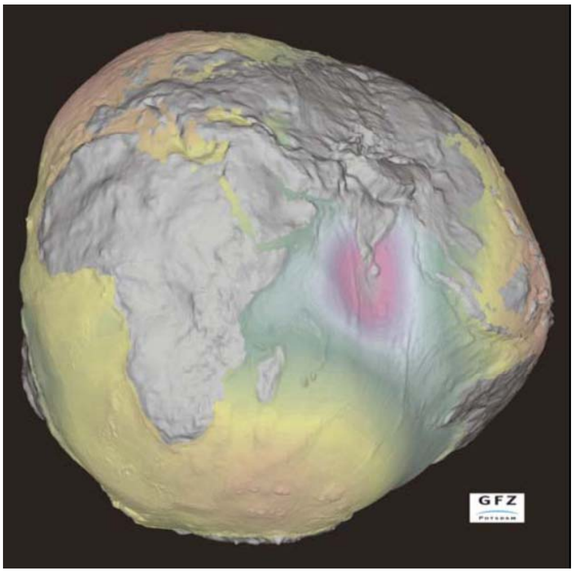
→ it is the geoid

These figures show the differences between the geoid and the reference ellipsoid/spheroid



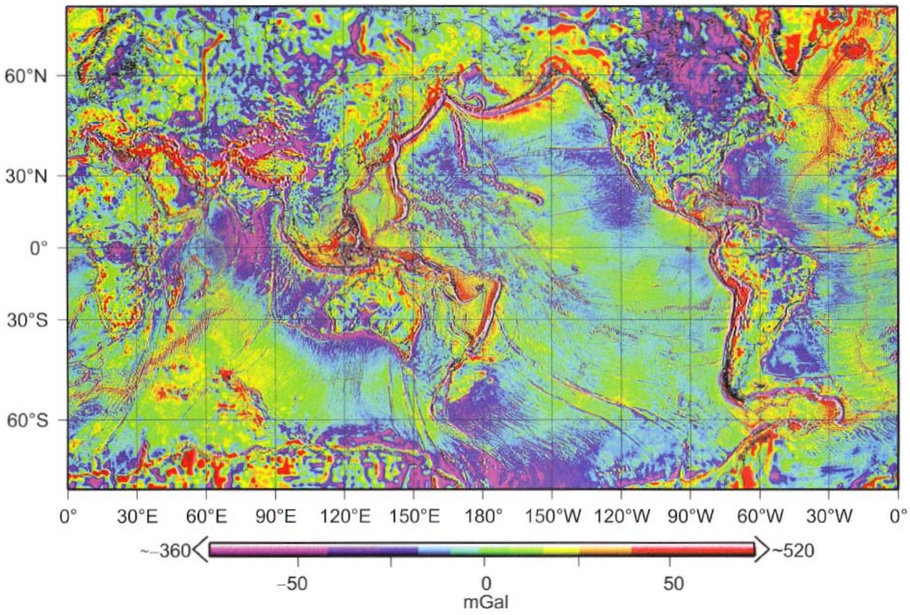


5.4 Shape of the Earth
The geoid



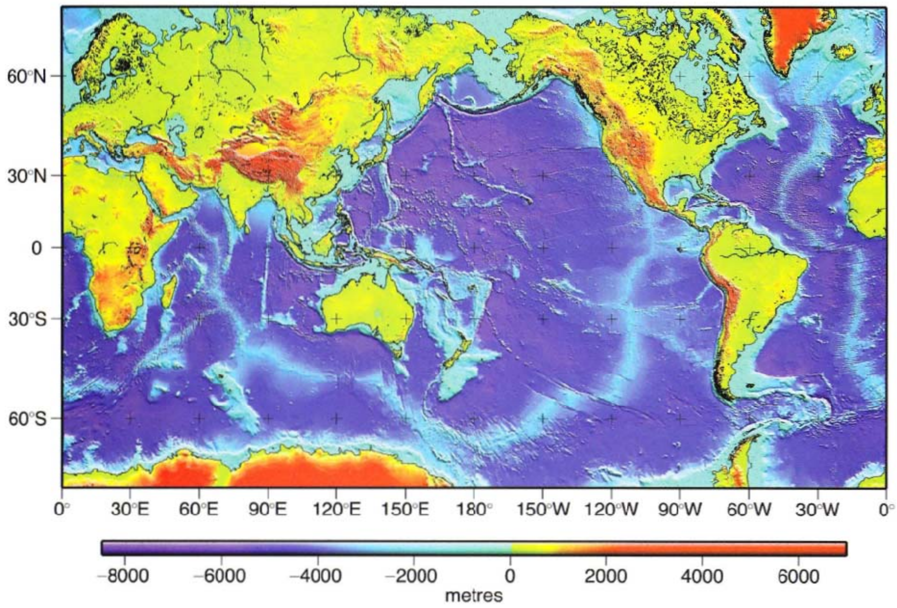
5.4 Shape of the Earth

Earth's gravity anomaly field

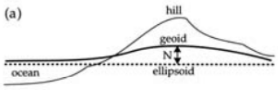


5.4 Shape of the Earth

Global topography (ETOP02)

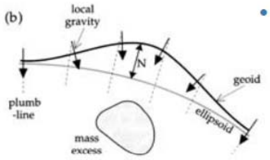


5.5 Gravity anomaly
Geoid anomaly



Differences between the geoid and reference ellipsoid are due to lateral density anomalies

- Due to topography



- Due to higher and lower density material in the crust or mantle

Instrument sensitivity:

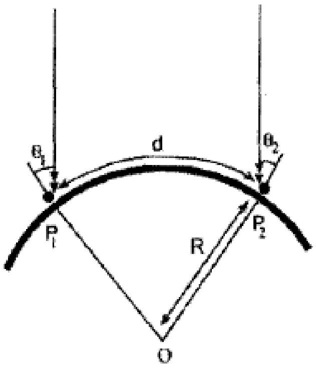
John Milne (1906): When a squad of 76 men marched to within 16 or 20 feet of the Oxford University Observatory it was found that a horizontal pendulum inside the building measured a deflection in the direction of the advancing load

Measured gravity anomalies are small compared to mean gravity

- mean g = 9.81 m/s^2
- measure gravity anomalies of 10^{-8} of the mean surface value

5.5 Gravity anomaly
French Expedition (Pierre Bouger and Everest)

- Try to measure the length of a degree of latitude in Peru and near Pairs in order to determine the shape of the Earth (1735-1745)



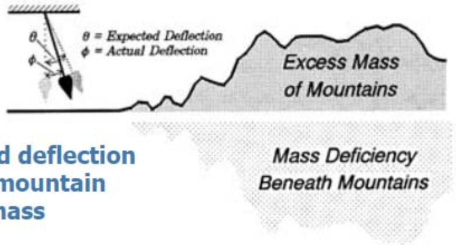
$$d = R(\theta_1 + \theta_2)$$

5.5 Gravity anomaly
5.5.2 Isostasy

- 18th and 19th century surveys set out to measure the shape of the Earth.

- They used plumb bobs and expected them to be attracted toward adjacent mountain chains eg the Andes and Himalayas

- But the plumb bob was not attracted as much as expected



They calculated that the observed deflection could be explained if the excess mountain mass was matched by an equal mass deficiency beneath

➔ The mountains were in **isostatic equilibrium**

The Earth's lithosphere is "floating" on the denser asthenosphere

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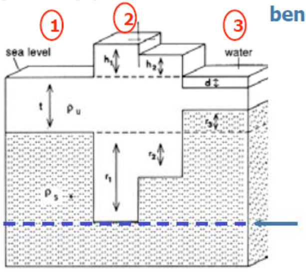
5.5 Gravity anomaly
Airy's hypothesis

Airy's hypothesis

to account for the mass deficiency beneath mountains



- Two densities, that of the rigid upper layer, ρ_u, and that of the substratum , ρ_s
- Mountains therefore have deep roots

The **compensation depth** is the depth below which all pressures are hydrostatic

Equating the masses in vertical columns above the compensation depth:

① $t\rho_u + r_1\rho_s = (h_1 + t + r_1)\rho_u$ ②

① $t\rho_u + r_1\rho_s = d\rho_w + (t - d - r_3)\rho_u + (r_1 + r_3)\rho_s$ ③

A mountain height h_1 is underlain by a root of thickness:

$$r_1 = \frac{h_1\rho_u}{\rho_s - \rho_u}$$

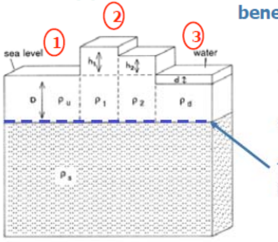
Ocean basin depth, d, is underlain by a anti-root of thickness:

$$r_3 = \frac{d(\rho_u - \rho_w)}{\rho_s - \rho_u}$$

5.5 Gravity anomaly
Pratt's hypothesis

Pratt's hypothesis

to account for the mass deficiency beneath mountains



- The depth of the base of the upper layer is constant
- Mountains therefore have low density roots

The **compensation depth** is the depth below which all pressures are hydrostatic

Equating the masses in vertical columns above the compensation depth:

① $D\rho_u = (h_1 + D)\rho_1$ ②

① $D\rho_u = d\rho_w + \rho_d(D-d)$ ③

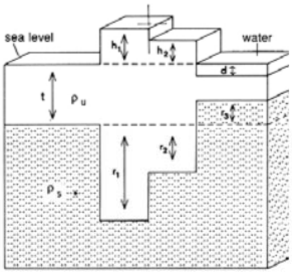
A mountain height h_1 is underlain by low density material, density ρ_1:

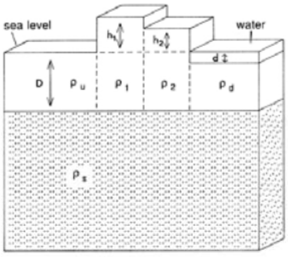
$$\rho_1 = \rho_u\left(\frac{D}{h_1 + D}\right)$$

Ocean basin depth, d, is underlain by a high density material, density ρ_d:

$$\rho_d = \frac{\rho_u D - \rho_w d}{D-d}$$

5.5 Gravity anomaly
Airy vs. Pratt





For a given location we must ask ourselves
- Is there isostatic equilibrium?
- Which process is operating?
- Is it a combination of the two?
  Pratt and Airy are end members

We can start to address some of these questions with gravity measurements

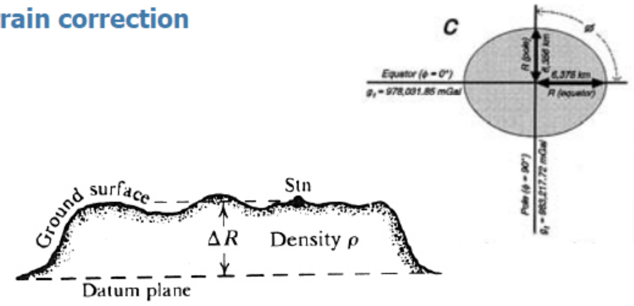
But first, we must correct our measurements...

5.5 Gravity anomaly
Gravity corrections

Observations still subject to extraneous effects unrelated to subsurface geology

➔ Must make corrections…

1. **Latitude correction**
2. **Free-air correction**
3. **Bouguer correction**
4. **Terrain correction**



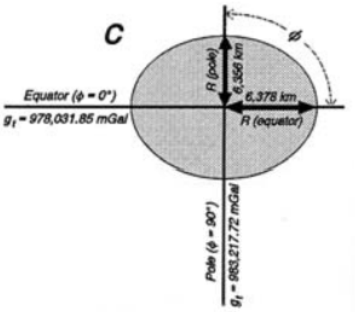
5.5 Gravity anomaly
Latitude corrections

Reference formula:

$$g(\lambda) = g_e(1 + \alpha \sin^2 \lambda + \beta \sin^4 \lambda) \quad \text{m/s}^2$$

Where λ is the latitude, g_e the gravitational acceleration at the equator g_e = 9.7803185 m/s^2, and constants α = 0.005278895 β = 0.000023462

- **This formula provides the variation in g due to the spheroid**
- **It is a function of latitude λ only**
- **Calculate the correction for latitude of observation point**



5.5 Gravity anomaly
Free-air correction

Accounts for the $1/r^2$ decrease in gravity with distance from the center of the Earth. A given gravity measurement was made at an elevation h, not at sea level, recall:

$$g = \frac{GM_E}{{R_E}^2}$$

The gravity at elevation h above sea level is approximated by:

$$g(h) = g_0\left(1 - \frac{2h}{R_E}\right)$$

g_0 is the gravity at sea level, ie

$$g_0 = g(\lambda)$$

The free-air correction is therefore:

$$\delta g_F = g_0 - g(h) = \frac{2h}{R_E} g_0$$

5.5 Gravity anomaly
Free-air anomaly

A gravity "anomaly" suggests the difference between a theoretical and observed value

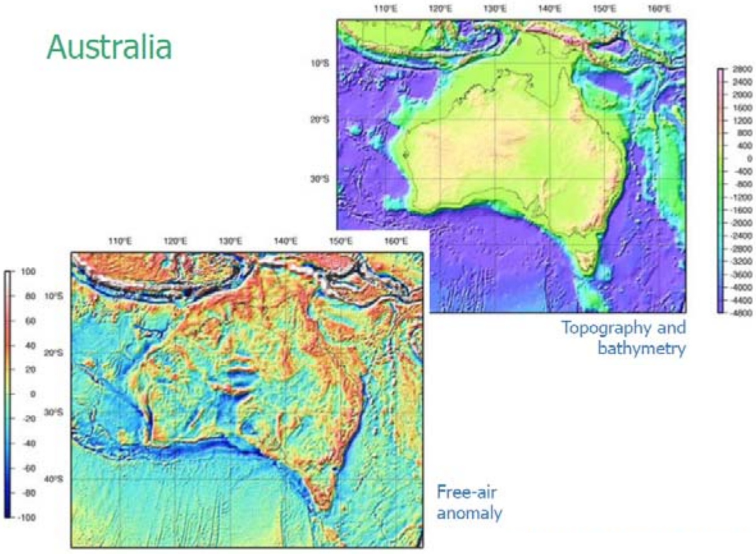
The free-air anomaly is calculated by correcting an observation for expected variations due to (1) the spheroid and (2) elevation above sea level

Then the free-air anomaly is:

$$\begin{aligned} g_F &= g_{obs} - g(\lambda) + \delta g_F \\ &= g_{obs} - g(\lambda) + \frac{2h}{R_E} g(\lambda) \\ &= g_{obs} - g(\lambda)\left(1 - \frac{2h}{R_E}\right) \end{aligned}$$

5.5 Gravity anomaly
Austrailia

Australia

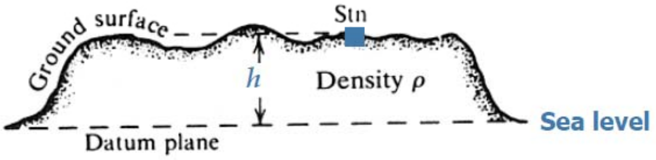


Topography and bathymetry

Free-air anomaly

5.5 Gravity anomaly
Bouguer correction

Accounts for rock thickness between observation and sea level



Treat the rock as an infinite horizontal slab, the Bouguer correction is:

$$\delta g_B = 2\pi G\rho h$$

- This additional slab of rock between the observation point and sea level causes an additional attraction

5.5 Gravity anomaly
Bouguer anomaly

Apply all the corrections:

$$\begin{aligned} g_B &= g_F - \delta g_B + \delta g_T \\ &= g_{obs} - g(\lambda) + \delta g_F - \delta g_B + \delta g_T \end{aligned}$$

...watch the signs!

With the Bouguer anomaly

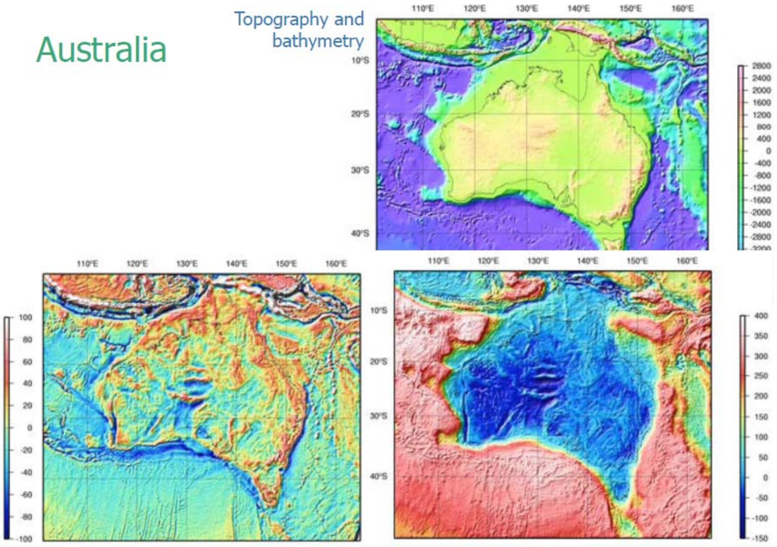
- We have subtracted theoretical values for the latitude and elevation
- We have removed the rock above sea level so the anomaly represents the density structure of material below sea level
- This is comparable to the free-air anomaly over the oceans and both have been corrected to sea level

Bouguer anomaly for offshore gravity:

- **Replace the water with rock**
- **Apply terrain correction for seabed topography**

5.5 Gravity anomaly
Austrailia

Australia



Free-air anomaly

Bouguer anomaly

5.5 Gravity anomaly
Isostatic equilibrium

Can we use gravity anomalies to tell if a region is in isostatic equilibrium?

Isostatic equilibrium means no excess mass.
Does this mean no gravity anomaly?

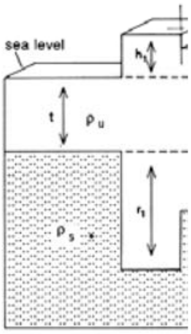
Not quite!

Consider the figure:

- Assume isostatic equilibrium
- Bouguer anomaly will be large and negative
- Free-air anomaly: small but positive

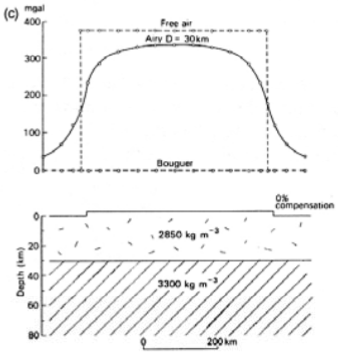
WHY?

WHY?

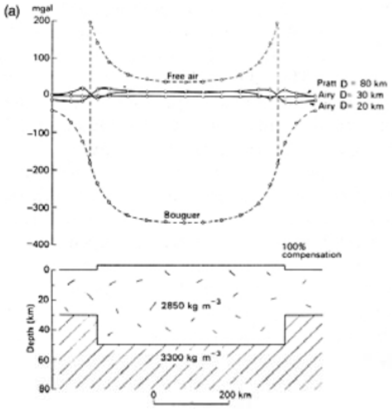


5.5 Gravity anomaly
Isostatic equilibrium

Isostatic equilibrium



Uncompensated
- Large positive Free-air
- Zero Bouguer

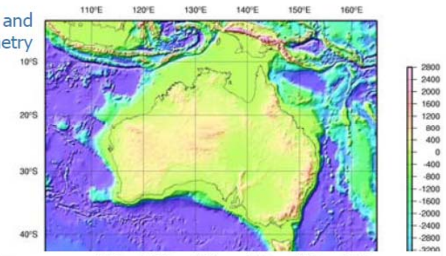


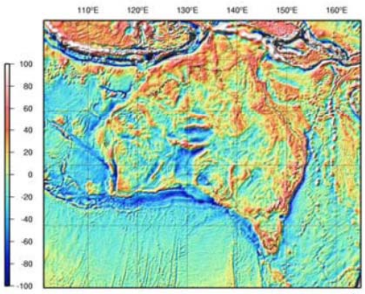
Compensated
- Small positive Free-air
- Large negative Bouguer

5.5 Gravity anomaly
Austrailia

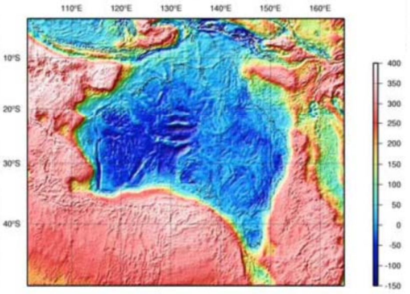
Australia

Topography and bathymetry





Free-air anomaly



Bouguer anomaly

5.5 Gravity anomaly
Buried sphere

Gravity anomalies

Analytical expressions for simple shapes: Buried sphere

Only the density contrast is important

$$\Delta\rho = \rho_1 - \rho_2$$

Gravitational acceleration toward sphere

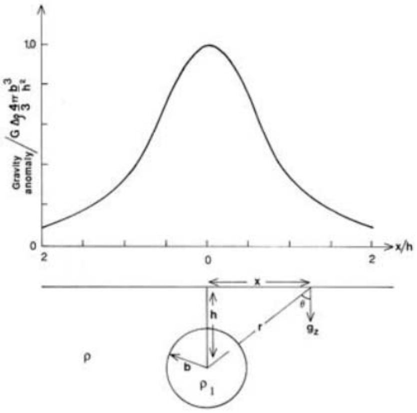
$$g = \frac{Gm}{r^2}$$

Gravimeters measure the vertical gravitational acceleration

$$g_z = \frac{Gm}{r^2}\cos\theta$$

$$= \frac{Gmh}{r^3}$$

$$= \frac{Gmh}{\left(x^2 + h^2\right)^{3/2}}$$



Mass excess of sphere $m = \frac{4}{3}\Delta\rho\pi b^3$

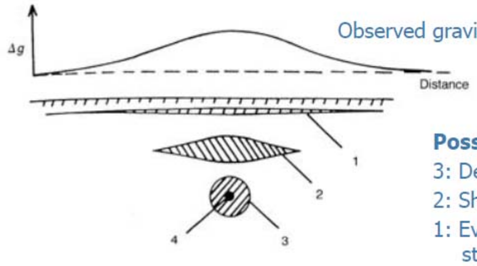
Gravity anomaly:

$$\delta g_z = \frac{4G\Delta\rho\pi b^3 h}{3\left(x^2 + h^2\right)^{3/2}}$$

5.5 Gravity anomaly
Ambiguity

Ambiguity

An observed gravity anomaly can be explained by a variety of mass distributions at different depths



Observed gravity anomaly

Possible causal structures:

3: Deep sphere
2: Shallower elongated anomaly
1: Even shallower, more elongated structure

Ambiguity in formula for a sphere:

$$\delta g_z = \frac{4G\Delta\rho\pi b^3 h}{3\left(x^2 + h^2\right)^{3/2}}$$

- Trade-off between density and radius

5.6 Observed gravity and geoid anomalies
Mid-ocean ridge

Gravity observations

Mid-ocean ridge

Four density models adequately satisfy the observations

Small positive free-air anomaly, large negative bouguer anomaly: close to isostatic equilibrium

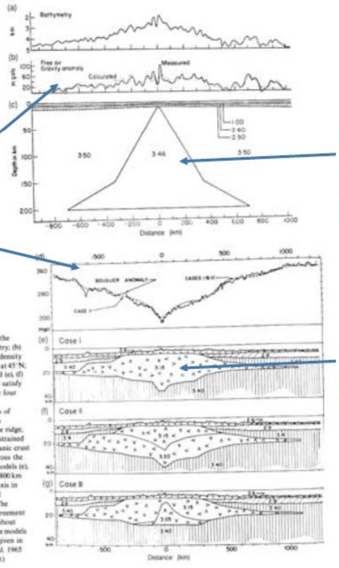


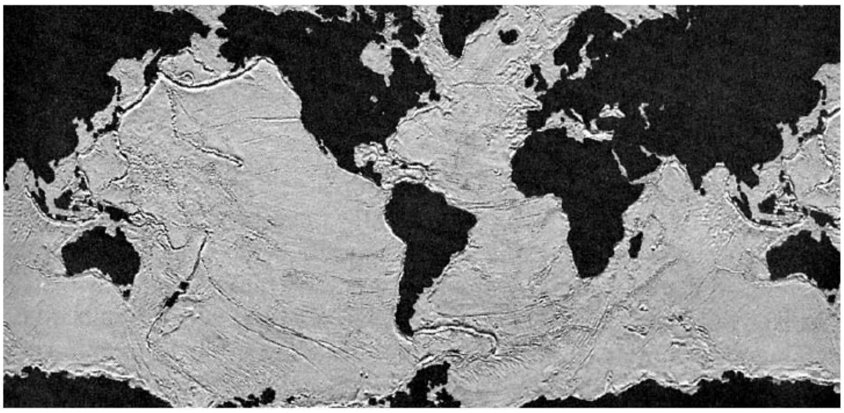
Figure 8.11. Gravity models for the Mid-Atlantic Ridge: (a) bathymetry; (b) free-air gravity anomaly and (c) density model for the Mid-Atlantic Ridge at 45°N; (d) Bouguer gravity anomaly and (e), (f) and (g) density models which all satisfy the anomaly shown in (d). These four density models – (c), (e), (f) and (g) – illustrate the nonuniqueness of models based on gravity data. A low-density zone lies beneath the ridge, but its dimensions need to be constrained by other methods also. The oceanic crust is assumed to be continuous across the ridge axis in model (c), but in models (e), (f) and (g) there is a zone some 800 km wide and centred on the ridge axis in which normal oceanic crust and uppermost mantle are absent. The density model in (c) is in better agreement with everything that is known about midocean ridge structure than the models in (e), (f) and (g). Densities are given in 10^3 kg m^{-3}. (From Talwani et al. 1965 and Keen and Tramontini 1970.)

Deep model has small density contrasts

Shallow models have larger density contrasts

5.6 Observed gravity and geoid anomalies

Ocean trenches



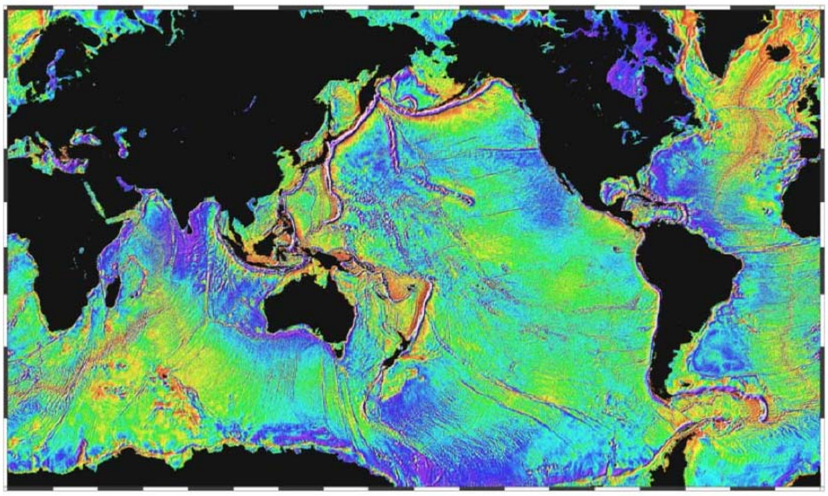
SEASAT Gravity Map

Largest anomalies are associated with the trenches

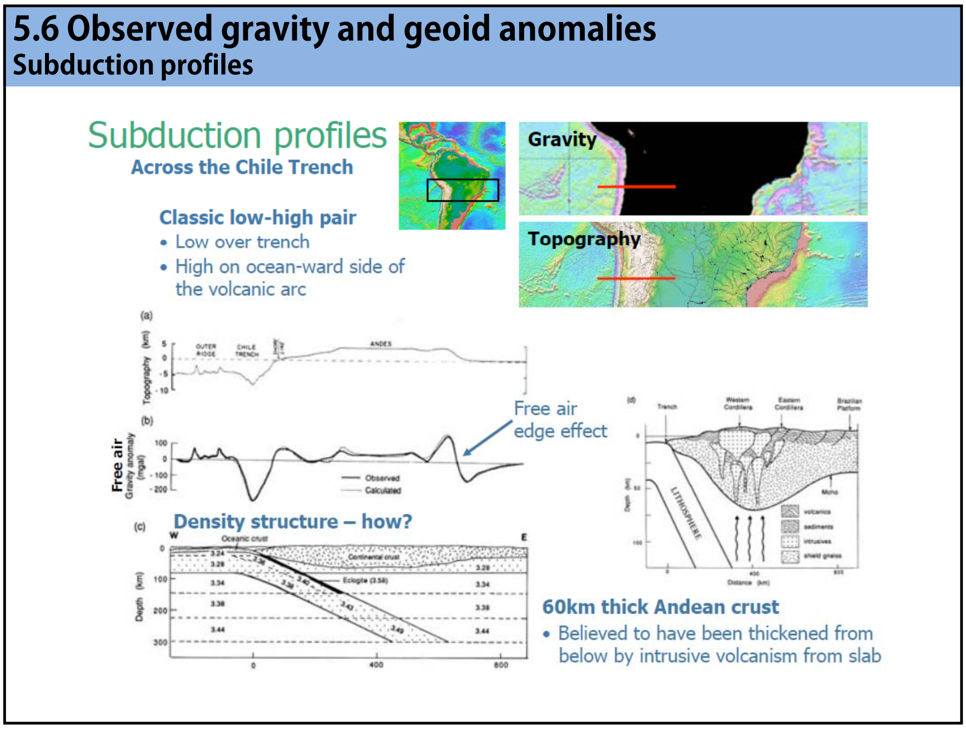
- 10 km deep and filled with water rather than rock
- Not compensated as they are being loaded down dip

5.6 Observed gravity and geoid anomalies

Ocean trenches



SEASAT Gravity Map



5.6 Observed gravity and geoid anomalies

Subduction profiles

Subduction profiles

Across the Chile Trench

Classic low-high pair

- Low over trench
- High on ocean-ward side of the volcanic arc

Free air edge effect

Density structure – how?

60km thick Andean crust

- Believed to have been thickened from below by intrusive volcanism from slab

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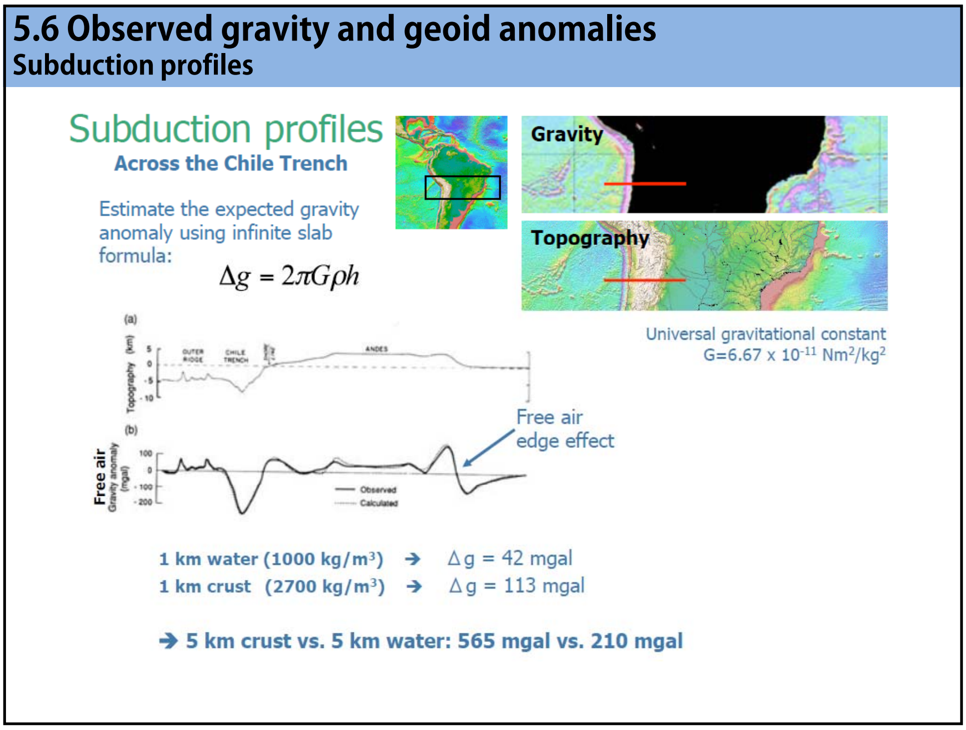


5.6 Observed gravity and geoid anomalies

Subduction profiles

Subduction profiles

Across the Chile Trench

Estimate the expected gravity anomaly using infinite slab formula:

$$\Delta g = 2\pi G\rho h$$

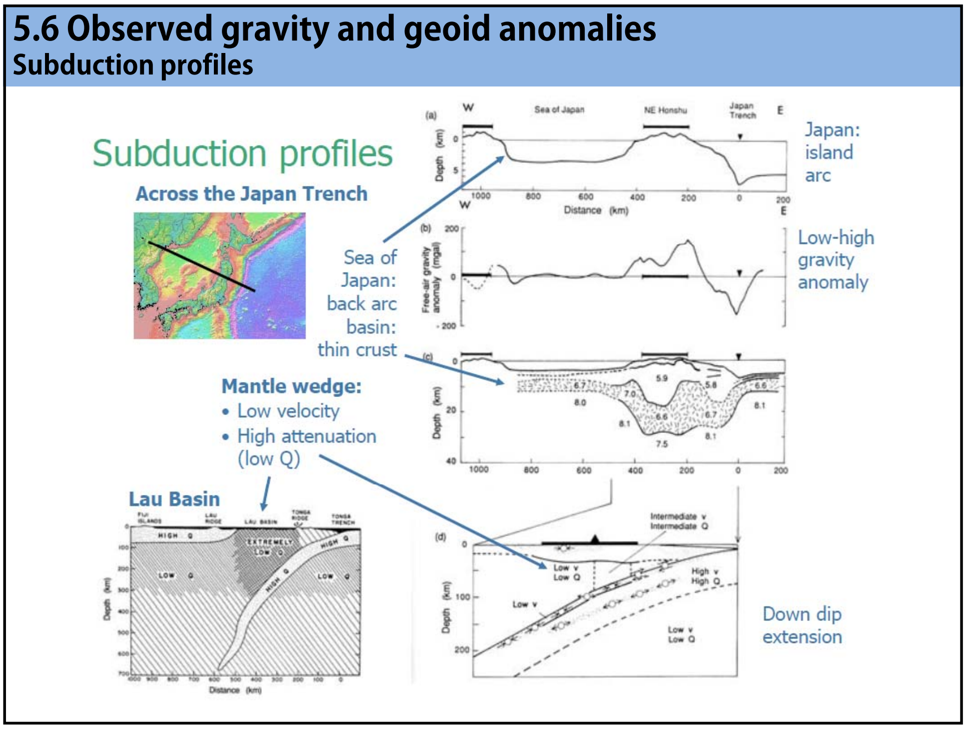
Universal gravitational constant $G=6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$

Free air edge effect

1 km water (1000 kg/m^3) → Δg = 42 mgal

1 km crust (2700 kg/m^3) → Δg = 113 mgal

→ 5 km crust vs. 5 km water: 565 mgal vs. 210 mgal



5.6 Observed gravity and geoid anomalies
Subduction profiles

Subduction profiles

Across the Japan Trench

Japan: island arc

Low-high gravity anomaly

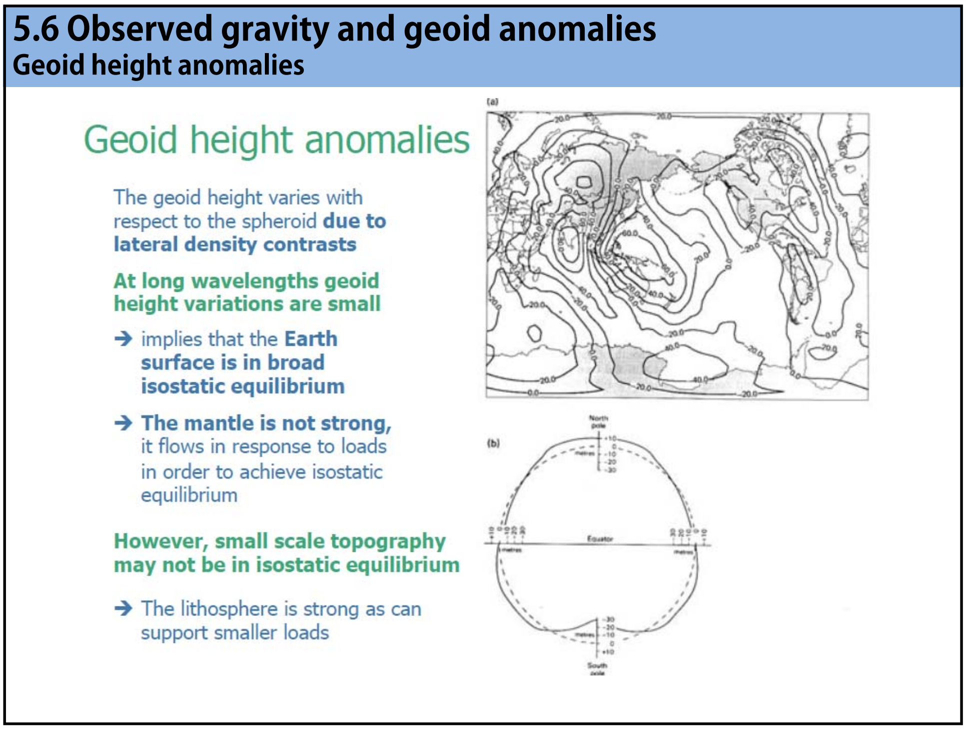
Sea of Japan: back arc basin: thin crust

Mantle wedge:

- Low velocity
- High attenuation (low Q)

Lau Basin

Down dip extension



5.6 Observed gravity and geoid anomalies
Geoid height anomalies

Geoid height anomalies

The geoid height varies with respect to the spheroid **due to lateral density contrasts**

At long wavelengths geoid height variations are small

➔ implies that the **Earth surface is in broad isostatic equilibrium**

➔ **The mantle is not strong,** it flows in response to loads in order to achieve isostatic equilibrium

However, small scale topography may not be in isostatic equilibrium

➔ The lithosphere is strong as can support smaller loads

5.6 Observed gravity and geoid anomalies

Gravity and convection

Our knowledge of the temperature and material properties of the mantle lead us to believe that there is convection

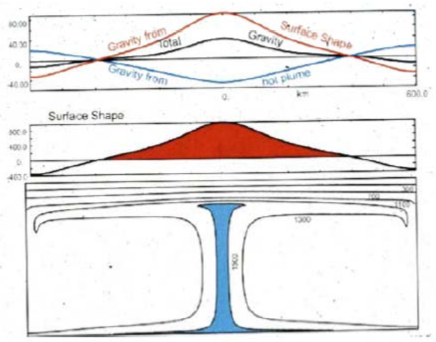
We would expect to see this in the global gravity field:

Upwelling of low density material

1. Gravity low due to low density
2. Gravity high caused by bulge due to upwelling

The winner: 2 > 1 so get bulge

See opposite effect above downwellings of cold dense material



5.6 Observed gravity and geoid anomalies

Gravity and convection

Our knowledge of the temperature and material properties of the mantle lead us to believe that there is convection

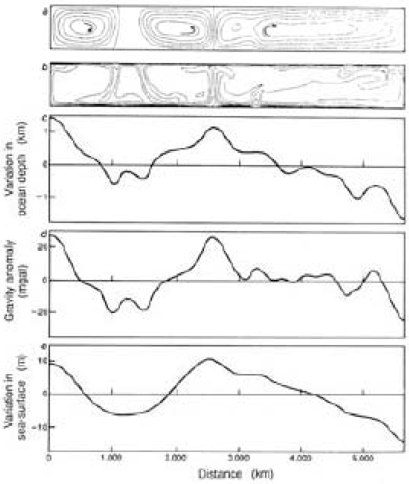
We would expect to see this in the global gravity field:

Upwelling of low density material

1. Gravity low due to low density
2. Gravity high caused by bulge due to upwelling

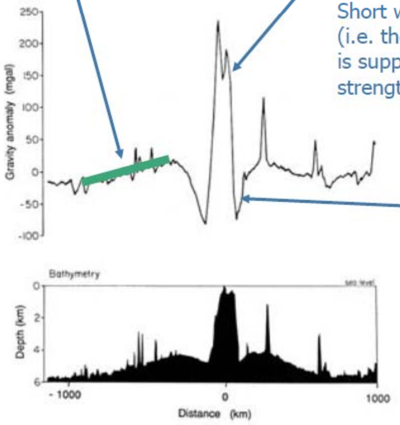
The winner: 2 > 1 so get bulge

See opposite effect above downwellings of cold dense material



5.6 Observed gravity and geoid anomalies

Gravity over Hawaii



Broad swell due to dynamic buoyancy

Lithosphere has no strength on this scale and is responding to dynamic forces

Gravity high due to thick islands

Short wavelength features (i.e. the islands: ~200 km) is supported by plate strength

Trough due to regional compensation

5.6 Observed gravity and geoid anomalies

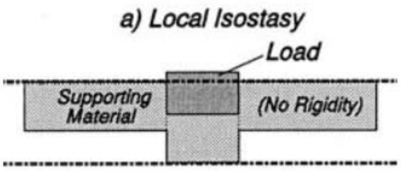
Lithospheric flexure

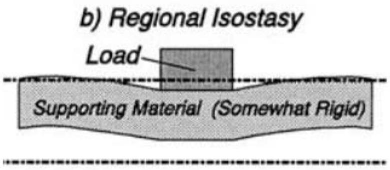
Local isostasy

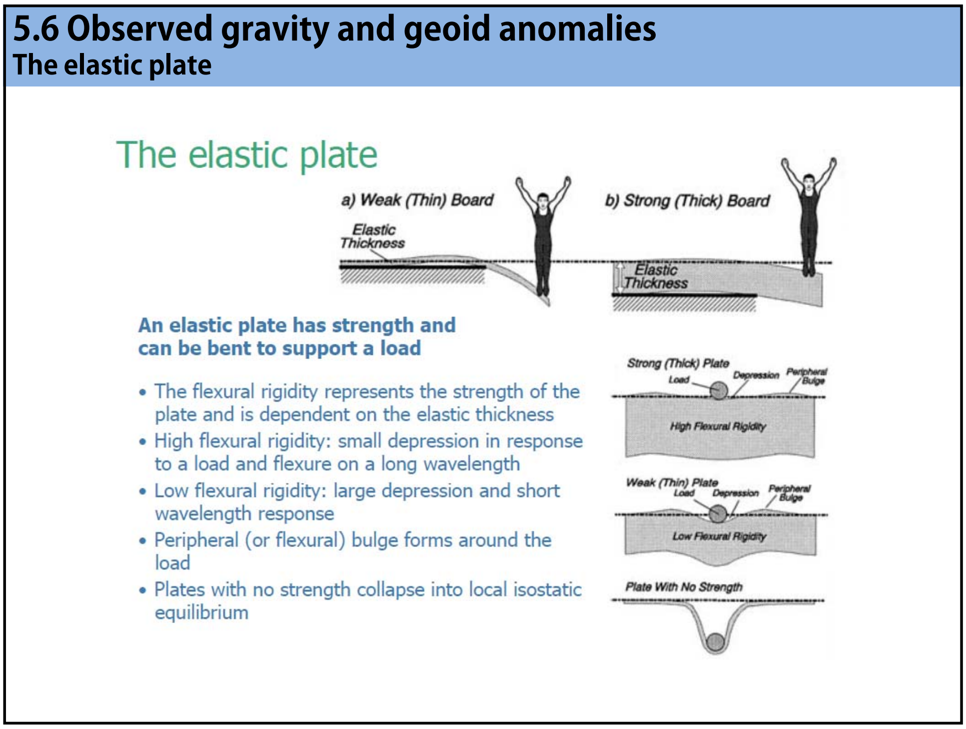
- Pratt and Airy are about local isostasy
- Any load is perfectly compensated
- Therefore there is no rigidity

Regional isostasy

- Some of the load is supported by the strength of the lithosphere
- Isostatic compensation still occurs on a larger regional scale







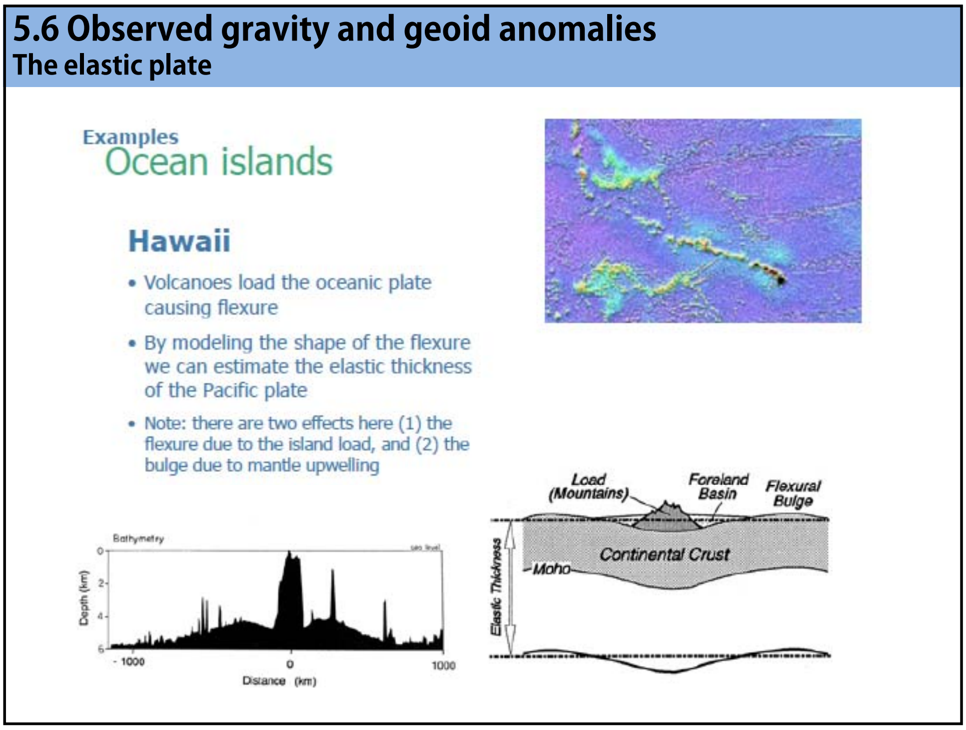
5.6 Observed gravity and geoid anomalies

The elastic plate

The elastic plate

An elastic plate has strength and can be bent to support a load

- The flexural rigidity represents the strength of the plate and is dependent on the elastic thickness
- High flexural rigidity: small depression in response to a load and flexure on a long wavelength
- Low flexural rigidity: large depression and short wavelength response
- Peripheral (or flexural) bulge forms around the load
- Plates with no strength collapse into local isostatic equilibrium



5.6 Observed gravity and geoid anomalies

The elastic plate

Examples

Ocean islands

Hawaii

- Volcanoes load the oceanic plate causing flexure
- By modeling the shape of the flexure we can estimate the elastic thickness of the Pacific plate
- Note: there are two effects here (1) the flexure due to the island load, and (2) the bulge due to mantle upwelling

5.6 Observed gravity and geoid anomalies
The elastic plate

Examples

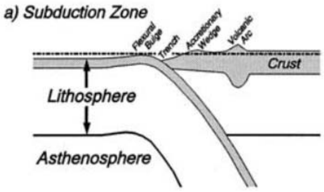
The elastic plate

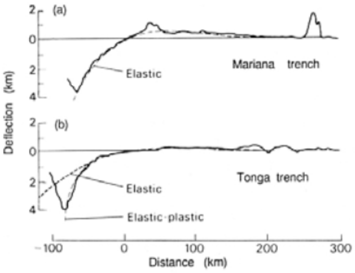
Subduction zones

- The accretionary wedge loads the end of the plate causing it to bend
- A flexural bulge is often observed adjacent to the trench

Mariana trench

- Topography matched with elastic plate, elastic thickness 28 km





Tonga trench

- Not all subduction zones can be modeled in this way eg the Tonga trench
- Instead this topography needs an elastic plate which deforms plastically once some critical yield stress is applied

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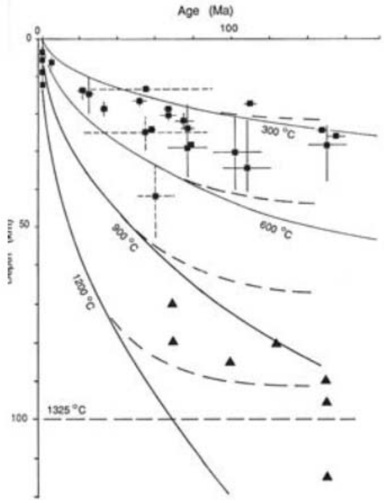
5.6 Observed gravity and geoid anomalies
Elastic thickness of oceanic plates



By modeling the flexure of the plates in response to loads we can estimate the elastic thickness

This shows an age dependency

- The elastic thickness increases with age and corresponds to the ~450°C isotherm
- Plate strength increases with age
- This is due to the gradual cooling of oceanic lithosphere

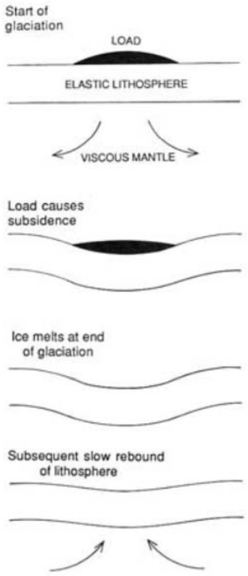
Elastic thickness

- **of oceans: 10-40 km**
- **of continents: typically 80-100 km**

Note: this is not the lithospheric thickness

5.6 Observed gravity and geoid anomalies
Isostatic rebound

Isostatic rebound



The rate of deformation after a change in load is dependent on the flexural rigidity of the lithosphere and the viscosity of the mantle

Need a load large enough which is added or removed quickly enough to observe the viscous response of the mantle

1. **Smaller loads:** ~100 km diameter
   tell us about uppermost mantle viscosity

   Lake Bonneville, Utah
   - dried up 10,000 years ago: 300 m of water load removed
   - Center of the lake has risen 65 m
   - Viscosity: 10^{20} to $4x10^{19}$ Pa s for 250 to 75 km thick lithosphere

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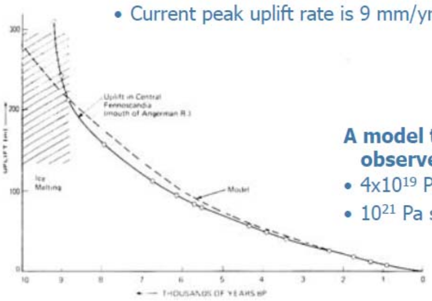
5.6 Observed gravity and geoid anomalies
Isostatic rebound

Isostatic rebound

2. **Larger loads:** ~1000 km diameter
   tell us about upper mantle viscosities

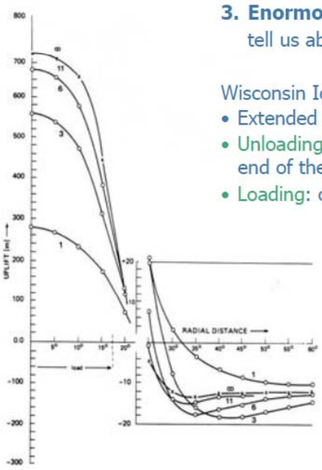
   Scandinavia
   - Removal of ice sheet at the end of the last ice age 10,000 years ago: ~2.5 km if ice removed
   - Current peak uplift rate is 9 mm/yr



A model that satisfies the observed deformation:
- $4x10^{19}$ Pa s asthenosphere overlaying
- 10^{21} Pa s mantle

5.6 Observed gravity and geoid anomalies
Isostatic rebound

Isostatic rebound



3. **Enormous loads:** thousands km diameter
   tell us about upper and lower mantle viscosities

Wisconsin Ice Sheet
- Extended over Northern US and Canada, up to 3.5 km thick
- Unloading: of continent due to removal of ice sheet at the end of the last ice age 10,000 years ago
- Loading: of the ocean basins due to additional water

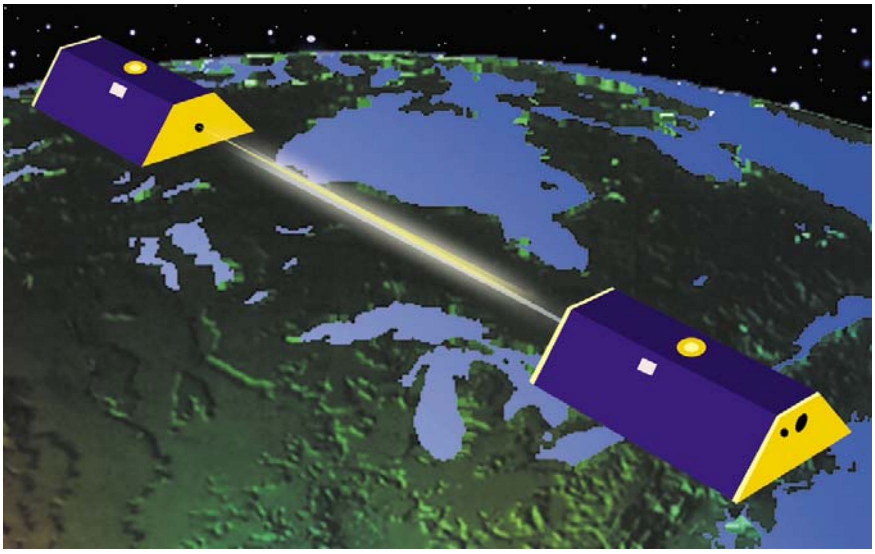
Mantle viscosity estimates from isostatic rebound:

Table 5.1. *Estimates of the viscosity of the mantle as determined from studies of postglacial rebound*

	Depth (km)	Viscosity (Pa s)
Lithosphere	0–100	Elastic (rigidity = 5×10^{24} N m)
Asthenosphere	100–175	4×10^{19}
Mantle	175–2885	10^{21}

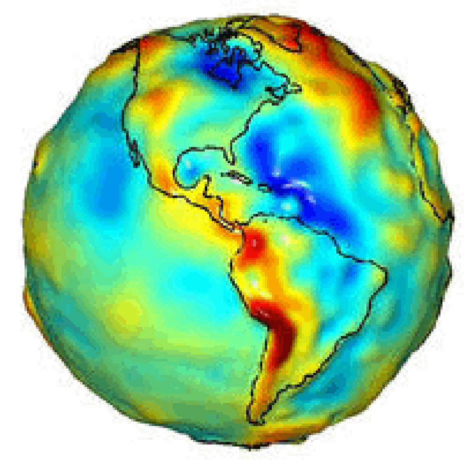
Source: From Cathles (1975).

Measuring Gravity
GRACE (Gravity Recovery And Climate Experiment)



Measuring Gravity

GRACE (Gravity Recovery And Climate Experiment)



Earth's Gravity Field Anomaly

- Global ocean circulation
- Slow current of Magma
- Thinning of ice sheets
- Earthquake
- and much more