Microseismicity: Beyond dots in a box

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Special Section

Microseismicity: Beyond dots in a box — Introduction

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This special section of GEOPHYSICS is a consequence of a very successful SEG workshop titled Microseismicity: Beyond dots in a box. The workshop was held 22 October 2010, following the SEG Annual Meeting that was held in Denver. More than 120 people attended the meeting, and nine oral papers and 22 posters were presented.

Small microseismic events, or acoustic emissions, occur naturally and as a result of anthropogenic influences in reservoirs. Sudden stress release leads to elastic rock failure, which serves as an effective seismic source. These microearthquakes may be the result of production or hydraulic stimulation, but they also may be a consequence of natural tectonic activity. They are usually detectable using only sensitive sensors and after careful data processing. Such passive seismic monitoring has been used in mining settings for more than 100 years, but its application in petroleum setting is relatively new. As such, it is a rapidly advancing field of technology, where the challenges are manifold and span issues associated with data acquisition, processing, and interpretation. Much can be learned from methods and case studies developed for microseismic monitoring in volcanological, geothermal, and mining settings, but one of the key advantages in oilfield monitoring is that a great deal is ordinarily already known about the reservoir. Comparatively good velocity models exist, and production or injection information is available — the same is obviously not true for a volcano.

The microearthquake or microseismic event location is useful in itself. Clusters of events delineate faults and fracturing, highlighting reactivation or the generation of new zones of failure. This has been very effective in helping assess the efficacy of hydraulic stimulation (i.e., frac monitoring). Longer term monitoring will no doubt provide an early warning system for detecting top seal leakage and fault reactivation in CO₂ sequestration projects, for example.

Until recently, most microseismic monitoring studies in petroleum settings have concentrated on detecting and locating events. An aim of the SEG workshop was to highlight progress in a wealth of other applications — ‘beyond dots in a box.’ For example, source characteristics and mechanisms provide helpful information about the stress field, especially as multiwell monitoring and surface arrays become more common. Magnitudes help quantify stress drop and focal mechanisms and provide insights into the magnitude and orientation of the stress tensor. Furthermore, microseismic events can be used to image the surrounding media. They can help refine velocity models, study attenuation, and are ideally suited to estimating anisotropy parameters. Insights into the nature of faults, estimates of the stress tensor, and velocity model refinement are all valuable inputs for reservoir simulators.

The breadth of potential applications of microseismic monitoring can be summarized as follows:

- estimating magnitude and orientation of the stress tensor
- predicting stress build-up and potentially mitigating wellbore failure
- imaging fault and fracture orientations and their reactivation
- characterizing seismic anisotropy, which can be used to determine anisotropy parameters but also can be used to assess lithology and fracture-set properties including orientation, density, and size.
- studying fluid-properties using frequency-dependent wave characteristics (e.g., Q estimation and frequency-dependent shear-wave splitting)
- monitoring injection fronts such as water, CO₂, and steam
- monitoring hydraulic fracturing, especially in tight-gas shales and sands
- studying compaction effects around reservoirs
- studying cap-rock integrity

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• studying sealing faults and reservoir compartmentalization
• identifying seismically active and potentially hazardous zones
• calibrating geomechanical models.

The papers presented in this special section summarize recent developments in six aspects of passive seismic monitoring: survey design and processing considerations, improving event locations, interpreting event magnitudes, source mechanisms, anisotropy estimation, and reservoir geomechanics.

Survey design and processing considerations

Microseismic event location and source mechanisms offer a rich source of information about a reservoir, but the accuracy of any such analysis needs to be treated carefully. It is easy to make meaningless estimates of such event attributes. Foulger and Julian describe factors affecting the typical accuracy of source locations and discuss methods to compute more precise relative locations. They also describe the various magnitude scales used to report source strength. Finally the accuracy of source mechanism studies using moment tensor inversions is discussed.

Zimmer provides a tutorial on the content and application of modeling and design studies for microseismic surveys with downhole geophones. Using only the information available prior to the microseismic survey, these studies provide a tool to optimize the acquisition, processing, and interpretation of microseismic data and to assess the potential of a given acquisition geometry to meet the survey objectives.

Improving event locations

Any subsequent data analysis is strongly influenced by the quality of event locations. Single-well monitoring is the norm, rather than the exception, so it is important to use as much information as possible to refine event location. Interferometry offers a new approach, and Poliannikov et al. consider a problem of localizing hydraulic fracture microseismic events using previously localized events in a reference fracture. They propose the use of single-well interferometry and show how event localization uncertainty can be reduced by averaging over a large number of reference events.

Another approach is to use seismic multiplets (recurring events whose waveforms are essentially identical) to refine locations. Moriya proposes a phase-only correlation of time-varying spectral representations of waveforms to identify similar microseismic events. The author tests its application to earthquake aftershock events and demonstrates the feasibility of this technique for identification of similar seismic waveforms.

Interpreting event magnitudes

Event magnitude is related to energy release and the stress drop associated with rock failure. The relationship between event magnitude and frequency is well known to be sensitive to the style of rock failure and can be described by the so-called Gutenberg-Richter ‘b-value’. Haney et al. calculate the magnitudes of several thousand small earthquakes at the German Deep Drilling Site (KTB) during an injection phase in 2004–2005 using data from a single three-component borehole geophone and extrapolating a scale determined for a much smaller number of events using near-surface stations. They determine a b-value for all events of 0.78. They find that the event distribution with time is consistent with prediction from theory assuming pore pressure diffusion as the underlying mechanism to trigger the events. The seismogenic index of −4 shows that the seismic hazard potential at the German Deep Drilling Site (KTB) is comparatively low.

Shapiro et al. discuss seismic hazard associated with fluid injection. The authors investigate the relationship between the geometric size of the cloud of microseismicity induced by fluid injection and magnitude of induced earthquakes, and they propose a relationship between the dimension of the fluid stimulated volume and discontinuities that controls the probability of triggering earthquakes.

Source mechanisms

The signature of the seismic source is held in the recorded seismic waveform. Microseismic data can be used to infer fault geometry and failure mechanism. With a good velocity model and favorable recording geometry, the entire moment tensor can be estimated, but practical considerations usually mean that neither is generally available.

Du and Warpinski discuss the accuracy of fault plane investigations. Uncertainties in inferred fault plane orientations and the associated slip directions are examined using moment tensor inversion of synthetic data sets. The authors add white noise to the theoretical seismic radiation pattern for typical downhole hydraulic fracture monitoring geometries and examine the errors in the resulting fault plane solutions.

Microseismic moment tensors have the potential to reveal important details of fracture processes that occur during hydraulic fracture treatments of tight reservoirs. Eaton and Forouhiideh show that careful survey design is required to be confident in inversion results. The paper reveals a simple design consideration, namely that the stability of the inversion is related to the solid angle subtended by the receiver array, and it shows that ill-conditioned inversions may produce biased results.

Li et al. describe a moment tensor inversion method and apply it to a case study of surface and downhole recording of microseismicity associated with oil production. A full waveform inversion method is used to compare the orientations of the seismically active faults with known faults in the reservoir.

Song and Toksöz discuss a new methodology for estimating source mechanisms of induced microseismic events. They show that full waveform inversion improves stability of the inversion; in particular, including the near-field terms enables inversion of the complete moment tensor even from a single vertical monitoring borehole. They demonstrate that such inversion is practical for events in close vicinity of the monitoring borehole and investigate methods of stabilization at greater distances.

Anisotropy estimation

Due to the recording geometry and distribution of sources, microseismic data generally are well suited to estimating anisotropy parameters. Furthermore, microseismic events normally are rich in S-waves and are therefore ideally suited to study shear-wave splitting. Such information is valuable input for data processing that considers anisotropy, but it also provides valuable information about rock properties.
Gei et al. use P-wave arrivals recorded by surface monitoring arrays to invert for a best fitting homogeneous transversely isotropic model with a vertical axis of symmetry (i.e., VTI symmetry). The Thomsen parameter $\delta$ and the anellipticity coefficient $\eta$ are estimated from traveltime picks. They explore the sensitivity of this inversion to picking errors and uncertainties in the P-wave vertical velocity and source depth.

Grechka and Duchkov develop a technique for estimating seismic anisotropy from perforation-shot data acquired in narrow-angle geometries. They apply their methodology to build a triclinic model that allows them to fit the traveltimes and simultaneously locate perforation shots recorded in a shale-gas field. Grechka et al. then present a case study in which they build an effective anisotropic model simultaneously with locating microseismic events. They show that anisotropy is necessary to explain the recorded data and its variation is suggestive of opening fractures in the course of hydraulic well stimulation.

Wuestefeld et al. present an analysis of seismic anisotropy using shear wave splitting measurements made on microseismic events recorded during hydraulic fracture stimulation. They then use an assumed rock-physics model to invert the splitting measurements for fracture parameters. They show a temporal variation in anisotropy that mimics fluid pumping volumes and present evidence for induced fracturing that follows preexisting fractures that are oblique to the general trend of the main frac.

**Reservoir geomechanics**

The long-term management of any reservoir requires a reliable geomechanical model that considers stress changes and the development of faults and fractures. This is a grand challenge, and microseismic monitoring provides a valuable means of calibrating and verifying geomechanical models. Numerical models of seismicity and its relation to fluid and mechanical properties provide a valuable link to observations.

Zhao and Young present a numerical modeling study using a 2D distinct element particle flow code that offers insights into fracture mechanics in naturally fractured reservoirs. They compare the geometry of hydraulic fractures with seismic source information (locations, magnitudes, and mechanisms). The numerical results qualitatively agree with laboratory and field observations and suggest possible mechanics for new fracture development and their interaction with a natural fracture (e.g., a tectonic fault).

McClure and Horne present an approach to modeling induced seismicity due to hydraulic fracturing through coupled rate and state friction with fluid flow. Their simulation focuses on fluid-only injections (no proppant) and understanding injection parameters that control the maximum magnitude of the induced seismicity. While the model is relatively simple, the authors show that decreasing injection pressure over time was a successful strategy for reducing the maximum magnitude.

The papers compiled in this volume illustrate the diversity in applications of microseismic monitoring. The technology has rapidly gone from a subject of abstract curiosity to a staple in the exploration seismologist’s toolbox. The number of research papers in this field is growing rapidly; the reader is also referred to the recent special issue of *Geophysical Prospecting* (vol. 5, 2010) and two review papers in the 75th anniversary issue of *Geophysics* (Maxwell et al.; Duncan and Eisner, vol. 75, no. 5, 2010). Passive seismic monitoring will no doubt help address many of the future challenges facing the petroleum industry, including optimizing enhanced oil recovery (EOR), heavy oil extraction, CO$_2$ storage in geologic reservoirs, exploiting unconventional hydrocarbons targets, and drilling hazard mitigation — to name a few.
Earthquakes and errors: Methods for industrial applications

Gillian R. Foulger¹ and Bruce R. Julian¹

ABSTRACT

The high accuracies and realistic confidence assessments demanded for seismic monitoring of hydraulic fracturing work require specialist experimental approaches. These include seismic network design based on quantitative modeling, high-quality instrument deployments, and accurate and detailed crustal models. Confidence estimates must take into account uncertainties about crustal structure, which may dominate error budgets. Earthquake size should be expressed in terms of scalar seismic moment or the associated moment magnitude $M_w$, which is related to fundamental physical source processes, and not as traditional earthquake magnitudes. Representing earthquake mechanisms in terms of seismic moment tensors allows for processes such as volume changes and complex types of shearing that are important in hydrocarbon and geothermal reservoirs. Traditional fault-plane solutions are based on simplifying assumptions such as shear slip on a planar faults, and isotropic crustal structures, which may introduce large uncertainties. Quantitative assessment of confidence regions for moment-tensor source mechanisms, a newly emerging field, is important for distinguishing computational artifacts from real physical phenomena. We review methods currently available for realistic error estimation for earthquake locations and moment tensors, with particular emphasis on surface sensor arrays in geothermal areas.

INTRODUCTION

The use of hydraulic fracturing as an aid to hydrocarbon extraction and for creating enhanced geothermal systems (EGS) is becoming increasingly widespread. Such operations often induce large numbers of small earthquakes (known as “microearthquakes,” “microseismic events,” or “fracking-induced events”). Like natural earthquakes, these events occur in response to rock failure and fluid motion. There is no evidence that they differ fundamentally in any way from naturally occurring earthquakes in geothermal and hydrocarbon reservoirs or, for that matter, other contexts, e.g., on tectonic faults. The techniques applicable to studying both industrially induced and naturally occurring earthquakes are thus fundamentally the same.

The sizes of earthquakes are fractally distributed, regardless of the context in which they occur. As a result, swarms of many small earthquakes are accompanied by the occasional larger event which is not technically a “microearthquake” (i.e., an earthquake that is not felt at the surface). For this reason, we use the terms “earthquake” or “event” in this paper. Hydraulic fracturing operations aim to manage the sizes of induced events so they do not cause ground shaking at the surface that is strong enough to be a societal nuisance.

Earthquake seismology (“passive seismology”) has a history of technological development about a century long. During this time, methods have been developed to use recordings of earthquakes to calculate earth structure, hypocenter locations, magnitudes, and source mechanisms. For most of this time, work has primarily targeted academic and hazard-reduction objectives. Achieving these objectives does not necessarily require high levels of accuracy. For example, the exact locations of earthquakes at the level of a few meters are not critical for tectonic studies or hazard reduction. Likewise, although earthquake sizes and source mechanisms are required to inform those applications, extreme levels of accuracy are not. Adequate approaches may include using single-component sensors, 1D crustal models, station corrections, compressional-waves only for locations, simple amplitude- or coda-length magnitudes, and the shear-faulting-only assumption to derive fault-plane solutions.

Such approaches are not, however, adequate to deliver the levels of accuracy, either in results or in error estimates, that modern industrial hydraulic fracturing applications demand. These may include, for example, hypocenter locations accurate to a few meters in cases where earthquake locations are used to guide drilling. For these purposes, specialist approaches are necessary. In this paper,
we provide a brief overview of some aspects of the advanced, state-of-the-art methodologies that can deliver results to the required standards. We focus particularly on surface seismometer arrays deployed to monitor hydraulic fracturing in geothermal settings.

EARTHQUAKE LOCATIONS

Maximizing location accuracy

Hypocentral locations are arguably the most fundamental information about earthquakes. Nevertheless, hypocenters are often surprisingly poorly determined. Even when earthquakes actually lie on planar structures, computed hypocenters typically show only diffuse "dots in a box," and have limited ability to reveal geological features such as faults that may be important drilling targets. Furthermore, derived hypocenters are often subject to significant systematic biases, causing clouds of locations to be displaced from their true positions. The problem for the end user may be exacerbated where independently computed locations differ by more than the stated formal errors. This inconsistency results from the errors being calculated incorrectly, ignoring what is often the major contributor to the error budget, which is uncertainty about crustal structure. As a result, there is an urgent requirement for tools that produce locations that are not only accurate, but also have realistic error estimates.

Seismic network design

A geometrically well-distributed network of 3C seismometers is a prerequisite for computing hypocenters accurately. Ideally, seismometers should surround the earthquakes uniformly in three dimensions, and include stations below the events, but this is not usually practical. In general, a distributed, near-surface array is the closest approximation to this that can economically be achieved. Deploying strings of sensors in multiple boreholes is also a good approach, but usually prohibitive on grounds of expense.

Where the earthquakes of interest are small or ground noise, e.g., from operations, is high, near-surface sensors may be unable to record events of interest sufficiently well to contribute data to locations. In some experiments, strings of sensors in boreholes are used. With such an approach, it may be possible to deploy instruments within a few hundred meters of the earthquakes of interest. However, a single linear string of sensors is a geometrically weak configuration, which limits potential hypocentral accuracies. The network geometry can be improved by deploying strings in several boreholes. This is expensive, and thus rarely done, and brings with it increased uncertainties in sensor positions and orientations if the boreholes are deep (Bulant et al., 2007).

The stations of near-surface arrays should preferably be installed in boreholes at least a few tens of meters deep that traverse superficial layers of unconsolidated material and penetrate bedrock. Experience in geothermal areas shows that such deployment tends to reduce noise from operations, weather, and seismic-wave reverberations in soft shallow layers.

Seismometers should be distributed uniformly over the upper focal hemisphere, the top half of a conceptual sphere surrounding an earthquake hypocenter, through which seismic waves pass on their way to stations. The accuracies of both hypocenter locations and moment tensors are enhanced if the focal sphere is well sampled by rays.

In real situations, crustal structure is not homogeneous. As a result, seismic wavefronts from earthquakes are distorted from ideal shapes. Seismic waves leaving the hypocenter and penetrating the upper focal hemisphere thus do not impinge on the surface of the earth in a regular, symmetric way. This must be taken into account in the placing of seismic stations if an optimal geometry is to be achieved. Figure 1 shows the upper focal hemisphere of an earthquake hypocenter mapped onto the earth’s surface. The distortion of the impinging wavefront was calculated by tracing rays through a 3D crustal model derived earlier using local-earthquake tomography. To achieve uniform coverage of the focal sphere, the stations of near-surface arrays should be sited with reference to calculations of seismic-ray distortion of this sort; this method has so far been applied to two geothermal areas (Miller et al., 1998; Julian et al., 2009).

Crustal models

In the case of well-designed networks, the dominant source of location error in geothermal areas is usually imperfectly known crustal structure (e.g., Maxwell, 2009). Whereas random errors associated with timing and arrival-time measurements may result in a few tens of meters of error in locations, the systematic errors that result from poor knowledge of crustal structure may be several times larger. There are several ways of reducing this problem.

Initial, 1D models are generally obtained using controlled-source seismology. To penetrate to several kilometers depth in refraction experiments, profiles several tens of kilometers long, and relatively large explosions are needed. Sophisticated experiments involving many profiles and multiple explosions are necessary to acquire spatially extensive, 3D crustal models from such work. In oil and gas reservoirs, sophisticated 3D seismic reflection surveys are sometimes done to obtain accurate crustal structure information, but this is rarely done in geothermal areas. In those environments,
usually, a few profiles only are shot and the result is a relatively simple, 1D model.

Such models may be improved using the local earthquakes themselves. One-dimensional inversions are conducted of arrival-times from shots (explosions for which the origin time is not measured), explosions (explosions for which the origin time is measured) and earthquakes, to calculate the 1D crustal model that minimizes the root-mean-square misfit of the arrival-time measurements (e.g., Kissling, 1995). Locating earthquakes using 1D models improved in this way may reduce location errors by a few percent.

If earthquakes are well-distributed beneath a seismometer network, it may be possible to use local-scale seismic tomography to determine a 3D crustal model. Seismic tomography has been successfully applied to several exploited volcanic and geothermal areas (Foulger and Toomey, 1989; Foulger and Arnott, 1993; Arnott and Foulger, 1994; Foulger et al., 1995a, 1995b; Julian et al., 1996; Ross et al., 1999).

In such areas, it is not uncommon for wave speeds to vary laterally on a scale of kilometers by more than 10%. The resulting traveltime anomalies may then be an order of magnitude larger than measurement errors and dominate the hypocentral-location error budget. Significant errors may also be introduced into moment-tensor solutions if this affect is not accounted for (Foulger and Julian, 1993). Tomographic inversions typically solve for corrected hypocenter locations, in addition to a 3D crustal model. Locating local earthquakes using such a model can greatly reduce these errors. In practice, only errors resulting from inhomogeneities on the scale of structural parameterization are reduced. This is typically 1–2 km at best.

A variety of physical processes can cause the large-scale structure in geothermal and hydrocarbon reservoirs to vary with time. These include changes in the stiffness of the rock matrix caused by drying or wetting of clay minerals (e.g., Boitnott and Boyd, 1996), changes in pore-fluid compressibility caused by CO₂ flooding (e.g., Wang et al., 1998; Daley et al., 2007) and fluid extraction (e.g., Gunasekera et al., 2003). Particularly strong changes in \( V_P/V_S \) occurred in The Geysers geothermal reservoir, California in the 1990s, as a result of production of steam and consequential drying of the geothermal reservoir. Changes in anisotropy are also commonly observed in hydrocarbon reservoirs (e.g., Al-Harrasi et al., 2011; Meersman et al., 2009).

Temporal changes in structure can be detected using conventional tomography methods to invert seismic-wave arrival-time data sets for different epochs independently. This approach is only reliable under ideal experimental conditions, however. This requires that the area is continuously seismically active and has been monitored by a dense, high-quality seismic network for a period spanning major operations. Most situations fall short of this ideal. In those cases, the assumption that differences in tomographic structure calculated independently from year to year correspond to real changes in structure is unsafe. The results of repeated tomography experiments differ even if the structure does not change, simply because of variation in the seismic ray distribution caused by natural variation in earthquake locations (Foulger et al., 1995a). These problems have been addressed by the recent development of a new tomography method, tomo-4d, which inverts multiple data sets simultaneously, imposing constraints to minimize the differences between the models for different epochs (Julian and Foulger, 2010).

Calibration explosions

Only one method can almost completely eliminate errors due to poorly known crustal structure in the case of regions of interest on the scale of kilometers. This is to use calibration explosions to determine exact traveltime corrections corresponding to the raypaths between the earthquake source volume and each seismic station. In the simplest case, a single explosion is fired in a borehole, near to the anticipated position of the hydraulic-fracturing induced earthquakes. This explosion is recorded on the seismic stations of the network that will monitor the earthquakes. An alternative, equivalent approach that avoids the necessity of firing a shot in the borehole is to deploy a sensor downhole and to fire timed explosions at each seismic station. This technique is currently in the early pioneer stage at geothermal areas monitored by surface seismometer arrays.

Absolute traveltimes for waves traveling between the anticipated seismogenic volume and the stations may then be measured. These may be compared with the traveltimes calculated using the best crustal model available. The differences between the two sets of traveltimes represent corrections that must be applied to earthquake arrival times, to correct for the imperfection of the crustal model.

Relative locations

Relative hypocenter location methods can greatly reduce the errors in location between individual earthquakes in a cluster, thereby improving the resolution of seismically active structures (e.g., De Meersman et al., 2009; Jansky et al., 2009). It is important to realize, that the absolute location of the entire cluster is not improved by this method, but only the errors in the differences between earthquake locations.

Relative location methods use, for each seismic station, the differences in arrival times of waves from closely spaced earthquakes. For such earthquakes, the biases caused by imperfectly modeled geological structure along the raypaths are almost identical. They thus nearly cancel out when arrival times are differenced, and the variations in arrival times remaining are largely a result of true, small variations in location between the earthquakes (Waldhauser and Ellsworth, 2000). The relative location method can be applied to arrival times obtained using any method, including automatic measurements, hand-measurements, and arrival times improved by waveform crosscorrelation. Waveform crosscorrelation is most helpful where waveforms are similar, and may be included automatically in the relative-location process. The results of relatively relocating events in a tight cluster can be a spectacular improvement in the clarity of delineated structures — a move from “dots in a box” to “faults in a box” (Julian et al., 2010b) (Figure 2).

Assessment of hypocentral uncertainty

Just as important as accurate hypocenter locations are reliable assessments of their accuracy, an aspect of earthquake seismology that is still an area of active research. By far the most common method of estimating hypocenter locations involves fitting the arrival times of seismic body-wave phases such as P and S using conventional least-squares methods (e.g., Aki and Richards, 1980, Box 12.3; Press et al., 2007, Chapter 15). Implicit in the use of least-squares fitting is the assumption that the errors in the data are normally distributed. For a linear (or, as in this case, linearized) inverse problem, the derived parameters are then (asymptotically, as the...
confidence level decreases to zero) normally distributed and their joint confidence regions are (asymptotically) hyperellipsoids. Furthermore, if the data errors are statistically independent and equal (or if their relative magnitudes are known), then their values can be estimated from the quality of the least-squares fit obtained.

It has long been evident, however, that the hypocentral confidence regions computed by most commonly used earthquake-location programs are unrealistic. In an early, but very careful, computer-based study of the subject Flinn (1965) obtained a mean depth of 39 ± 2 km for 12 nuclear explosions that had been fired just below the surface in Nevada. Numerous subsequent studies using increasingly large high-quality data sets and increasingly sophisticated analysis methods gave similar results (for example, Chang et al., 1983; Yang et al., 2004).

The cause of such absurdly optimistic confidence estimates lies in two frequently made but incorrect assumptions: that hypocentral errors are caused entirely by observational (seismogram-reading) errors, and that the errors for different observations are statistically independent. These assumptions confer significant computational advantages. The \( m \times m \) covariance matrix \( S \) of the observational errors (see below) becomes diagonal, with only \( m \) independent elements, where \( m \) is the number of observations, and inverting \( S \) becomes trivial. Because of the greatly enhanced computing power now commonly available, these computational advantages are, however, no longer as important as they once were.

In reality, for experiments on the scale of kilometers, variations in travel times caused by the imperfectly known structure of the earth usually are much larger than observational errors. More importantly, these structure-related errors are strongly correlated: raypaths from an event to seismometers near one another are close together, and their associated travel times are affected similarly by heterogeneities in the seismic wave speeds. In such cases, hypocenter-location algorithms can commonly reduce the arrival-time residuals by “mislocating” the earthquake. For example, if wave speeds are generally higher to the east than to the west of an earthquake, the arriving waves will be earlier to the east than at the same distances to the west, and they will “pull” the computed location to the east. Such mislocation produces a fit to arrival-time data that is deceptively good, and leads to unrealistically small computed confidence regions.

An occasionally used and simple, but approximate, solution to this problem is to use a priori standard errors for data when estimating hypocenters, and to make these values large enough to account for both the reading errors and traveltime uncertainties (Julian, 1973). The covariance matrix is still diagonal, so negligible extra computational effort is required. This tactic is not ideal, however. It enlarges the sizes of the computed confidence regions, but does not change their shapes, as it should to account for the effect of traveltime correlation. We outline an improved strategy below.

In addition to distorting the sizes and shapes of computed confidence regions, the spatial correlation of traveltime anomalies also biases computed hypocenters. This effect results from the inevitably nonuniform distribution of observable seismic rays around hypocenters. The rays oversample the wave-speed anomalies in some directions and these rays therefore have a disproportionate effect and introduce bias into computed hypocenters.

To avoid this difficulty, we propose incorporating into hypocenter-location methods a stochastic model of traveltime anomalies caused by earth heterogeneity. This tactic can greatly improve the quality both of hypocentral estimates, and of computed hypocentral confidence regions.

**Method**

Arrange the four origin coordinates (three spatial coordinates and origin time) of an earthquake in a column vector \( x \), and similarly arrange the \( m \) observed arrival-time residuals (observed minus predicted times) with respect to an assumed origin in a column vector \( b \). The functional relation between the origin coordinates and the predicted arrival times is nonlinear, and the standard solution method is to iteratively solve a linearized problem in which small changes in the origin \( \delta x \) are related to predicted changes \( \delta b \) in the residuals by a linear operator (the first term of a Taylor-series expansion). A good starting guess is necessary. This operator takes the form of an \( m \times 4 \) matrix \( A \), such that first-order changes in the residual vector are given by the “design equation”

\[
\delta b = -A \delta x. \tag{1}
\]

In least-squares fitting, we seek to minimize the quantity

\[
\chi^2 = (A \delta x - b)^T S^{-1} (A \delta x - b), \tag{2}
\]

where the superscript \( T \) indicates vector/matrix transposition and \( S = <bb^T> \) is the symmetric \( m \times m \) covariance matrix of the observational errors. The bracket symbols \( <> \) indicate the mathematical expectation of a random variable.

Most existing hypocenter-location programs assume that the errors in the observed arrival times are statistically independent, so that the covariance matrix \( S \) is diagonal, with elements equal to the variances of the \( m \) observations:

\[
S_{ij} = \sigma_i^2 \delta_{ij} \text{ (no summation over } i). \tag{3}
\]

In this case, the effect of the covariance matrix \( S \) in equation 2 is to divide each design equation by the corresponding standard error, \( \sigma_i \), which is equivalent to weighting each design equation by

![Figure 2. Left: Conventionally determined individual earthquake locations (dots in a box); right: relative relocations (faults in a box). But do we know the absolute location of the new fault well enough to drill through it? The green box is 4 x 4 x 4 km in size, and the white cross is 2 x 2 x 2 km from side to side.](image)
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\( \langle \delta t(t_1)\delta t(t_2) \rangle \approx \int_0^P \int_0^Q R(r^{(1)}(p), r^{(2)}(q)) dp dq \). \hspace{1cm} (8)

The correlation function \( R(a, b) \) contains all the statistical information needed to compute the contribution of traveltime uncertainties to the covariance matrix \( S \). Computing this double integral using complete raypaths for every pair of observations would be laborious, even if all the required information, such as the function \( R(a, b) \), were available. To simplify the problem, we approximate the raypaths as straight-line segments and also assume that the correlation function depends only upon the vector difference of its arguments (is spatially stationary), and further, that it depends only on the length of the difference vector (which is spatially isotropic)

\[ R(a, b) = R(r), \quad \text{where} \quad r = |a - b|. \hspace{1cm} (9) \]

We assume a mathematically tractable exponential analytic form for \( R \),

\[ R(r) = \sigma^2 e^{-r/a}, \hspace{1cm} (10) \]

where \( \sigma \) is the standard deviation of the slowness and \( a \) is its “correlation distance,” which may be thought of as the typical size of the slowness heterogeneities. Actually, this assumption involves a significant oversimplification. In reality, the correlation distance usually is greater in horizontal directions than in the vertical direction, especially in sedimentary basins. We only intend to outline a general approach here, not a fully developed theory.

Even with these simplifying assumptions, it is not possible to express the covariance of the traveltimes in closed form as a function of the angle \( \alpha \) between the rays and their length \( L \), which we take as the same for both rays in a pair. We can, however, determine the approximate shape of this function. If the angle \( \alpha = 0 \), then the rays are coincident and the variance of the traveltime is

\[ \langle (\delta t)^2 \rangle = 2\sigma^2 \{ e^{-L/a} + L/a - 1 \}. \hspace{1cm} (11) \]

For short paths (\( L/a \ll 1 \) this function is quadratic, with \( \langle (\delta t)^2 \rangle \approx \sigma^2 L^2 \), and for long paths (\( L/a \gg 1 \) it is linear, with \( \langle (\delta t)^2 \rangle \approx 2\sigma^2 aL \). For rays departing in opposite directions (\( \alpha = \pi \)) the covariance \( \langle \delta t_1 \delta t_2 \rangle = \sigma^2 a^2 \{ 1 - e^{-L/a} \}^2 \). Figure 3 shows the covariance function for these cases. For infinitely long paths, the covariance is

\[ \langle \delta t_1 \delta t_2 \rangle = \frac{\sigma^2 a^2}{\sin \alpha} \int_0^\infty \int_0^\infty e^{-\sqrt{x^2 + y^2 - 2xy \cos \alpha}} dx dy \]. \hspace{1cm} (12) \]

(Figure 4). This function increases without limit as \( \alpha \to 0 \), but for finite path lengths it must be bounded by the values given by equation 12, indicated by dotted lines on Figure 4. The use of a full covariance matrix, even one based on an ad hoc combination of the curves and dotted lines of Figure 4, will yield major improvements in both hypocentral accuracy and in the accuracy of computed hypocentral confidence regions. Improving our knowledge about the actual statistics of structural fluctuations in the earth will yield even greater improvements.

The quantity \( 1/\sigma^2 \). Frequently, programs constrain the relative values of the standard errors \( \sigma_i \), and use the best-fit value of \( \chi^2 \) to estimate their overall level, a procedure that would be justified if the errors were statistically independent. However, this leads to hypocentral bias and misleading confidence estimates when they are not, for reasons explained above. A possible solution to this problem is to recognize that the error in each observed arrival time \( \delta t \) has two components: a true measurement error \( \delta t_m \), and a traveltime anomaly \( \delta t_a \)

\[ \delta t = \delta t_m + \delta t_a. \hspace{1cm} (4) \]

This approach leads to a nondiagonal covariance matrix in the least-squares fitting problem, and inverting such a matrix requires \( O(m^3) \) operations, which even today may be prohibitive for problems with many hundreds of observations. Many current industrial applications, however, involve tens of observations, so inverting the full covariance matrix in these cases is practical.

Rodi and Meyers (2008) recently undertook such an approach, estimating the statistics of \( \delta t_a \) for long paths (up to 3000 km) from models of earth’s crust and upper mantle. We propose here an approach that may be more appropriate for the short paths relevant to industrial applications, treating the wave speeds as spatially constant with additive stochastic perturbations.

A stochastic model of traveltime anomalies

The traveltime along a ray is

\[ t = \int_0^L s(r(l)) dl, \hspace{1cm} (5) \]

where \( s(\xi) \) is the wave slowness (inverse of wave speed) as a function of position \( \xi \) and \( r(l) \) is the parametric representation of the raypath, with the parameter \( l \) being path length and ranging from zero to \( L \) along the ray.

Now let the slowness field \( s(\xi) \) consist of two components: a deterministic component \( s_0(\xi) \), upon which the earthquake location computations are based, and a random perturbation \( \delta s(\xi) \), about which only statistical information is available: \( s(\xi) = s_0(\xi) + \delta s(\xi) \). The perturbation \( \delta s \) will cause a first-order change in the raypath, but to a first-order the change in the traveltime can be computed using the unperturbed raypath

\[ \delta t_a \approx \int_0^L \delta s(r(l)) dl \hspace{1cm} (6) \]

(Julian and Anderson, 1968). The covariance of these traveltime perturbations along two different raypaths \( r^{(1)} \) and \( r^{(2)} \), of lengths \( P \) and \( Q \), can be expressed as

\[ \langle \delta t^{(1)} \delta t^{(2)} \rangle \approx \int_0^P \int_0^Q R(r^{(1)}(p), r^{(2)}(q)) dp dq \]

(7)

Introducing the correlation function of the slowness perturbations \( R(a, b) = \langle \delta s(a) \delta s(b) \rangle \), we get

(8)
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The sizes of earthquakes were originally expressed using magnitude scales defined in terms of seismogram characteristics, not source characteristics. This is the basis for scales such as the local (or "Richter") magnitude scale, for example. They are poorly suited for industrial work for a number of reasons. They result in the magnitude of an earthquake varying from seismogram to seismogram, and the seismic phase measured is sometimes not defined, so it is impossible, even in principle, to improve measurements of $M_L$ as knowledge improves about the earth and about seismic-wave propagation. Empirical magnitude scales for local networks are often constructed by correlating measured wave amplitudes with magnitudes from existing regional stations. This requires downward extrapolation over several magnitude units to apply them to micro-earthquakes, potentially resulting in large systematic variations between different scales.

The most accurate measure of earthquake size, and the one that should be used, is the low-frequency scalar seismic moment, $M_0$. Seismic moment is a measure of earthquake size that is based on fundamental physics of the source, and not simply seismogram characteristics. Because $M_0$ varies by more than a factor of $10^{18}$, it is conventional to define a logarithmic moment-magnitude

$$M_W = \frac{2}{3} \log M_0 - 10.7,$$

where $M_0$ is measured in Newton-meters (Kanamori, 1977; Hanks and Kanamori, 1979). For simple shear faulting, $M_0 = \mu \bar{a} a$, where $\mu$ is the rigidity modulus at the hypocenter, $a$ is the fault area, and $\bar{a}$ is the average displacement on the fault. For more general source types, seismic moment is regarded as a symmetric tensor with elements $M_{ij}$. The scalar moment is $M = \sqrt{\frac{2}{3} \sum_{i,j=1}^{3} M_{ij}^2}$ (Silver and Jordan, 1982), and $M_0$ is the low-frequency limit of $M$. The theory of the excitation of elastic waves by moment-tensor sources is well developed (e.g., Aki and Richards, 2002, equations 4.29, 4.97, 7.148–7.150), so determining moment magnitude depends only on our knowledge of the propagation medium and our ability to solve the wave equation. Moment magnitude thus anticipates, rather than precludes, exploitation of advances in knowledge.

SOURCE MECHANISMS

Seismic failure can involve several physical processes that are important in hydrocarbon or geothermal reservoirs. These include (1) simple shear slip on planar faults, (2) tensile cracking, (3) simultaneous shear slip and/or tensile cracking on multiple faults, and (4) rapid fluid motion. To these might be added artificial sources such as (5) explosions, which can additionally trigger all of the above processes. Traditional seismological "fault-plane solutions," based on the "double-couple" equivalent force system, describe only the first of these processes, and then only under simplifying assumptions about rock homogeneity and isotropy. The more general moment-tensor source model of seismic source mechanisms can represent any combination of processes (1), (2), and (3), and offers in addition the computational advantage of a linear theory for seismic-wave excitation.

Fault-plane solutions usually are derived from the polarities of first-arriving compressional waves ($P$ phases). If these polarities are projected back along seismic rays to an infinitesimal "focal
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spheres” surrounding the hypocenter, the simple-shear-faulting (double couple) model predicts that compressional and dilatational polarities (outward and inward motions) will occupy alternating quadrants separated by a pair of orthogonal great circles. One of these defines the fault-plane. The only property a fault-plane solution has, therefore, is its orientation. Furthermore, the fault-plane solution does not identify which great circle represents the fault and which the “auxiliary plane.”

Obtaining accurate fault-plane solutions from P-phase polarities requires dense sampling of the focal sphere. In the case of surface arrays, examination of real data suggest that networks of 20 to 30 well-distributed seismometers are sufficient (Miller et al., 1998). When, as is commonly the case in routine seismic monitoring of geothermal areas, fewer stations are used (sometimes fewer than 10), errors in the orientation of the derived fault plane, and in the direction of slip, may be several tens of degrees.

In the 1980s, some earthquakes in volcanic and geothermal areas were found to have non-double-couple source mechanisms (Julian, 1983; Foulger and Long, 1984; Foulger et al., 1989). These required volume changes caused by the opening or closing of cracks or other cavities, and more general kinds of shear deformation, which might indicate tensile cracking or simultaneous shear slip on multiple faults. The first such discoveries involved events with large non-double-couple components and dense seismometer networks. As high-quality seismometer networks in such environments became more numerous, however, reports of non-double-couple source mechanisms became more common and various methods were developed for inverting observed data to retrieve them (e.g., Šílený et al., 1992). Seismic studies of mines also detected non-double-couple earthquake mechanisms (e.g., Feignier and Young, 1992; McGarr, 1992). Studying such sources can yield information of potential operational value, distinguishing, for example, shear-faulting from crack opening and closure. For earthquakes induced by EGS-related hydraulic fracturing, moment-tensor mechanisms can reveal the nature of the fractures created or reactivated.

Non-double-couple source mechanisms are difficult to identify using P-phase polarities alone, however. Even with dense seismometer networks, such data usually are highly ambiguous (Julian et al., 1998). Additional information is needed, which can include either full waveforms (e.g., Dreger and Helmberger, 1993) or, more simply, the polarities and amplitudes of P- and S-phases (Julian and Foulger, 1996). The polarities of horizontally polarized shear waves (SH) are fairly easy to determine because S H and P/S V waves are only weakly coupled (under idealized conditions they are uncoupled), but for instruments distributed on the surface S H polarities provide much more information than P polarities do.

The shapes alone of even a small number of seismic waveforms may contain enough information to resolve the complete moment tensor of a seismic event. Existing waveform-inversion methods (Dreger and Helmberger, 1993), however, are applicable only to comparatively large earthquakes recordable at regional distances. For small earthquakes such as those induced by hydraulic fracturing, a practical method is inversion of P- and S-wave polarities and amplitude ratios (Julian and Foulger, 1996). Like waveforms, amplitude ratios can be less severely distorted by wave-propagation anomalies than are amplitudes themselves because effects such as anelastic attenuation and geometric spreading behave similarly for appropriately chosen paths, reducing their distorting effect upon the corresponding ratios. Using this method, good results may be obtained with networks containing 10–15 3C seismometers. The separate channels must have well matched (or at least well known) gains and known orientations. If borehole sensors are used, they must meet these requirements.

Displaying moment tensors graphically, including both source types and orientations, requires complicated plots that are difficult to understand. Therefore, separate plots are usually used, one displaying source-type information only (Hudson et al., 1989) (Figure 5 — top), and the other displaying only orientation information (Figure 5 — bottom).

Moment tensor analysis has now been used to study the source mechanisms of many natural and industrially induced earthquakes. These include several geothermal and hydrocarbon reservoirs areas and EGS stimulation experiments (Julian and Foulger, 1996; Ross et al., 1996; Julian et al., 1997; Miller et al., 1998; Šílený et al., 2009; Baig and Urbancic, 2010; Julian et al., 2010b).

Interpreting moment tensors and hypocenter locations jointly

Interpreting moment tensors in terms of physical processes is, unfortunately, nonunique. Different source processes can produce identical seismic wave fields, and thus have identical moment tensors (Julian et al., 1998). For example, the deviatoric part of any moment tensor can be decomposed into a pair of double-couple (shear-faulting) sources, or alternatively, into a combination of a (different) double couple and a CLVD. Even worse, neither type of decomposition is unique. More information than the moment tensor provides is thus needed to reduce this ambiguity.

One useful approach is to interpret moment tensors along with structural information obtained from earthquake locations. Relatively located hypocenters can provide high-resolution images of the spatial distributions of earthquakes, revealing the geometry of seismic failure regions.

We have applied this method to two cases so far. In 1997, a swarm of earthquakes in the south moat of Long Valley caldera, west of Casa Diablo Hot Springs was recorded by a network of 69 digital, 3C seismic stations (Foulger et al., 2004). Well-constrained moment tensors obtained for several of these earthquakes have mechanisms that are close to double couples, but contain a net volume increase. High-resolution relative hypocenter locations for the entire sequence of several hundred earthquakes clearly define a failure plane that bisects the dilatational polarity fields of the earthquake radiation patterns. This fact rules out shear faulting and requires that the primary failure mechanism is tensile cracking, probably caused by hydraulic fracturing (Foulger et al., 2004).

In the second case, a hydraulic injection in the Coso geothermal area in 2005 induced a swarm of several tens of microearthquakes. Relative locations determined using 27 surface and shallow-borehole 3C seismometers delineate a clear fault plane, 600 m in length, striking N 20°E, and dipping at 75° to the west-northwest. Surface geology and borehole televiwer observations show that this plane coincides with a preexisting fault. The earthquakes had similar non-double-couple mechanisms involving volume increases, and the fault plane bisects the dilatational P-phase polarity fields (Julian et al., 2010a) (Figure 6). The additional constraining information from the orientation of the fault plane shows that the source process was dominated by tensile failure.
Confidence bounds for moment tensors

Source mechanisms such as those shown in Figure 5 and Figure 6 are of little use without additional information about their probable uncertainties. Inevitable errors in measuring wave polarities and amplitudes are mapped in a complicated manner into errors in the derived moment tensors. If these errors are strongly correlated, they might produce systematic artifacts that could be mistaken for real geophysical phenomena. To date, only a few moment-tensor analyses have included any kind of error analysis (Baker and Young, 1997; Trifu et al., 2000; Šilený et al., 2009; Baig and Urbancic, 2010).

We therefore extended the linear-programming method of Julian and Foulger (1994) to compute confidence regions for moment tensors. After finding the minimum value of an objective function that measures the L1 norm of the residuals between the observed and computed polarities and amplitude ratios, we constrain this objective function to lie below a somewhat larger value chosen on the basis of a priori estimates of uncertainty caused by measurement errors and earth-model uncertainty. We then move the solution in 6D moment-tensor space in various specified directions as far as the constraint allows. In this way, we obtain a suite of solutions that fit the data adequately (Figure 7).

Results from applying this method to amplitude-ratio data from several geothermal areas in Iceland, Indonesia, and California show that moment-tensor confidence regions often, but not always, are elongated along a trend between the +Dipole and −Dipole points on the source-type plot. This observation suggests that part of the systematic trend that is frequently found for geothermal earthquakes may be an artifact of measurement error. For earthquakes that comprise a combination of shear slip and tensile crack opening, a distribution along a trend between the +Crack and −Crack points is predicted theoretically (Julian et al., 1998). Further work is required to fully understand why the moment tensors of natural earthquakes do not follow this distribution also. Importantly, error assessments need to be incorporated into routine moment-tensor results, both to give more information on the reliability of the results, and to build a set of case histories that may be used to increase our understanding of the response of reservoirs to hydraulic fracturing.

This same analysis shows that volume changes are well resolved by polarity/amplitude-ratio data. The observation that the source mechanisms involve systematically smaller volume changes than are expected for combined shear-plus-tensile faulting, is thus apparently real — the trend between the +Dipole and −Dipole points on the source-type plot remains, even after error analysis. A volume-compensating process, such as rapid fluid flow into opening cracks, must thus be at work. Such unsteady fluid flow would contribute net forces to source mechanisms, which might be detectable by inversion methods that include lower-order errors and earth-model uncertainty. We then move the solution in

Figure 5. Moment tensor results for 37 earthquakes that were induced by a hydraulic injection in a borehole in the Coso geothermal field in 2005. Top: “Source-type” plots depict the moment tensor in a form that is independent of source orientation. All simple shear-faulting mechanisms, whether strike-slip, normal, or reverse, plot at the central point labeled DC (double couple). The vertical coordinate \( P \) characterizes the type of shear deformation, and ranges from \( -1 \) at the bottom of the plot to \( +1 \) at the top. Mechanisms with volume increases lie above the horizontal line through the central point DC, and mechanisms with volume decreases lie below it. Pure (spherically symmetric) explosions lie at point \( +V \) and pure implosions lie at \( -V \). The left-right coordinate \( T \) characterizes the type of shear deformation, and ranges from \( -1 \) on the left to \( +1 \) on the right side of the plot. Simple shears (\( T = 0 \)) lie on the vertical line through the central point DC, and more complex pure shears lie to the right or left of this line. In particular, opening (closing) tensile cracks, which involve both shear and volumetric deformation, lie at the point \( +Crack \) (−Crack). The points \( ±CLVD \) and \( ±Dipole \) represent mathematically idealized force systems that could be caused by the opening or closing of cracks accompanied by volume-compensating fluid flow. Bottom: Pressure (\( P \)) and tension (\( T \)) axes for the pre-, co-, and post-swarm earthquakes, plotted on upper-hemisphere stereographic projections. The \( P \) and \( T \) axes give a rough indication of the orientation of the greatest and least principal stresses, respectively.
moments (Backus and Mulcahy, 1976a, 1976b; Julian et al., 1998).
Net forces may be significant components of earthquake mechanisms in geothermal and hydrocarbon reservoirs, and including them in source studies may contribute to understanding the volumetric components of geothermal and other reservoir-related earthquakes (Foulger et al., 2004).

DISCUSSION

Earthquake seismology has been used for a century to monitor regional earthquakes for academic and public hazard-reduction purposes. However, high accuracies and rigorous error estimates in hypocenter locations, magnitudes, and source mechanisms have tended to be neglected because they do not seriously influence decision-making for these applications. This is not the case for small-scale industrial applications, that include the mining industry and hydraulic stimulations, that have become a growing sector of earthquake seismology over the last several decades. In those cases, both high accuracies and correct error estimates are required. To achieve such accuracies, specialized experiment design is necessary — approaches traditionally used for regional monitoring do not suffice.

Many modern geothermal industrial applications require hypocenter-location accuracies of meters or tens of meters. To achieve such accuracies, several design features must be incorporated into experiments, in particular in the case of near-surface seismometer arrays. Network geometry must take into account the crustal model, since this governs the mapping of seismic station locations onto the focal sphere. Sensors should preferably be deployed in boreholes, below poorly consolidated surface layers, to improve the signal-to-noise ratio. A crustal model as accurate as possible should be obtained, preferably a 3D model. Nevertheless, despite these measures, to reduce location errors to the level of a few meters or tens of meters, calibration explosions are required to obtain traveltime corrections corresponding to the specific source-station raypaths applicable to the planned experiment. Relative location methods can improve the accuracy of earthquake locations with respect to one another, thus improving the clarity of seismically active structures, but they do not improve the absolute location of the structure as a whole.

Rigorous uncertainty estimates are also needed. In the case of regional earthquake monitoring, little attention has traditionally been paid to the qualities of hypocenter locations because significant decisions have not depended on high accuracies. The most commonly used earthquake location programs assume that uncertainties are caused entirely by errors in the P- and S-wave arrival-time measurements, that these errors are statistically independent, and that simple arrival-time residuals give meaningful estimates of them. In fact, unknowns in crustal structure dominate the error budget, and must be accounted for in calculating hypocentral confidence regions. A hypocenter-location method based on a simple stochastic model of crustal heterogeneity, described in this paper, is applicable to industrial scenarios involving small seismometer networks and short raypaths.

The magnitudes of earthquakes are important in most industrial applications. Conventional magnitudes do not meet industrial standards. Moment magnitude should be used because it is related to fundamental physical processes at earthquake sources — it is not defined in terms of seismograms properties that ignore factors such as source and path effects.
In the case of source mechanisms, fault-plane solutions have been traditionally used for regional earthquake monitoring. This approach assumes shear slip on a planar fault surface and does not allow nonshear source components such as the opening and closing cracks, to be determined. To derive possible nonshear components, source mechanism studies must invert high-quality data for full moment tensors. The seismic networks that are suitable for such analyses are the same as those best suited for accurate locations. They comprise dense arrays of calibrated, 3C sensors, well-distributed over the focal sphere. Moment tensors are more complex to display graphically than simple fault-plane solutions. An effective approach is to use both a source-type plot, which displays the orientation-free information, along with a plot of P- and T-axes, which displays stress orientation information.

Interpreting moment tensors in terms of physical processes is fundamentally ambiguous. This ambiguity can be reduced by additional types of information. A promising approach is to infer structural information from relative hypocenter locations within earthquake sequences. These can provide independent constraints on the sizes, shapes, and orientations of causative structures. These, in turn, can be used to reduce the ambiguity in the sense of motion deduced from the moment tensors.

Realistic assessment of probable errors in derived moment-tensor source mechanisms is important to distinguishing spurious systematic trends from real physical source phenomena. A method involving calculating extremal solutions that fit the data within a priori estimates of measurement error is an effective way forward.

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Microseismic design studies

Ulrich Zimmer

ABSTRACT

Microseismic monitoring has become an important part of borehole completions in tight-reservoir formations. Usually, clear objectives for a microseismic survey are set prior to the data acquisition. The possibility of meeting these objectives is determined by the acquisition geometry, the target formation, the completion schedule, and only to a lesser extent, by the data quality itself. Provided is a tutorial on the content and use of pre-job modeling and design studies as a tool to anticipate viewing distances, data quantity, location accuracy, event magnitudes, achievable mapping distances, expected waveforms, and noise levels. In addition, potential challenges in meeting the survey objectives can be identified and solutions to these challenges can be devised prior to the survey. For downhole surveys, this involves the evaluation of different sensor array geometries and their impact on the location accuracy in different parts of the expected model. The sensitivity of the event location on the velocity model can be estimated using an initial log-based model. Recently, the detailed characterization of the event mechanism in form of a moment tensor inversion has received increased attention. The accuracy of the inverted moment tensor depends largely on the coverage of the focal sphere, i.e., the distribution of the sensors around the event location. Based on the sensor positions, areas with high- and low-quality moment tensor inversion results can be identified prior to data acquisition through the distribution of the condition number. Depending on the survey objectives and the given constraints, the microseismic design study might show that the survey objectives cannot be met. In this case, it is possible to evaluate alternate technologies, e.g., distributed temperature sensing (DTS), ahead of the project for their potential to meet these challenges.

INTRODUCTION

The potential of a microseismic survey to meet the objectives is partly independent from the data quality itself. Acquisition geometry, velocity structure, target formation, and treatment parameters combined with some assumptions based on experience from similar settings can be used to optimize the acquisition and to create realistic expectations on the outcome of the survey. As the understanding of microseismic activity increases, the expectations on the microseismic results have grown significantly. In the last few years, modeling and design studies have been used to check if a given acquisition geometry can meet the survey objectives. In cases where the available acquisition geometry and processing flow cannot meet the objectives, the expectations on the survey have to be adjusted (Greehka, 2010; Zimmer, 2010a). Typical objectives for a microseismic survey monitoring a hydraulic fracturing treatment can include the determination of the geometry and development of the fracture network, proof of successful isolation in staged treatments, monitoring casing integrity, determining stage overlap, source characterization of the events, etc.

The question of the maximum observational distance of the microseismic events is discussed before any project. But these estimates were often very informally based on previous experiences in approximately the same target information and with similar treatment parameters, i.e., pressure, flow rate, and proppant concentration. Only recently has this approach been more formalized, often using a magnitude versus distance plot (Zimmer, 2010a) or by trying to estimate the number of expected events for a given observational distance (Maxwell, 2009). Both approaches require assumptions based on analogues from previous surveys, but these assumptions are much more clearly stated than in the previous informal approach.

The individual accuracy of the event locations for downhole receiver arrays in a single observation well has been discussed in
numerous papers. Warpinski (2009) provides a general overview of the acquisition, processing, and interpretation of microseismic events, including the calibration of the velocity model and a discussion of the location uncertainties. Fuller et al. (2010) and Monk et al. (2011) comment on the low-depth accuracy achieved when relying on critically refracted waves for the event localization. In such cases, the location accuracy can be significantly improved by combining the use of first arrivals and direct waves (Zimmer, 2011). Jones et al. (2010) show how the particle motion can be used to resolve ambiguities in the event locations. Eisner et al., (2009, 2010) use mainly synthetic examples to compare the theoretical uncertainties of data recorded by sensor arrays installed in observation wells and on surface. Using data from the German KTB deep drilling project, Gajewski et al. (2009) investigate the impact of different model simplifications, e.g., neglecting anisotropy or heterogeneity, on the event location. The calibration of the velocity model can be improved by using absolute traveltimes, i.e., “perforation timing,” as shown in Warpinski et al. (2005). Pei et al. (2009) describe a fast process for velocity model calibration under multiple constraints using the simulated annealing algorithm. Some easy ways to identify location uncertainties in the event distribution of microseismic maps and how to avoid misinterpretations are provided in Zimmer et al. (2009).

Microseismic monitoring is often used on tight shale formations that might show an anisotropic velocity distribution. In addition, the fracture network created during hydraulic injections also can create oriented fracture sets adding another degree of anisotropy. Verdon and Kendall (2011) show how to detect the induced anisotropy in the microseismic data where Wuestefeld et al. (2010) provide an automated workflow to analyze the degree of anisotropy in large microseismic data sets.

Moment tensor inversions

If the acquisition geometry is favorable, the failure mechanism creating the microseismic events can be described by calculating the event’s moment tensor (Trifu, 2002; Eisner et al., 2010; Baig and Urbanic, 2010; Warpinski and Du, 2010). The moment tensor has been used to characterize microseismic events in long term monitoring projects of mines (e.g., Trifu and Shumila, 2001, Šílený and Milev, 2008) and geothermal fields (e.g., Miller et al., 1998).

However, it is important to note that the stress changes creating these events can be significantly different from the type of stress changes induced by hydraulic fracturing injections. Geothermal injections are almost always done below the frac pressure, trying to induce shear slippages rather than cracking the rock open. In mining applications, stress relaxation around underground openings is often the cause of microseismic activity. For hydraulic injections above the frac pressure, the mechanism of creating microseismic events is likely to be very different requiring a different interpretation. Potentially, the moment tensor can be used to determine differences in the stress field of the mapped area. Calculating a stable moment tensor depends largely on the acquisition geometry and the underlying velocity structure. The spatial distribution of the condition number summarizes these factors. For areas with large (i.e., >20) condition numbers, no stable moment tensor can be calculated even if the signal quality is excellent (Petitt, 1998). Areas with low condition numbers indicate favorable acquisition geometry. But the quality of the inverted moment tensor also depends on the data quality of the individual event. Poor data quality or weak signals can produce unreliable moment tensor solutions with high condition numbers even in areas where the acquisition geometry is favorable.

Although microseismic monitoring provides some important information about the development of fracture networks, some questions can only be answered by combining the microseismic results with other technologies. Surface and downhole tiltmeters have been successfully employed to monitor hydraulic fracture treatments as standalone applications and as an integrated part of joint tiltmeter and microseismic surveys (e.g., Warpinski et al., 2006). Distributed temperature sensing (DTS) on fiber-optic cables has proven as a technology that can track fluid movements close to a wellbore with high resolution (Holley et al., 2010) showing where the fluid enters the formation in hydraulic fracture stimulations or during other injections. This information about the near wellbore area complements the more regional description of the microseismic event distribution.

Detection limit and mapping distance

The ability of a downhole array of microseismic sensors with a given sensitivity to detect a microseismic event depends largely on the magnitude of the event, the ambient noise, and the observational distance. Many microseismic events contain a large shear slippage component, creating a nonuniform radiation pattern. Hence, the event’s slippage mechanism can also influence detection of an event and the ability to locate the event. An effective way to visualize the relations between these factors is the magnitude versus distance plot (Figure 1).

The minimum detection limit depends on the specific detection algorithm and defines the lowest magnitude event that can be detected at a given observational distance. The magnitude number used here is a scaled $S_H$ amplitude that accounts for losses due to geometric spreading but does not include effects of elastic attenuation or radiation pattern. Currently, the displayed minimum detection limit uses a minimum horizontal shear-wave amplitude ($S_H$) of 40% above the ambient noise level (N) at the sensor, which

![Figure 1](image.png)
has been empirically shown in numerous projects to be an adequate estimate for downhole receiver arrays and the applied processing flow. Different acquisition geometries and processing flows might need to apply a different threshold for the minimum detection limit. The amplitude is corrected for the geometric spreading associated with the observational distance \((d)\).

\[
M = 0.65 \times \log_{10}\left( S_b \left( \frac{\mu m}{s} \right) \times d(m) \right) - 4.89 \quad (1)
\]

\[
\Rightarrow M_{\text{min}} = 0.65 \times \log_{10}\left( 1.4 \times N \left( \frac{\mu m}{s} \right) \times d(m) \right) - 4.89 \quad (2)
\]

The minimum detection limit for a constant amplitude follows a logarithmic line in this plot. Any events above this line are large enough and close enough to the receivers to be detected. Any events below this line are too small or too far away from the receivers to be detected since the signal amplitude has degenerated below the 1.4 signal-to-noise ratio (S/N) value due to geometric spreading. Elastic attenuation of the rock further increases this effect. The distant dependent detection of events creates an observational bias where at large observational distances only a few large events are detected. Without additional filtering of the small events, a symmetric fracture network can appear asymmetric in the microseismic event distribution. This observational bias can be reduced by using multiple observation wells.

For a given formation and given treatment parameters, i.e., pump rate, pressure, proppant concentration, the largest magnitude of the created events can be estimated from previous experiences in the same field. Almost always there will be a few events that have a significantly larger magnitude than most of the events. By choosing a magnitude number that is representative for most of the events, the maximum viewing distance, i.e., the distance where the minimum detection limit reaches this number, can be estimated. Since the interpretation of microseismic maps requires the detection of more than just a few events, the maximum mapping distance is significantly less than the maximum viewing distance. There is no exact value for the maximum mapping distance since this limit depends also on the individual survey objectives. Estimating the limiting magnitude numbers in new fields is based on analogies from other fields. But this estimate can be replaced by a much more reliable number once at least one project has been mapped in that particular formation.

The minimum detectable amplitude also depends on the ambient noise level. Increasing the ambient noise level will shift the minimum detection limit upward, effectively reducing the maximum viewing (and mapping) distance. Sources for increased noise levels are bad cement bond at the receiver positions, pump trucks directly above the receivers, leaking bridge plugs in the observation well, etc. The S/N can be improved by grouping several downhole receivers close together and stacking the signal (e.g., Shemeta et al., 2009). Since the number of downhole receivers is limited, this stacking rarely exceeds 3–5 receivers per level. Stacking only reduces the randomly distributed part of the ambient noise.

The effect of the radiation pattern is not included in Figure 1. Assuming that the radiation pattern for a cluster of events is sufficiently random, the overall radiation pattern effect of that cluster will become negligible. In areas with a very pronounced fracture direction this assumption might not be valid, which can lead to the preferred recording of S-wave only or P-wave only signals depending on the relative position of the receivers to the event location. If a location algorithm is used that requires the detection of P- and S-waves, the events would be detected but not necessarily located on the map.

**LOCATION ACCURACY**

To evaluate different acquisition geometries it is necessary to quantify the maximum achievable location accuracy in different parts of the model. For a grid search algorithm, each grid point is assigned a value that measures the difference between the real data parameters, e.g., P- and S-wave arrival times, and the theoretical parameters, e.g., theoretical arrival times, associated with the individual grid point. The grid point with the smallest value is considered to be the most likely location. In this case, this traveltime residual value is defined as

\[
t_{\text{res}} = \sqrt{\frac{\sum_{2n} \left( t_i - t_{i,\text{calc}} \right)^2}{2n - 3}}
\]

This number depends on the number of sensors \((n)\), the measured arrival time \((t_i)\), and the theoretical arrival times \((t_{i,\text{calc}})\). Equation 3 is only one of numerous options on how to define a “fit” between the observed and the calculated arrival times. The term \((t_i - t_{i,\text{calc}})\) would provide this fit already if absolute traveltimes were known. But since the origin time of the microseismic event is unknown before the event localization, there will be a constant shift between the identified and calculated arrival time. This shift is the same for all sensors and phases and can be eliminated by removing the average of the difference, i.e., second term. The term “2\(n\)” denotes the number of picked phases. If not all the P- and S-wave arrival pairs are picked on each of the \(n\) sensors, the term \(2n\) needs to be replaced by the number of phases (P- or S-wave) picked. For the modeling studies, it is assumed that the P- and S-wave arrival times are identified at every available receiver. For real data, this is only true for strong events with a large S/N. Almost all uncertainty estimations assume that the underlying velocity model and the given downhole receiver positions are correct. For real data, both parameters have an individual level of uncertainty that is hard to quantify and, therefore, often neglected.

Figure 2 shows the distribution of the traveltime residuals for three test locations in the vicinity of a high velocity layer and at different distances from the receiver array. In the top figure, only the first arriving P- and S-waves are used to locate the event, where in the bottom figure only the direct waves are included in the localization process. P-wave raypaths are also shown from the event to the receivers and their distribution has a large impact on the size of the traveltime residuals. The different contours indicate the uncertainty space for events of different quality, i.e., events with different levels of accuracy in the identified arrival time information. Where the blue contours indicate the uncertainty space for events with accurately measured arrival times, the increasingly larger contours (green, yellow, orange, red) show the uncertainty space for events with an increasing uncertainty in the determined arrival time information, e.g., due to low S/N.

The size of the uncertainty space for a given quality of the input data depends on the number of receivers, the length of the receiver array or the distribution of the receivers if more than one receiver
string is deployed, and the wave phases involved in the localization process. Generally, longer receiver arrays produce smaller uncertainty spaces as long as the signal can be detected on all levels with comparable accuracy. To minimize the logistical impact and costs, it appears reasonable to use the smallest sensor string configuration that is able to meet the survey objectives. Depending on the survey objectives, receiver arrays can contain 11–40 (+) 3C sensor and extend over hundreds of meters. Extended receiver arrays have the added advantage that single-phase events, i.e., events where only the P- or S-waves are detected, can be located as well using the shape of the moveout to determine the distance of the event.

The size of the uncertainty space changes significantly depending on the phases involved in the localization (Zimmer, 2010b). In Figure 2, the three test locations use the first arriving waves and direct waves separately for localization. For location one, the direct wave is the first arriving phase on all sensors creating a relatively small uncertainty space. For location two, critically refracted head wave from the underlying high velocity layer arrives before the direct wave at some sensors, creating an even smaller uncertainty space as long as the traveltimes information is accurate. For location three, the first arrivals are dominated by the head wave on all receiver levels. Since the travelpath for the head wave is nonunique and only constrained by the separation of the P- and S-wave arrival, the uncertainty space, even with accurate traveltimes information, is very large. In such a case, it is better to use the later arriving direct waves to locate the event. Although the direct wave arrives later in the wavetrain, its amplitude is much larger than the head wave amplitude, therefore, the accuracy of the direct wave arrival times is likely to be greater than the head wave arrival times due to signal-to-noise differences. The uncertainty space can be reduced even further by using first arrivals and direct wave arrivals combined. Figure 3 shows the residual distribution using a velocity model (Figure 4) from a real project with a vertical sensor array on the left. Compared to Figure 4, the bottom three sensors in the high-velocity layer are omitted in Figure 3 to enhance the effect of first arrivals versus direct arrivals in the residuals. For most projects, it is also more common to have the complete sensor array situated above the target formation. In cases where sensors are placed inside the high velocity layer (Figure 4), it is easier to identify and demonstrate the moveout pattern for direct waves and headwaves. The top figure shows the distribution of the traveltime residuals when using P- and S-wave first arrivals. For the shown event locations, the first arriving waves are headwaves for the P- and the S-phase on all sensors. Since the headwaves follow a nonunique wavepath, the event location is only determined by the P- and S-wave separation, constraining the distance to the event, and the general moveout, constraining the depth of the event. For events shallower than the shown test event, the first arriving waves are becoming the direct waves at more and more sensors, which changes the moveout pattern from linear (headwaves) to more hyperbolic (direct waves). The constraint to larger depth is not as good since the transmitted waves originating in the high velocity layer can mimic the linear moveout pattern of the headwaves. The blue area in Figure 3 (top) shows the uncertainty space for the event indicating a large depth uncertainty. When using the later arriving direct waves for the same event (Figure 3, middle), the uncertainty space has a different shape. The depth uncertainty can be significant for events that are further than a few array lengths away from the sensors (Kidney et al., 2010). Using either of the arrival types introduces a significant depth uncertainty in the...
location of distant event. But since the two uncertainty spaces have different shapes, the uncertainty can be reduced by combining the use of direct waves and first arriving headwaves (Figure 3, bottom). The results in Figure 3 require that the headwaves can be identified with the same accuracy as the direct waves. Headwaves can be hard to identify due to their small amplitude (Figure 4), but the onset of later arriving direct waves can also be masked by interferences with other arrivals. This is especially true for the late arriving S-waves. In addition, using headwaves requires accurate calibration of the velocity in the underlying layer where log information is often very limited. If the velocity model can be calibrated accurately, adding observation wells is another way to increase location accuracy and area coverage at the same time. The modeling and design studies are able to quantify the benefit to meeting the survey objectives in these cases, allowing a reliable cost-benefit analysis.

For the correct location of microseismic events, it is important to correctly identify the different wave phases used in the localization. Figure 4 shows a waveform example where the sensors were placed in the low-velocity target layer (layer three) and an underlying high velocity layer. The event creating the shown waveform originated in the target layer and shows direct waves and headwaves for the P- and S-wave arrivals. The P-headwaves are visible as small amplitude signals arriving in front of the larger amplitude direct wave. For the sensors in the high-velocity layer, the headwave arrival transitions into the direct arrival. The arrival times of the direct wave have a discontinuity at the high velocity interface. For the S-waves, the direct arrivals are clearly visible. The smaller amplitude headwave arrivals of the S-wave are much harder to identify due to their interference with other secondary arrivals. After the arrival of the P- and S-waves, the amplitude does not return to the ambient noise level, but shows a rather complex pattern where individual secondary arrivals, e.g., reflected or converted waves, are impossible to identify with sufficient accuracy from the shown waveforms. In cases where the headwave and direct waves are not easy to identify from the original recordings, forward finite-difference modeling of the expected waveforms has proven to be an effective tool to avoid misidentifications during the processing of microseismic events.

**VELOCITY MODEL CALIBRATION**

The accuracy of the underlying velocity model has a significant impact on the accuracy of the event location, although, in many error analyses this factor is largely neglected. Without any real calibration data prior to the data acquisition, the modeling and design studies can show the sensitivity of the event locations in different parts of the model to slight changes in the velocities. This sensitivity analysis determines the requirements on the accuracy of the model calibration for meeting the set survey objectives. It also shows the influence of a slightly miscalibrated model, which is of special interest to the interpretation of real-time results, as often the velocity model...
is less accurate in these circumstances. Figure 5 shows the effect of small changes in the velocity model on the location of three test events. One hundred random variations of up to 3% of the original velocity value were introduced and the effect on the event location was observed. The random variations were chosen along a symmetric triangular probability function where the probability of choosing a variation higher than 3% is zero. In each of the 100 realizations, each layer is assigned a new velocity up to 3% different from its original value. With increasing observational distance, the effect of changes in the velocity model increase. Depending on the relative position of the event to the receiver array, these variations in event location are asymmetric with better constraints in areas of high receiver concentration.

The initial velocity model is usually set up using P- and S-wave velocities from available log information or from seismic surveys. This model is then modified to locate the calibration events, usually perforation shots or string shots, in their known places of origin. Single Vibroseis shots from the surface can be used to determine the downhole orientation of the receivers. Using them to calibrate the velocity model often requires additional shot locations and potentially different positions of the sensor array. However, in the absence of any other downhole calibration shots, small VSP surveys can be useful to constrain the velocity model. If multiple observation wells are used, the velocity model can be calibrated using the single array solutions from each observation well. For any given event that is recorded simultaneously on multiple observation wells, the locations obtained using only data from the individual observation wells, i.e., single array solutions, have to match. This method has the additional advantage that the model can be calibrated in any area where clear multiple array events are available, and the calibration is not tied to the immediate vicinity of the treatment well as it is the case when using only perforation shots.

**MOMENT TENSOR INVERSION**

For the purpose of these modeling and design studies, it is assumed that the moment tensor inversion is calculated following the approach of Vavrycuk (2001) and Warpinski and Du (2010). Once calculated, the moment tensor can be visualized by using T-k source-type plots according to Hudson et al. (1989) or ternary diagrams where the moment tensor is decomposed into its percentages of isometric (ISO), double couple (DC), and compensated linear vector dipole (CLVD), or by using the angle alpha describing the out-of-plane slippage direction (Vavrycuk, 2001). Although the quality of the moment tensor inversion can vary greatly, uncertainty analyses are almost never performed, as they are challenging due to the nonlinear nature of the inversion process. Changes in event location or slight changes in the amplitude information can have a significant impact on the inverted moment tensor result. An indication for the sensitivity of the inverted result to the input data is the condition number of the Green’s function matrix. The Green’s function describes the displacement at the receiver location due to a unit pulse at the source location. In a modeling and design study, it is assumed that the theoretical displacement at each sensor is correctly resolved by seismogram amplitude at the picked arrival time. The condition number is given by the ratio of the largest over the smallest eigenvalue of the Green’s function matrix. If the condition number is large, the inverted moment tensor solution is very sensitive to the exact input parameters. Since the identified seismic signal can contain noise, interferences, and other uncertainties, the measured amplitude deviates from the theoretical amplitude. If the condition number of the Green’s function matrix is high, these slight changes will create large changes in the inverted moment tensor. Condition numbers larger than 20 indicate increasingly uncertain results, even if the S/Ns of the signals are large.

The maximum achievable quality of the moment tensor inversion, i.e., assuming accurate amplitude information from all sensors in the array, is largely determined by the distribution of the raypaths from the event to the receivers, which depends on the acquisition geometry and the velocity model. Both factors can be reasonably approximated prior to the data acquisition and summarized in the spatial distribution of the condition number. Figure 6 shows a map view (top) and side view (bottom) of the spatial distribution of the condition number for a synthetic dual array setup. The velocity model is an approximation of a typical tight shale scenario where the low-velocity target formation has high velocity layers above and below. The 3D distribution of the condition number depends on the velocity model, the sensor distribution, and the method used for the inversion. In this case, only the amplitudes of first-arriving direct waves are used for the inversion. If the first arriving phase for a given location-sensor combination is a critically refracted head wave, the sensor is excluded from the calculation. This method creates high condition numbers for short sensor arrays and event locations close to high velocity layers, since in such cases the first arrivals at the sensor array are dominated by head waves and only few sensors can be used for the inversion. Extending the sensor array or repositioning the sensor array can avoid this problem and the issue can be effectively addressed during the planning and design phase of the project. If direct wave information is available, even if they are not the first arriving phase, the distribution of the condition number is much more favorable in most cases, but for the real data the amplitude information of the later arriving direct waves can be compromised by interferences with other waves. The condition number displayed in the modeling and design studies assumes excellent S/N on all receivers. Since this can vary significantly for real data, the quality of the individual moment tensor inversion needs to be assessed once the data is available. A low
condition number in the modeling and design study does not guarantee a low condition number for a specific event in this area. But for areas with high condition numbers in the design phase, no reliable moment tensor solutions can be calculated even if the amplitude information on all receivers is very accurate.

**ALTERNATE TECHNOLOGIES**

Surface and downhole tiltmeter surveys are technologies that can either complement microseismic surveys or help achieving survey objectives that cannot be achieved otherwise. Tiltmeters measure the deformation caused by the hydraulic injections and can provide information about the general fracture azimuth, fracture height, and the percentage of horizontal versus vertical fractures. The latter cannot be obtained from microseismic surveys unless very reliable moment tensor inversions are available. A thorough description of the tiltmeter method and its value for fracture monitoring can be found in Wright et al. (1997) and Davis and Marsic (2010).

In recent years, fiber-optic based DTS has emerged as another technology that complements microseismic surveys. Where microseismic surveys map a large area with limited resolution, DTS monitors fluid movements in the immediate vicinity of the treatment wellbore. Current DTS technology provides a temperature profile with a spatial resolution of 1 m and a temperature resolution of 0.01° C every few minutes. Since the treatment fluid is (typically) cooler than the rock formation, the temperature profile can be used as a direct indicator for fluid movements. In addition, once installed, the fiber-optic line can be used during the production phase to determine points of gas inflow into the wellbore as these inflows lower the temperature due to the Joule-Thompson effect. Figure 7 shows a snapshot of a combined Microseismic/DTS survey after the treatment of stages one to three. Stage one did not yield many microseismic events and is not shown. The complete microseismic and DTS results of the first five stages in this project are shown and discussed in detail by Holley et al. (2010). The results of stages two and three of the microseismic results show a significant overlap between the stages, which could lead to the interpretation that the perforation intervals are too closely spaced. When looking at the DTS temperature trace, though it becomes obvious that the fluid injection is not confined to the immediate vicinity of the perforation, but rather that fluid enters the formation over a broad interval as indicated by the large interval of relatively low temperatures. This is indicated by the cool temperatures in a wide interval. Combined with the microseismic information, this would suggest an interpretation where the overlap in the microseismic events is a result of a stage isolation issue and the perforation interval might still be adequate if this problem is sufficiently addressed. The warm temperatures between the stage one and two perforations show that stage one was successfully isolated from the stage treatment through a bridge plug. A leaking bridge would be indicated by cool temperatures in this area. Playing back the development of all the data in time visualizes dynamic changes in the treatment. Since almost no processing is involved in the displaying of the DTS data, the final results are available with minimal delay allowing for a detailed interpretation even during the injection. This example shows briefly how other technologies can complement and improve the interpretation of microseismic maps. Holley et al. (2010) provide a full in-depth discussion of this example.

![Figure 6](image1.png)

Figure 6. Map view (top) and side view (bottom) of the distribution of the condition number for a dual array setup indicating the areas where high-accuracy, moment-tensor inversions (dark blue areas, condition number <20) are possible.

![Figure 7](image2.png)

Figure 7. Map view of combined microseismic and DTS survey results showing and apparent overlap in the microseismic event distribution. The DTS curve proves that the actual injection interval during the treatment of stage three is much larger than the individual perforation suggesting stage isolation problems, which is creating the overlap in the MS data.
DISCUSSION

The tools described in this paper can be effectively used not only to optimize the microseismic data acquisition, but also to create realistic expectations on the outcome of the survey. Often the main emphasis of project feasibility assessment is on detection of a sufficient number of events and the ability to locate them with sufficient accuracy. The calculation of location uncertainties is not standardized and estimates can vary widely depending on what measure is used. Using processing parameters that locate a microseismic event less than 5 m from its known origin, e.g., perforation shot, does not mean that the location accuracy for this event or any other event that uses the same processing parameters, i.e., sensor position, velocity model etc., is 5 m. For realistic estimates of the uncertainty of any calculated parameter, e.g., event location, magnitude, moment tensor, the standard approach is to quantify the uncertainty of the input parameters and propagate these uncertainties to the calculated parameter. Where this cannot be done in an analytical way, statistical methods, e.g., Monte-Carlo simulations, bootstrapping, jackknifing, can be used, but even then it is important to state clearly what input parameters are included. For microseismic event locations, often only the uncertainty in the traveltine arrival picks is considered in the uncertainty analysis. Almost always, the uncertainties in the underlying velocity model are ignored since they are difficult to specify. Calibrating the velocity model by using multiple events of known origin with clear P- and S-wave arrivals, i.e., perforation shots, only provides a model that is proven valid in the immediate vicinity of the calibration data. Depending on the formation, the microseismic events for hydraulic treatments can be hundreds or even thousands of meters away from the calibration dataset, and often there is no proof that the velocity distribution does not change over this area. The modeling and design studies help in quantifying the potential impact of these uncertainties in the velocity model on the event location. Although the calculated event location, i.e., “dot on the map,” is supposed to be the most likely location, realistic uncertainty estimates are essential for a correct interpretation of the microseismic event distribution.

The presented elements of a modeling and design study encourage the definition of specific objectives for the survey and clear statements of certain limitations, e.g., number of observation wells, sensors or sensor positions, when considering specific acquisition geometries or processing methodologies. If multiple observation wells are available, the design studies can quantify the value of adding additional sensor arrays in these wells toward meeting the survey objectives. If only a single observation well is available these studies can quantify the value of adding additional sensor arrays in these wells toward meeting the survey objectives. If only a single observation well is available these studies can quantify the expected location accuracy. If this accuracy does not meet the survey objectives, either the acquisition geometry or the objectives need to be adjusted.

Many of the shown calculations and figures require certain assumptions. For a correct interpretation of the modeling results, it is always important to properly document these as the results are only as good as the validity of these assumptions. Comparisons between the design studies and the actual results after the project is finished have shown a generally good agreement of the prediction and the results as long as all the assumptions were valid. Predicting the largest magnitudes for a formation without previous experience in that formation is often the most challenging of the assumptions.

The main input parameters for a modeling and design study are survey objectives, limitations on sensor placement, e.g., number of sensors, observation wells, installation depths, velocity model information, e.g., velocity logs, seismic surveys, formation tops, and anisotropy parameters.

The length and details of the study can vary depending on the requirements, but typical elements of a prejob modeling and design study include expected ambient noise level, maximum viewing/mapping distance, estimates of magnitude and quantity of events detected, expected location uncertainties, expected waveforms, velocity model calibration plan, distribution of the condition number, assessment if the survey can meet the objectives, and consideration of alternate technologies or setups if necessary to meet the survey objectives. The interpretation of microseismic event properties and distributions is a fast advancing process. New or existing processing techniques, e.g., double differencing (Waldhauser, 2001), quantification of fracture densities using shear-wave splitting data, will be developed and integrated into the standard processing flow or they will be available as options where the design studies show some valuable contribution to meeting the survey objectives. Integration of microseismic data with other mapping technologies (e.g., DTS, tillmeters), log information, geo-mechanical models, reservoir characterization techniques, etc. will also likely create new demands and objectives for microseismic surveys, requiring new elements for the modeling and design studies.

CONCLUSIONS

Even before data is acquired, the possibilities and limitations of a given acquisition geometry and processing flow can be assessed for their ability to meet survey objectives. Although some of the calculations are specific to the applied detection and localization algorithm, the general principles still apply and it should be easy to adapt the procedures accordingly. By pointing out the limitation of a certain survey setup, it is possible to either adjust the survey objectives or the acquisition geometry well ahead of the data collection. The content of the modeling and design studies should be detailed to the specific survey objectives and the acquisition and processing tools available at the time. New elements will likely enter the microseismic processing flow, and interpretation in the years ahead. Quantitative interpretation of shear-wave splitting to determine fracture geometry and density is only one of the potential new applications currently under development. Modeling and design studies appear to be an excellent tool for proving the project specific value of a certain setup, processing flow and processing method before any receivers are installed in the ground.

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Interferometric hydrofracture microseism localization using neighboring fracture

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ABSTRACT

Hydraulic fracturing is the process of injecting high-pressure fluids into a reservoir to induce fractures and thus improve reservoir productivity. Microseismic event localization is used to locate created fractures. Traditionally, events are localized individually. Available information about events already localized is not used to help estimate other source locations. Traditional localization methods yield an uncertainty that is inversely proportional to the square root of the number of receivers. However, in applications where multiple fractures are created, multiple sources in a reference fracture may provide redundant information about unknown events in subsequent fractures that can boost the signal-to-noise ratio, improving estimates of the event positions. We used sources in fractures closer to the monitoring well to help localize events further away. It is known through seismic interferometry that with a 2D array of receivers, the traveltime between two sources may be recovered from a crosscorrelogram of two common source gathers. This allowed an event in the second fracture to be localized relative to an event in the reference fracture. A difficulty became evident when receivers are located in a single monitoring well. When the receiver array is 1D, classical interferometry cannot be directly employed because the problem becomes underdetermined. In our approach, interferometry was used to partially redatum microseismic events from the second fracture onto the reference fracture so that they can be used as virtual receivers, providing additional information complementary to that provided by the physical receivers. Our error analysis showed that, in addition to the gain obtained by having multiple physical receivers, the location uncertainty is inversely proportional to the square root of the number of sources in the reference fracture. Because the number of microseism sources is usually high, the proposed method will usually result in more accurate location estimates as compared with the traditional methods.

INTRODUCTION

Hydraulic fracturing enhances the production of hydrocarbons and other fluids from rock formations. It is a critical tool in many applications, including shale gas and geothermal energy production. The process of hydraulic fracturing involves injecting fluids under high pressure into a reservoir formation with the purpose of creating additional fluid pathways leading to the production well. These pathways may have complicated shapes; however, fractures aligned with the direction of maximum stress are commonly observed (Zoback et al., 2003). Knowing where hydraulic fractures have been created helps in predicting fluid flow, designing additional fractures, and positioning additional production wells.

Hydraulic fracture creation is normally accompanied by microseismic events (microseisms) thought to be generated by the cracking of the rock. Locating these microseisms is an indirect method of fracture imaging and fracture growth monitoring in near real time. To locate these microseisms, an array of three-component receivers is often installed in one or several monitoring wells. Arrivals from each microseismic event are recorded for each receiver. These recordings are then used to locate the microseisms. The problem of localizing events from this type of data has received considerable attention (Michaud et al., 2004; Huang et al., 2006; Bennett et al., 2006). Despite significant progress, the problem remains a challenge in need of further investigation due to large localization uncertainties.

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Most techniques currently employed by the industry locate events one by one. The estimate of the location of one event is not used to improve the estimate of another. For microseism localization from a single observation well, the traveltime from each microseism to the receiver is picked, and the wave’s polarization is estimated. The polarization provides the direction of the arriving wave as it impinges on the receiver. The polarization and the traveltime along with an assumed velocity model, allow us to ray trace to the inferred event location. We shall refer to this as the classical localization technique. The event excitation time is not known in practice, but this problem may be eliminated by considering the time difference between P and S arrivals (Pearson, 1981; House, 1987). In our analysis of the classical approach, we will assume for simplicity that the event excitation time is known. This time is not needed for the interferometric method proposed in this paper. The goal of this paper is not to characterize the performance of state-of-the-art proprietary algorithms of the non-interferometric type, but to contrast controlling mechanisms for localization uncertainty in the classical approach with those for our new interferometric approach.

In practice, multiple fractures are created sequentially. One fracture can often be created close to the monitoring well, with other fractures appearing further away, as shown in Figure 1. This geometry dictates that sources corresponding to the closest fracture can be imaged better with traditional methods than those in further fractures because the velocity is likely to be better constrained near the observation well, and the receivers form a larger angular coverage relative to the fracture event locations. Instead of locating events in a more distant fracture independently, we would like to use available information about the reference (closest) fracture to improve the estimated locations in the more distant fractures. We will use seismic interferometry to couple together events from both fractures. Under idealized assumptions, interferometry recovers the Green’s function between any two source locations. Those assumptions are rarely satisfied in practice, and for a single borehole, the full recovery of the Green’s function between two source locations is fundamentally impossible. The signal recorded in the well can be only partially redatumed to an event in the first fracture. The end result of the redatuming process is not the complete Green’s function, but partial information about it. We show in this paper, however, that this information, along with an assumed velocity model, can significantly reduce uncertainty around the estimated event location as compared to the classical method.

Interferometry provides additional advantages by leading to stable imaging in the presence of velocity uncertainty (Borcea et al., 2005). We will not explore the full range of potential benefits provided by interferometry, but rather, how it can be used to improve microseismic event localization in a canonical case and explain its fundamental limitations.

We propose using interferometry to partially redatum every noisy record of an event from a second fracture onto a reference fracture. Using this approach, in addition to measuring events in the second fracture with physical receivers in the monitoring well, we obtain additional information from the many virtual receivers located in the reference fracture. This redundancy boosts the signal-to-noise ratio (S/N) and reduces the localization uncertainty. For classical methods, a higher S/N is achieved by appropriately stacking over all available receivers. In the method proposed in this paper, additional information coming from events in the reference fracture is also stacked over. Because the number of events in the reference fracture is typically much higher than the number of physical receivers in the observation well, we can expect a considerable improvement over the performance of the classical algorithm. We present the theory and show numerical examples that illustrate this result.

**PROBLEM SETUP**

Assume that the velocity model is known with sufficient accuracy to compute traveltimes. A monitoring well is instrumented with $N_{\text{rec}}$ three-component receivers with locations $x_{\text{rec}} \in \mathcal{W}$, where $\mathcal{W}$ denotes the monitoring well. The signals recorded at the receivers are seismograms that are assumed to contain direct arrivals from each event in each of two fractures. We also assume that the observed seismograms are perturbed by additive uncorrelated Gaussian noise that models measurement errors. Denote the two fractures as $\mathcal{F}_1$ and $\mathcal{F}_2$ and assume that $\mathcal{F}_1$ is closer to the monitoring well than $\mathcal{F}_2$. For convenience, we think of the two fractures as being roughly parallel to one another (Figure 1). This is not a necessary assumption, however, and deviations from it are allowed. Indeed, in our usage here, a fracture is merely a collection of source events.

We first assume for simplicity that the events of the reference fracture $\mathcal{F}_1$, have been located precisely. These microseismic locations are denoted by $x_{\text{r,1}} \in \mathcal{F}_1$. In a later section, we will generalize our results to a more realistic situation where the event locations in the first fracture have some uncertainty. Our goal is to localize events $x_{\text{r,2}} \in \mathcal{F}_2$.

**IMPROVED LOCALIZATION FROM INTERFEROMETRY**

In this section, we present the interferometric method of event localization using data recorded from a single monitoring well. We begin by presenting a simplified localization technique, which we use as a benchmark. This is followed by a brief summary of classical interferometry, which inspired our method. When only single well data is available, classical interferometry is no longer fully applicable. However, by performing a stationary phase analysis of 1D correlogram events, we are able to extract partial information about unknown event locations from reference event locations. The proposed method is illustrated using both a homogeneous and layered velocity model.
Classical localization

In this paper, we use a simplified version of a classical localization technique as a point of comparison for our proposed interferometric method. We pick the traveltime $t_p$ of the P-wave from the event to the receiver using a crosscorrelation method with a known source wavelet. Then, we estimate the unit polarization vector $\hat{p}$ of the P-wave using a method based on a singular value decomposition analysis of the arrival of the P-wave (de Franco and Musacchio, 2001). If the velocity model is homogeneous, then the polarization vector is given by

$$\hat{p} = \frac{x_{\text{rec}} - x_s}{||x_{\text{rec}} - x_s||},$$

(1)

where $|| \cdot ||$ denotes the vector length ($L^2$ norm). The polarization is a unit-length vector pointing from the event location $x_s$ in the direction of the receiver location $x_{\text{rec}}$. Denoting the homogeneous P-velocity by $V_p$, an estimate of the source location is given by

$$x_s = x_{\text{rec}} - V_p t_p \hat{p}.$$  

(2)

For a general velocity model, the source location is found by tracing a ray that leaves the receiver in the $\hat{p}$ direction and stops at time $t_p$. This localization method is perfect if the medium is known exactly, and the observed signal contains no noise. Random noise in the seismograms results in localization uncertainty, which can be reduced by stacking over multiple receivers. More detailed analysis of this uncertainty will be presented in the Uncertainty analysis section.

Seismic interferometry

Seismic interferometry allows physical sources to be redatumed to receiver locations (Rickett and Claerbout, 1996; Derode et al., 2003; Bakulin and Calvert, 2004; Schuster et al., 2004; Wapenaar et al., 2005; Drijkopp et al., 2009). Receivers can likewise be redatumed to source locations according to the principle of reciprocity (Curtis et al., 2009).

Specifically, suppose that two sources located at $x_{s1}$ and $x_{s2}$ inside a 3D medium are surrounded by a closed 2D surface of receivers in the set $\mathcal{W}$. Although the theory we are developing works in the elastic case, we pick the P- and S-wave traveltimes separately, essentially using acoustic theory for both arrivals. Following Wapenaar (2004), Wapenaar et al. (2005) and Schuster and Zhou (2006), the interferometrically derived Green’s function between two source locations may be recovered from the representation theorem, and has the form

$$G(x_{s2},x_{s1},\omega) + G^*(x_{s1},x_{s2},\omega)$$

$$\propto \int_{x_{\text{rec}} \in \mathcal{W}} G^*(x_{s1},x_{\text{rec}},\omega) G(x_{s2},x_{\text{rec}},\omega) dS,$$  

(3)

where $G(x_{s2},x_{s1},\omega)$ is the Green’s function between the locations $x_{s1}$ and $x_{s2}$ written in the Fourier domain, and the star (*) denotes the complex conjugate. While interferometry may generally be used to redatum entire waveforms, in this work, we are only interested in recovering traveltimes. In particular, we ignore the issue of the source mechanisms of the microseismic events. Note that in this context, even when the surface $\mathcal{W}$ does not completely enclose the medium, traveltimes along certain rays may be recovered. Specifically, traveltimes may be recovered along rays that are emitted by one source, pass through the other, and are finally received at some receiver location, as shown in Figure 2 (Lu et al., 2008). Mathematically, the traveltime between the sources will come from a stationary phase point in the crosscorrelogram of two common source gathers (Snieder, 2004). The crosscorrelation lag of two direct arrivals from two different sources is a function, $\tau(x_{\text{rec}})$, of the receiver position $x_{\text{rec}}$ that belongs to the 2D surface $\mathcal{W}$. The stationary phase point $(x_{\text{rec}}^0, \tau(x_{\text{rec}}^0))$ is defined by the extremum of the function $\tau$. The stationary receiver location $x_{\text{rec}}^0$ marks the receiver that records the ray connecting the two sources. The stationary value, $\tau(x_{\text{rec}}^0)$, has the physical meaning of the traveltime between the two sources along that ray.

In classical interferometry, receivers enclosing the two sources must span a 2D surface $\mathcal{W}$. The stationary phase point is found by setting the two partial derivatives of $\tau$ in orthogonal directions to zero. Because only one partial derivative can be estimated with a 1D receiver array, the stationary phase condition becomes underdetermined (Figure 2). Stationary points along a 1D receiver array, i.e., single monitoring well, are thus not stationary in the classical sense, but they still give useful information for source localization.

Figure 2. (a) The ray that connects two sources is received at a stationary receiver location within a 2D receiver-array aperture (the intersection of the two green lines). (b) The stationary receiver location is the stationary point of the 2D correlogram. The stationary point is shown as the intersection of two common source gather lags plotted as green curves. A correlogram calculated over a 1D receiver array (red line) may exhibit an extremum, but it need not correspond to any physical ray or yield a physical traveltime.
**Interferometric localization using a single monitoring well**

The method that we introduce here is applicable to a general velocity model and an arbitrary well geometry subject only to minor technical assumptions. Specifically suppose that receivers are given by their locations:

$$x_{\text{rec}}(\ell) \equiv (x_{\text{rec}}(\ell), y_{\text{rec}}(\ell), z_{\text{rec}}(\ell)), \quad (4)$$

where $\ell$ is the well arc length measured from the top of the well. Suppose further that we are given a velocity model, from which we compute the traveltime function, $T(x, \ell) \equiv T(x, x_{\text{rec}}(\ell))$. In what follows, $\ell$ is a continuous parameter, but in practice, a receiver array consists of a finite number of receivers. We use spline interpolation to obtain traveltimes at locations between physical receivers.

Let $x_{s,1}$ be an event with a known location in the reference fracture. By jointly analyzing the signal emitted from that known location with another signal coming from an event in the second fracture whose location is unknown, we would like to recover at least partial information about the location $x_{s,2}$ of that second fracture.

The correlogram of the direct arrivals from the two events contains a peak with lag:

$$\tau(x_{s,1}, x_{s,2}, \ell) = T(x_{s,2}, \ell) - T(x_{s,1}, \ell). \quad (5)$$

Assuming this lag has a stationary point, $\ell^0$, with respect to the receiver location, we call this a stationary receiver and denote it by $x_{\text{rec}}^0 \equiv x_{\text{rec}}(\ell^0)$. Hence,

$$\partial_\ell \tau(x_{s,1}, x_{s,2}, \ell^0) = \frac{\partial \tau(x_{s,1}, x_{s,2}, \ell)}{\partial \ell} \bigg|_{\ell = \ell^0} = 0. \quad (6)$$

The stationary lag of the event is consequently

$$\tau^0 \equiv \tau(x_{s,1}, x_{s,2}, \ell^0). \quad (7)$$

Observe that the pair consisting of the stationary receiver location and the stationary lag, which we call a stationary condition, are observed directly from the data. Their values tie together the known location $x_{s,1}$ with the unknown location $x_{s,2}$ in the following way.

The locus of all points $x_{s,2}$ giving rise to the same stationary condition, $(x_{\text{rec}}(\ell^0), \tau^0)$, is a subset of 3D space defined by

$$\mathcal{R}(x_{s,1}, \ell^0, \tau^0) = \{x_{s,2} | \partial_\ell \tau(x_{s,1}, x_{s,2}, \ell^0) = 0 \} \quad \text{and} \quad \tau^0 = \tau(x_{s,1}, x_{s,2}, \ell^0). \quad (8)$$

Provided that the two constraints defining $\mathcal{R}$ are sufficiently smooth, the set $\mathcal{R}$ will be a 1D curve. This curve can be determined numerically, or in some cases, analytically. Because a point $x_{s,2}$ can travel along the curve $\mathcal{R}$ with no change to the stationary condition, we cannot completely localize it using interferometry. We can, however, constrain two out of three coordinates in a suitable coordinate system, i.e., we can place the source on a particular 1D curve but cannot say where on this curve the event lies.

The stationary condition from equation 8 applied to multiple reference events $x_{s,1}$ produces a corresponding number of curves $\mathcal{R}$. All of those curves by construction contain the unknown event location $x_{s,2}$. If their intersection consisted of a single point, the event $x_{s,2}$ would be successfully localized. However, in some cases, including the example below, those curves may intersect along a common stretch, which makes the complete event localization impossible. Instead, the method produces multiple estimates of the same stretch of a 1D curve. Appropriate averaging over these estimates should greatly reduce the uncertainty in the direction perpendicular to the stretch and have little effect on the uncertainty along the stretch.

**Example: Localization in homogeneous medium**

Our general method outlined in previous sections can be easily applied to a horizontally stratified velocity model, as illustrated in the Numerical results section. For clarity purposes, we consider here the case of a homogeneous velocity model with a constant P-wave velocity $V_p$ and a vertical monitoring well. As the general theory predicts, interferometry allows us to use known events in a reference fracture to constrain two out of the three location parameters of a microseism in $F_2$, and those parameters have a very clear intuitive meaning in this example.

Consider two event locations, $x_{s,1} = (x_{s,1}, y_{s,1}, z_{s,1})$ and $x_{s,2} = (x_{s,2}, y_{s,2}, z_{s,2})$, and a receiver location, $x_{\text{rec}} = (0, 0, z_{\text{rec}})$. The traveltimes from the sources to the receiver are given by

$$T(x_{s,i}, x_{\text{rec}}) = \sqrt{\frac{x_{s,i}^2 + y_{s,i}^2 + (z_{s,i} - z_{\text{rec}})^2}{V_p}}, \quad i = 1, 2. \quad (9)$$

The crosscorrelogram of the two common event gathers contains an event from the correlations of the two direct waves with the lag

$$\tau_{x_{s,1}, x_{s,2}}(z_{\text{rec}}) = \sqrt{\frac{x_{s,1}^2 + y_{s,1}^2 + (z_{s,1} - z_{\text{rec}})^2}{V_p}} - \sqrt{\frac{x_{s,2}^2 + y_{s,2}^2 + (z_{s,2} - z_{\text{rec}})^2}{V_p}}. \quad (10)$$

The receiver located at $x_{\text{rec}}$ is stationary in the $z$-direction for the pair of events $x_{s,1}$ and $x_{s,2}$ if

$$\frac{\partial \tau_{x_{s,1}, x_{s,2}}(z_{\text{rec}})}{\partial z_{\text{rec}}} = 0. \quad (11)$$

Combining equation 9 with equation 10, we arrive at the following formula for the stationary receiver depth:

$$z_{\text{rec}}^0 = \frac{r_{s,2} z_{s,1} - r_{s,1} z_{s,2}}{r_{s,2} - r_{s,1}}, \quad r_{s,2} > r_{s,1}, \quad (11)$$

where $r_{s,i} = \sqrt{x_{s,i}^2 + y_{s,i}^2}$ is the horizontal offset from the receiver line to the event location $x_{s,i}$.

We can interpret equation 11 geometrically as follows. Suppose that the two events and the receiver are in the same vertical plane, and both sources are on the same side of the receiver. We may assume, for example, that they both have zero azimuth: $x_{s,1} = y_{s,2} = 0$, and $x_{s,2} > x_{s,1} > 0$. It then follows from equation 11 that

$$z_{\text{rec}}^0 = \frac{z_{s,2} - z_{s,1}}{x_{s,2} - x_{s,1}} (x - x_{s,1}) + z_{s,1} \bigg|_{x = 0}, \quad (12)$$

and then from equation 9 that
\[ \tau_{x_i, x_2}(z_{\text{rec}}) = \frac{\sqrt{(x_{i,2} - x_{i,1})^2 + (z_{i,2} - z_{i,1})^2}}{V_p}. \] (13)

The physical interpretation of the stationary receiver location and the stationary lag is identical to that in classical interferometry. The stationary receiver lies on the ray that connects the two event locations \( x_{i,1} \) and \( x_{i,2} \), and the stationary lag is the physical travel-time of the wave between those two points.

Now, consider sources \( x_{i,1} \) and \( x_{i,2} \) in general 3D positions. Because the solution in equation 11 depends only on the horizontal offset between the two sources, the two sources located at \( x_{i,1} \) and \( x_{i,2} \) will produce a stationary point at the receiver location \( x_{\text{rec}} \) so long as the three points \( (x_{i,1}, x_{i,2}, z_{\text{rec}}) \) can be made collinear by appropriate rotations of both sources about the receiver line (Figure 3a). The curve \( R_i \), whose general form is given in equation 8, in this simple example, is a ring lying inside a horizontal plane.

It follows from equation 11 that both sources \( x_{i,1} \) and \( x_{i,2} \) must have the same dip angle when viewed from the receiver location \( x_{\text{rec}} \), and that \( \tau \) is the traveltime between the circle containing \( x_{i,1} \) and the circle containing \( x_{i,2} \) (see Figure 3b). Write both event locations in spherical coordinates centered at the receiver \( x_{\text{rec}} \):

\[ x_{i,i} = x_{\text{rec}} + R_i (\cos \theta_i \cos \phi_i, \cos \theta_i \sin \phi_i, \sin \theta_i), \]
\[ i = 1, 2. \] (14)

where \( \theta_i \) is the dip angle, \( \phi_i \) is the azimuthal angle, and \( R_i \) is the radial distance. Then knowing the location \( x_{i,1} \) allows us to recover the dip angle of location \( x_{i,2} \), because

\[ \theta_2 = \theta_1. \] (15)

Its radial distance from the receiver is given by

\[ R_2 = R_1 + V_p \tau_{x_{i,1}, x_{i,2}}(z_{\text{rec}}). \] (16)

In this simple example, these are the two recoverable coordinates from a vertical receiver array using interferometry.

Figure 4 illustrates how event localization using a neighboring fracture works when the known fracture is planar. Any event \( x_{i,1} \) lying at the intersection of fracture \( F_1 \) with the cone \( \{ \theta = \theta_1 \} \) will produce a stationary point at the same stationary receiver depth \( z_{\text{rec}} \).

The stationary lags, \( \tau_{x_{i,1}, x_{i,2}}(z_{\text{rec}}) \), will vary depending on \( x_{i,1} \). Equation 16, however, always holds true.

We observe that the traveltime from any source location to the vertical receiver array does not depend on the azimuth of the source. One therefore cannot obtain constraints on the azimuth beyond those already provided by classical methods through polarization analysis.

Because the total number of sources in fracture \( F_1 \) is typically large, we can expect to have many redundant measurements of the dip angle and radial distance of \( x_{i,2} \). We can use these to boost the

![Figure 3](image1.png)

Figure 3. (a) The receiver in a 1D array is vertically stationary with respect to two given sources (red stars) if the source locations can be rotated about the receiver line into collinear positions (black stars). (b) All pairs of sources such that each source belongs to a corresponding circle (shown in a map view) produce a correlogram with identical stationary receiver location and with the same stationary lag.

![Figure 4](image2.png)

Figure 4. For (a) homogeneous or (b) layered medium, the unknown dip angle (or horizontal offset) and distance along the ray of a source (red star) can be estimated with the help of many stationary sources in the neighboring fracture (vertical plane). Any source along the red curve provides an independent measurement of the distance and horizontal offset of an unknown microseism.
the nature of the noise. However, in the following analysis the
transverse direction of the polarization angle.

**UNCERTAINTY ANALYSIS**

In this section, we quantify microseismic event localization uncertainty for the classical and interferometric methods so that these uncertainties may be compared. We perform the analysis for the case of a homogeneous medium. The approach may be easily generalized to the layered case. Extending it to more general models is a challenge. However, our basic conclusions about the reduction of errors (Figure 5):

\[ t_p = t_0 \pm \Delta t, \quad \theta = \theta_0 \pm \Delta \theta. \]  

(17)

The exact distribution of random variables \( t \) and \( \theta \) is dependent on the nature of the noise. However, in the following analysis the perturbations \( \Delta t \) and \( \Delta \theta \) may be simply thought of as standard deviations of the error in estimated parameters.

It is convenient to split the uncertainty in event location into two orthogonal directions: along the line of sight (range) and vertically perpendicular to that (transverse or cross-range). The uncertainty in range is determined by the error in picked traveltime and by the assumed velocity

\[ \sigma_R = V_p \Delta t. \]

The cross-range (transverse) uncertainty is proportional to the distance from the receiver to the true location of the event; it also depends on the uncertainty in the dip angle:

\[ \sigma_T = \| x_{rec} - x_{s,2} \| \tan \Delta \theta. \]

If the dip uncertainty is sufficiently small, then \( \tan \Delta \theta \approx \Delta \theta \), and

\[ \sigma_T \approx \| x_{rec} - x_{s,2} \| \Delta \theta. \]

Stacking these uncertainties over \( N_{rec} \) receivers gives

\[ \sigma_{R,rec}^N \approx \frac{V_p \Delta t}{\sqrt{N_{rec}}}, \quad \sigma_{T,rec}^N \approx \frac{d(W_i, x_{s,2}) \Delta \theta}{\sqrt{N_{rec}}}, \]  

(18)

where \( d(W_i, x_{s,2}) \) is the average radial distance between the monitoring well receivers and the event in the second fracture.

**Interferometry**

In the proposed method, we construct a correlogram of two common event gathers corresponding to the two event locations \( x_{s,1} \) and \( x_{s,2} \). The output of the stationary phase analysis of this correlogram is two quantities: the stationary receiver depth \( z_{rec} \) and the stationary lag \( \tau \). This presumes that all event pairs that do not produce a stationary point have been removed from consideration in a preprocessing step. Because the correlogram is insensitive to the azimuth of seismic events, we can assume without loss of generality in the following analysis that all events have zero azimuth, reducing the problem to two dimensions.

In a noisy environment, the picked stationary receiver depth is a perturbation of the true depth:

\[ z_{rec} = z_{rec}^0 \pm \Delta z, \]

where \( z_{rec}^0 \) is the true stationary receiver depth, and \( \Delta z \) is the error. Note that although receivers are located at discrete depths, the correlogram curve \( \tau(z_{rec}) \) can be smoothly interpolated between actual receiver locations to improve depth resolution. Similarly, the stationary lag is also picked with some error:

\[ \tau = \tau_0 \pm \Delta \tau. \]

A smooth least-squares interpolation of \( \tau(z) \) is expected to further reduce \( \Delta \tau \).

Because the location \( x_{s,1} \) is known, it is convenient to represent uncertainty in \( x_{s,2} \) relative to it (see Figure 6). The range uncertainty \( \sigma_R \) around the true distance \( \| x_{s,1} - x_{s,2} \| \) is determined by the error in lag picking and the assumed velocity:
\[ \sigma_R = V_p \Delta \tau. \]

The cross-range (transverse) uncertainty is principally determined by the uncertainty in dip angle of \(x_{s,2}\) relative to \(x_{s,1}\):

\[ \sigma_T = \frac{1}{2} \|x_{s,1} - x_{s,2}\| (\tan \Delta \theta^+ + \tan \Delta \theta^-), \]

where \(\theta^0 + \Delta \theta^+\) and \(\theta^0 - \Delta \theta^-\) are the dip angles of \(x_{s,1}\) relative to receiver depths \(z_{rec} - \Delta z\) and \(z_{rec} + \Delta z\), respectively, (see Figure 6)

\[
\begin{align*}
\tan(\theta^0 + \Delta \theta^+) &= \frac{z_{s,1} - z_{rec}^0 + \Delta z}{x_{s,1}}, \\
\tan(\theta^0 - \Delta \theta^-) &= \frac{z_{s,1} - z_{rec}^0 - \Delta z}{x_{s,1}}.
\end{align*}
\]

Applying the small error approximation \(\tan x \approx x\), valid for \(x \ll 1\), we obtain

\[ \theta^0 + \Delta \theta^+ = \arctan\left(\frac{z_{s,1} - z_{rec}^0 + \Delta z}{x_{s,1}}\right) \approx \theta^0 + \frac{\Delta z}{x_{s,1}} \left(1 + \left(\frac{z_{s,1}^0 - z_{rec}^0}{x_{s,1}}\right)^2\right). \]

Hence,

\[ \Delta \theta^+ \approx \frac{x_{s,1} \Delta z}{x_{s,1} + (z_{s,1} - z_{rec}^0)^2}. \]

Likewise, \(\Delta \theta^- = \Delta \theta^+\). Applying the small error approximation \(\tan x \approx x\), valid for \(x \ll 1\), we find that

\[ \sigma_T \approx \frac{x_{s,1} \|x_{s,1} - x_{s,2}\| \Delta z}{\|x_{s,1} - x_{rec}\|^2} = \frac{\cos \theta_0 \|x_{s,1} - x_{s,2}\| \Delta z}{\|x_{s,1} - x_{rec}\|}. \]

Stacking over all \(N_s\) events in the first fracture yields

\[ \sigma_{R,s}^N \approx \frac{V_p \Delta \tau}{\sqrt{N_s}}, \quad \sigma_{T,s}^N \approx \frac{\cos \theta_0 d(F_1, x_{s,2}) \Delta z}{\sqrt{N_s} d(W, F_1)}. \]

(19)

Here, \(d(F_1, x_{s,2})\) is the average distance between \(x_{s,2}\) and events in \(F_1\), and \(d(W, F_1)\) is the average distance from the receivers to the events in \(F_1\).

**Uncertainty in the reference fracture**

We have so far assumed that events in the reference fracture have been located precisely. A more realistic assumption is that, although events that are closer to the monitoring well are resolved better than those that are further away, there remains some uncertainty.

If the range of each event location \(x_{s,1}\) has been estimated with some error \(\sigma_{R,s}^{\text{ref}}(x_{s,1})\), then that error will propagate to the estimate of \(x_{s,2}\). The resulting range uncertainty follows directly from statistical independence of \(\Delta \tau\) and \(\sigma_{R,s}^{\text{ref}}(x_{s,1})\):

\[ (\sigma_{R,s}^N(x_{s,2}))^2 \approx \frac{(\sigma_{R,s}^{\text{ref}}(x_{s,1}))^2 + V_p^2 (\Delta \tau)^2}{N_s}. \]

(20)

The uncertainty in the transverse direction as a function of the localization errors in the reference fractures and of the errors in estimating the stationary lag can be derived geometrically as was done for the range error and is illustrated in Figure 7. It is given by

\[ (\sigma_{T,s}^N(x_{s,2}))^2 \approx \frac{1}{N_s} \left[ \left(1 + \frac{d(F_1, x_{s,2})}{d(W, F_1)}\right) (\sigma_{T,s}^{\text{ref}}(x_{s,1}))^2 + \frac{\cos^2 \theta_0 d^2(F_1, x_{s,2})}{d^2(W, F_1)} (\Delta z)^2 \right]. \]

(21)

**Understanding uncertainty**

Here, we give a heuristic interpretation of the uncertainty analysis developed in the previous subsections. We first observe that the number of microseismic events associated with a typical fracture is expected to be much larger than the number of receivers in the monitoring well, i.e., \(N_s \gg N_{rec}\). With this assumption, a comparison of equation 18 to equation 19 reveals that the uncertainty in the location obtained with the interferometric method is expected to be smaller than that obtained with the classical method.

![Figure 6](image-url)
The range uncertainty of the interferometric method depends on the error in picking the stationary lag in the correlogram. If the source mechanisms of different events are similar, we can expect very reliable picking in the correlogram. Also, certain types of noise, including additive white noise, are well suppressed by cross-correlation, which makes the proposed method even more robust. On the other hand, the combination of heterogeneous velocity, measurement errors, and differences in source mechanisms are likely to increase the error in recovered locations, both with this method and with classical approaches.

The cross range uncertainty is proportional to the error in picking the stationary receiver depth. For a small number of instruments, this error can be reduced by using interpolation of the correlogram lag times between the recorded depths.

Finally, uncertainty in the velocity model is another factor that will affect the performance of our algorithm. Although we do not address this issue in any detail, we note that the quality of the velocity model is an important factor for the classical method as well. Furthermore, velocity uncertainty between the receiver array and the reference fracture will be largely mitigated with the interferometric method because events in both fractures share much of the path from the first fracture to the observation well.

**NUMERICAL RESULTS**

In this section, we illustrate the performance of the proposed algorithm with a synthetic experiment. We compare the accuracy of the localization by the classical localization algorithm to the improved interferometric one.

The monitoring well is placed vertically at \((x_{\text{rec}}, y_{\text{rec}}) = (0, 0)\) m. Twenty-three-component receivers are placed in the well equidistantly at depths from 2150 to 2450 m (Figure 8a). The model consists of three layers with interfaces at depths 2200 and 2380 m (Figure 8b). The respective velocities are 3500, 3600, and 3700 m/s. Two vertical planar fractures are positioned next to a monitoring well at a depth of 2300 m. The reference fracture is positioned 100 m away from the well, and the second fracture is 200 m away. Both fractures are 300 meters wide and 100 meters tall. Microseisms are simulated by placing 625 sources on a rectangular grid inside the reference fracture (25 in each direction). All source locations in the first fracture are assumed to be known exactly.

For illustration purposes, we put just a single source in the second fracture at \(x_{s,2} = (200, 0, 2300)\) m. The source is a Ricker wavelet (the first derivative of a Gaussian) with a central frequency of 50 Hz. The seismograms are computed using the discrete wavenumber method (Bouchon, 1981) and the reflectivity method (Muller, 1985), and are then contaminated with additive, uncorrelated, Gaussian noise. The S/N, defined as the ratio of the peak amplitude to the standard deviation of the noise, is approximately three in our experiment. An example of a seismogram with and without additive noise is illustrated in Figure 9.

Localization of the event at \(x_{s,2}\) is attempted based on the noisy seismogram using the classical and the proposed method. The workflow is as follows. We generate 200 independent realizations of noisy seismograms. For each noisy realization, we localize the source using the classical approach (equation 2 and plot it as a blue dot in Figure 10. Because the proposed method is unable to improve the estimate of the azimuth, we present results in the horizontal offset-depth domain. The blue dots form a cloud centered around

![Figure 7. A schematic of the uncertainty variables in the transverse direction in the presence of uncertainty in the reference fracture. Knowing the uncertainty in the stationary receiver location \(\Delta z\) and in the location of the reference event, \(\sigma_T(x_{s,1})\), allows the uncertainty in the unknown location, \(\sigma_T(x_{s,2})\), to be computed through a simple geometric calculation. Only one-sided deviations from the true locations \(x_{\text{rec}}^0\) and \(x_{s,1}\) are shown.](image)

![Figure 8. The numerical model contains a monitoring well with two vertical fractures nearby. A source in the more distant fracture is localized using 625 microseismic sources in the nearer fracture.](image)
the true location of the source. The standard deviation of the error in estimated offset of the standard method in this case is approximately 4.5 m. The standard deviation of the depth error is approximately 3.36 m.

We locate the same source with the same geometry using the interferometric method presented here and the microseism locations in the reference fracture. According to the theory, for all events $x_{i1}$ in the reference fracture, we correlate their seismograms with those of the unknown microseism, and we find the stationary condition of the event in the correlogram, which consists of a stationary receiver and a stationary lag.

A ray is traced from the stationary receiver through the event $x_{i1}$, and the location $x_{s1}$ is measured on it using the stationary lag as the traveltime between the two points. The location of $x_{s2}$ can be estimated in a layered medium only up to an unknown azimuth. However, both the horizontal offset and the depth are recovered. We show the offset and depth of $x_{s2}$ so obtained as green stars in Figure 10.

Plotting the interferometric estimates shows a big improvement over the traditional method. The cloud is distributed much closer to the true values. The standard deviation of the error in estimated offset of the interferometric method is 0.52 m, and the standard deviation of the depth error is 0.94 m. Therefore, the improvement in localizing the source is about a factor of 3.6 in depth and nine in offset. Although specific results of this experiment may not translate to other experimental configurations, the superior performance of the interferometric method in this configuration is evident.

CONCLUSIONS

Microseism event localization remains an important and challenging problem. Classical algorithms tend to locate events individually without fully exploiting the coupling and redundancy that exists in the recorded data for multiple fractures. In this paper, we consider a problem with two fractures and a monitoring well. This prototype is typical in hydraulic fracture monitoring applications in which multiple fractures are sequentially created to improve fluid production. When some fractures are known better than others, we propose to use interferometry to image the less well-located fractures relative to those with more accurate locations. We derive our methodology in the context of general velocity heterogeneity and arbitrary well trajectory, and demonstrate its effectiveness on a layered model with a vertical well. Although we present the uncertainty analysis for a homogeneous medium, the method itself is general and can be expected to offer considerable improvement of event localization in much broader contexts. Applying classical interferometry in a 3D medium requires a 2D array of receivers. When the available data are 1D, basic concepts, such as the stationary phase point, are not uniquely defined, and consequently, standard techniques are not applicable. We have shown that for a vertical array of receivers, although the azimuth information is not improved, estimates of both dip angle and distance can be significantly improved using interferometric techniques. Errors present in the data as well as introduced during the crosscorrelogram analysis lead to localization uncertainty. Each event in the reference fracture, however, acts as an independent measurement. We have shown how to use this redundancy to boost the S/N and thus improve the localization.

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Phase-only correlation of time-varying spectral representations of microseismic data for identification of similar seismic events

Hirokazu Moriya

ABSTRACT

Identification of similar seismic events is important for precise estimation of source locations and for evaluation of subsurface structure. Phase-only correlation is well known as a real-time image-matching method for fingerprint identification. I applied the phase-only correlation in a geophysical context to identify similar waveforms among microseismic events. The waveforms were first transformed into time-varying spectral representations to express frequency content in the time-frequency domain. The phase-only correlation function is calculated between two time-varying spectral representations and similarity is evaluated using the peak value of the phase-only correlation function. This method was applied to arbitrarily selected waveforms from aftershocks of an earthquake in Japan to assess its ability to identify similar waveforms perturbed by white noise. The detection of similarity of the proposed algorithm was compared to the similarity as detected by a 2D crosscorrelation function of the time-varying spectral representation and a 1D cross-correlation of the raw waveform. This showed that the phase-only correlation function exhibits a sharp peak that quantifies similarity and dissimilarity over a wide range of signal-to-noise ratio (S/N) and remained unaffected by the length of the time window used to estimate time-varying spectral representations. Phase-only correlation may also have applications in other geophysical analyses and interpretations that are based on waveform and seismic image data.

INTRODUCTION

Image matching and pattern recognition are important tools in several geophysical fields, such as identifying similar seismic events (e.g., Moriya et al., 2002), detecting low-frequency earthquakes within tremor (Brown et al., 2008), seismic pattern recognition (e.g., Johann et al., 1996; Castro de Matos et al., 2007; Marroquín et al., 2009), and in the application of ground-penetrating radar (e.g., Ehret, 2010). The source location for a group of similar seismic events (a multiplet) can be determined with high resolution by using crosscorrelation or cross-spectrum analyses. These techniques are widely used to locate faults (Waldhauser and Ellsworth, 2000; Schaff, 2008) and to determine the orientation of rock fractures in geothermal fields (e.g., Rowe et al., 2002; Moriya et al., 2003) and petroleum fields (e.g., Arrowsmith and Eisner, 2006; De Meersman et al., 2009).

The aim of seismic facies analysis is to identify clusters of similar seismic waveforms (i.e., seismic traces that share similar attributes), where each cluster may represent particular rock properties (e.g., lithology, fluid content, porosity) of the strata being imaged (Marroquin et al., 2009). Cluster analysis is based on pattern recognition, and several pattern recognition algorithms to achieve this have been proposed, such as principal component analysis and neural networks (e.g., West et al., 2002). Crosscorrelation and coherence functions are commonly used to search for similar waveforms. Crosscorrelation techniques in the time domain involve time-consuming convolution of time-series data. Coherence analysis in the frequency domain provides shorter calculation times than analysis in the time domain, but the frequency range and length of the time window for analysis must still be defined. Furthermore, it is difficult to determine an appropriate frequency band because the observed seismic waves follow nonstationary stochastic processes. The selection of a time window of inadequate length or of an inappropriate frequency band will result in some similar seismic events not being identified.

Pattern recognition using neural networks has also been proposed as a method for identification of similar seismic events, but to achieve a stable result, this method requires parameter extraction for the identification of similar waveforms and a specific training data set before it can be applied to unknown data (e.g., Lippmann, 1987; Dorrington and Link, 2004).
Phase-only correlation is used in electrical and communication engineering for image-matching and is well known as a simple and robust technique for evaluating the similarity of two images (e.g., Maltoni et al., 2003; Takita et al., 2003; Ito et al., 2004). The phase-only correlation method deals well with noise and perturbations superimposed on signals (e.g., Ito et al., 2004) and has great potential for use in geophysical exploration, for instance, for identification of similar seismic events, similar seismic traces, and differences in seismic images. In this paper, I propose a new technique to evaluate waveform similarity by using 2D phase-only correlation of time-varying spectral representations of waveforms and demonstrate the feasibility of this technique for identification of similar seismic waveforms.

**METHODOLOGY**

**Phase-only correlation**

Two-dimensional phase-only correlation of time-varying spectral representations of seismic data makes it possible to evaluate in the time-frequency domain the similarity of waveforms for which both the frequency content and signal amplitude change with time. Figure 1 is a flowchart representing the method of calculation of phase-only correlation in this study.

Phase-only correlation is defined by 2D inverse discrete Fourier transform (IDFT) of the combined phase \( \hat{R}(k_1, k_2) \) obtained using 2D discrete Fourier transform (DFT) of two sets of image data, \( f(n_1, n_2) \) and \( g(n_1, n_2) \), as described by the following equations (e.g., Ito et al., 2004).

\[
F(k_1, k_2) = \sum_{n_1=-M_1}^{M_1} \sum_{n_2=-M_2}^{M_2} f(n_1, n_2) W_{N_1}^{k_1 n_1} W_{N_2}^{k_2 n_2} = A_F(k_1, k_2) e^{i\theta_F(k_1, k_2)},
\]

(1)

\[
G(k_1, k_2) = \sum_{n_1=-M_1}^{M_1} \sum_{n_2=-M_2}^{M_2} g(n_1, n_2) W_{N_1}^{k_1 n_1} W_{N_2}^{k_2 n_2} = A_G(k_1, k_2) e^{i\theta_G(k_1, k_2)},
\]

(2)

where \( k_1 = -M_1, \ldots, M_1 \), \( k_2 = -M_2, \ldots, M_2 \), \( W_{N_1} = e^{-i \frac{2\pi}{N_1}} \), \( W_{N_2} = e^{-i \frac{2\pi}{N_2}} \), \( A_F(k_1, k_2) \) and \( A_G(k_1, k_2) \) denote the amplitudes of \( F(k_1, k_2) \) and \( G(k_1, k_2) \), respectively, and \( e^{i\theta_F(k_1, k_2)} \) and \( e^{i\theta_G(k_1, k_2)} \) are their phases. Intervals of the samples, denoted as \( N_1 \) and \( N_2 \), are equivalent to discrete time interval of data sampling in Fourier transform, and \( M_1 \) and \( M_2 \) denote the numbers of samples in the \( n_1 \) and \( n_2 \) directions, and \( M_1 \) and \( M_2 \) are determined and limited by the size of image data.

Then, the function \( \hat{R}(k_1, k_2) \) can be defined as

\[
\hat{R}(k_1, k_2) = \frac{F(k_1, k_2)G(k_1, k_2)}{|F(k_1, k_2)G(k_1, k_2)|} = e^{i\theta(k_1, k_2)},
\]

(3)

where \( G(k_1, k_2) \) denotes the complex conjugate of \( G(k_1, k_2) \) and \( \theta(k_1, k_2) = \theta_F(k_1, k_2) - \theta_G(k_1, k_2) \). The phase-only correlation function can be represented by using inverse 2D IDFT as

\[
\hat{r}(n_1, n_2) = \frac{1}{N_1 N_2} \sum_{k_1=-M_1}^{M_1} \sum_{k_2=-M_2}^{M_2} \hat{R}(k_1, k_2) W_{N_1}^{-k_1 n_1} W_{N_2}^{-k_2 n_2},
\]

(4)

where the value of \( \hat{r}(n_1, n_2) \) has a range from 0 to 1. The peak value of \( \hat{r}(n_1, n_2) \) provides a measure of the similarity of the two images and the position of the peak shows the translational displacement between the two images.

**Identification of similar seismic events with the phase-only correlation function**

The similarity of waveforms can be evaluated by using the phase-only correlation function if the concept of image matching is introduced into the analysis of waveform signals. The observed seismogram is represented by the multiplication of the source function, the transfer function of the wave propagation path, and the receiver function. Then, the time-varying spectral representation can express the characteristics of a seismic wave by resolving the seismic waveform into its time-varying frequency component (e.g., Portnoff, 1980). In this study, I used only the phase of the 2D Fourier transform of time-varying spectral representations calculated from wave signals to evaluate the similarity of waveforms, and I quantified the similarity of waveforms by comparing the shapes of time-varying spectral representations. The time-varying spectral representations of two signals were input to the phase-only correlation algorithm as \( f(n_1, n_2) \) and \( g(n_1, n_2) \) of equations 1 and 2, where \( n_1 \) and \( n_2 \) are equivalent to time and frequency axes. An advantage of analysis in the time-frequency domain is that it deals well with noisy signals. For instance, the observation of spectra is needed to determine frequency band for filtering when we apply a filter in time domain. On the other hand, it is easy to limit the frequency band for analysis by multiplying the time-varying spectral representation by a weighting function, thus improving the S/N and providing a better phase-only correlation and a better estimate of similarity. It is future work to design an optimum weighting function such as a weighting function which maximizes the S/N in the
frequency band. Calculation of the phase-only correlation function over a limited frequency band also sharpens the peak of the function (Ito et al., 2004).

**APPLICATION TO SEISMIC DATA AND RESULTS**

I used repetitive aftershock events that followed an earthquake (M 5) in 1998 on the Nagamachi-Rifu fault in Sendai, Japan. The seismic events were detected by a wide-frequency band, three-component, downhole seismometer (Okada et al., 2001; Moriya, 2009).

Figures 2 and 3 show examples of two seismic events with similar waveforms and their time-varying spectral representations; the similarity was judged by visual inspection. Moving time windows with lengths of 128 points (1.28 s) (Hamming window) were used to calculate the time-varying spectral representations, and the window lengths for fast Fourier transform (FFT) were 256 points (2.56 s), with zeros inserted in the time-series data from points 129 to 256. This operation produces smoothed time-varying spectral representation. The frequency resolution was 0.39 Hz with a time-sampling frequency of 100 Hz. The time-step interval for the moving time window was 0.28 s, which corresponds to about 20% of the time window; thus the number of samples for each segment overlapped by 100 points (1.0 s). Consequently, time-varying spectral representations were obtained in 129 × 31 matrices. The above parameters were used to calculate the time-varying spectral representations of all events considered here.

Each time-varying spectral representation was regarded as 2D image data, and the phase-only correlation function was calculated from the two time-varying spectral representations shown in Figures 2b and 3b. Figure 4 shows the phase-only correlation function, 2D crosscorrelation coefficient, and 1D crosscorrelation coefficient, where the crosscorrelation coefficients are given by the crosscorrelation functions normalized by autocorrelations at a time lag of zero, calculated using the time-varying spectral representations and signals shown in Figures 2 and 3. Two-dimensional DFT was used to calculate the 2D Fourier transforms of time-varying spectral representations \(f(n_1, n_2)\) and \(g(n_1, n_2)\), where \(M_1 = 64\) and \(M_2 = 15\), respectively, and the 2D Fourier transforms were calculated using the data from \(-M_1\) to \(M_1\) and \(-M_2\) to \(M_2\). The spectra \(F(k_1, k_2)\) and \(G(k_1, k_2)\) were multiplied and normalized, and 2D IDFT was applied to \(\hat{R}(k_1, k_2)\) to obtain \(\hat{r}(n_1, n_2)\). The \(\hat{r}(n_1, n_2)\) represents the crosscorrelation between \(f(n_1, n_2)\) and \(g(n_1, n_2)\), where the two time-varying spectral representations were relatively shifted along their axes by \(n_1\) and \(n_2\) sample points.

The phase-only correlation function \(\hat{r}(n_1, n_2)\) (Figure 4a) has a sharp peak of amplitude 0.72, and it is clear that the two waveforms have a strong crosscorrelation. The 2D crosscorrelation coefficient (Figure 4b) has multiple blunt peaks aligned parallel to the \(n_2\) axis, and the gradient of the curved surface is gentler parallel to the \(n_1\) axis, which corresponds to the time axis (Figure 4b). The 1D crosscorrelation coefficient (Figure 4c) has a sharp peak at a time lag of 0.1 s, corresponding to the relative difference of the arrival times of the original waveforms.

Figure 5 shows a waveform that is dissimilar to those of Figures 2 and 3. Figure 6a shows the phase-only correlation function calculated using the time-varying spectral representation images of the waveforms shown in Figures 2 and 5. The distinct peak of the phase-only correlation function \(\hat{r}(n_1, n_2)\) for the dissimilar waveforms (Figure 5a) is not evident in Figure 6a, which implies that the two waveforms have weak similarity. The 2D crosscorrelation coefficient (Figure 6b) has a curved surface with a gentle slope and broad range, but the peak value is not markedly lower than that of the function for the similar waveforms (Figure 4b), even though the two waveforms are less similar. The 2D crosscorrelation coefficient was calculated by normalizing the IDFT of \(F(k_1, k_2)G(k_1, k_2)\), where the hamming window was used to suppress the side lobe of spectra \(F(k_1, k_2)\) and \(G(k_1, k_2)\). The 1D crosscorrelation coefficient (Figure 6c) does not show a peak that would indicate a strong correlation. For the dissimilar waveforms, the peak value of the
phase-only correlation function is 0.27 (Figure 6a), considerably lower than the peak value of 0.72 for the similar waveforms (Figure 4a). The peak value of the 2D crosscorrelation coefficient for the dissimilar waveforms (Figure 6b) is 0.67, compared to 0.96 for the similar waveforms (Figure 4b). The peak value of the 1D crosscorrelation coefficient for the dissimilar waveforms (Figure 6c) is 0.27, compared to 0.96 for the similar waveforms (Figure 4c).

To evaluate the effect of S/N, the phase-only correlation function was recalculated three times using the original waveform (Figure 2) and the same waveform with different levels of band-limited white noise added (Figure 7). The noise was derived from computer-generated random numbers. Figures 8, 9, and 10 show the resultant phase-only correlation functions and the 2D and 1D crosscorrelation coefficients, respectively. Without added noise (Figure 8a), the phase-only correlation function approximates a delta function \( \delta(n_1, n_2) \) with a peak value of 1.0. The shape of the correlation function is sharp with a clearly defined maximum. As the amount of noise added increases, the peak value decreases (Figure 8b, 8c, and 8d). The peak values of the 2D and 1D crosscorrelation coefficients also decrease with increasing added noise (Figures 9 and 10).

To examine the peak values of the three correlation methods as a function of S/N, I applied the three correlation methods to the waveform of Figure 2 and the same waveform with different amounts of added noise (Figure 11a), and to the waveform of Figure 2 and the dissimilar waveform of Figure 5 (Figure 11b) with different amounts of added noise. In the calculation, the band-limited white noise (<27 Hz) was added only to one side of the waveforms (slave waveforms shown in Figures 2a and Figure 5a) and not to the master waveform (Figure 2a).

When we evaluated by using the two similar waveforms (Figure 11a) and two waveforms differ only because noise has been added to one of them, the peaks of both the phase-only correlation function and the 2D crosscorrelation coefficient ranged from 0.4 to 1.0, and those of the 1D crosscorrelation coefficient ranged from 0.3 to 1.0. Both the phase-only correlation function and the 2D crosscorrelation coefficient produced higher peak values than the 1D crosscorrelation coefficient, even though the waveforms were polluted by superimposed noise. For the dissimilar waveforms (Figure 11b), the peak values also decreased with increasing noise and produced lower peak values than those of the similar waveforms (Figure 11a). The peak 2D crosscorrelation coefficients for the dissimilar waveforms were around 0.8, indicating weaker similarity than that of the similar waveforms. The peak values for the two other methods were below 0.3.

The results suggest that the phase-only correlation function shows stable and higher peak values for similar waveforms over a wide range of S/N although the waveforms were polluted by noise, and that the function shows stable and lower values for dissimilar waveforms as well as the 1D crosscorrelation coefficient. The 2D crosscorrelation coefficient shows stable and higher peak values for similar waveforms, but the 2D crosscorrelation coefficient have high peak values even for dissimilar waveforms and is unstable at low S/N. Therefore, the results suggest that the technique on phase-only correlation using the time-varying spectral representation gives a better estimate of similarity.
On the other hand, the maximum values are approaching a value when the S/N exceeds a value in Figure 11b. This phenomenon was also observed when the different waveforms were used for evaluation. In the case of Figure 11b, two waveforms shown in Figures 2 and 5 had a weak correlation even though they were judged as dissimilar waveforms by visual inspection. Therefore, the reason for the phenomenon is that the given two waveforms had a weak cross-correlation and the peak values approach a value when S/N exceeds a value.

To assess the effectiveness of the three methods used here over a broader data set, I randomly selected 30 events from the seismic data catalog and visually inspected their waveforms before applying separate phase-only correlation and 2D and 1D cross-correlations for the three orthogonal components of seismic signal for all possible pairs of waveforms within the selected data set. The relationships of the peak values of phase-only correlation with those of both 2D and 1D cross-correlations are shown in Figure 12. The peak values for each component differ slightly because both the shape of the waveform and the S/N are different for each component. The phase-only correlation function showed comparable peak values to those of the 2D cross-correlation coefficient and higher peak values than those of the 1D cross-correlation coefficient. The relationships of the means of the peak values of the three components (over components) in Figure 12 are shown in Figure 13.

The pair of waveforms with high peak value has similar waveforms as shown in Figure 13 (pair No. 1), and we can confirm that the pair of similar seismic events can be identified by the presented method. On the other hand, it is difficult to judge which pair (pairs No. 2–6) is more similar or less similar by visual inspection because the judgment depends on the person who visually inspects the similarity and dissimilarity.

For another example, I selected 15 seismic events from the seismic data catalog referring to the source locations, where the waveforms were not inspected before the analysis. Figures 14 and 15 show the relationships of the peak values of phase-only correlation with those of both 2D and 1D cross-correlation coefficients and the waveforms. The six events with similar waveforms in Figure 15 have adjacent source locations (<1 km), and they are considered to be the seismic events on the same fault plane. The plotted points in Figure 14 are clearly divided into two groups (groups A and B). The group with higher values denotes the data by the seismic events with adjacent source locations (group A), and the peak values of phase-only correlation function here has a broad range from 0.4 to 0.85, compared to the peak values of the 2D cross-correlation coefficient and the 1D cross-correlation coefficient. This result implies that the phase-only correlation function has better performance to distinguish the seismic events with relatively higher cross-correlation (more similar waveforms), and that the 2D and 1D cross-correlation coefficient in this paper are better to distinguish the seismic events with relatively lower cross-correlation (less similar waveforms).

**DISCUSSION**

The phase-only correlation method deals with waveforms as images and evaluates their similarity by using phase-only spectra.
of time-varying spectral representations. Therefore, the concept of time in waveform data is not much important in the calculation of \( \hat{r}(n_1, n_2) \), and the phase-only correlation function is not influenced greatly by time shifts in the signals. For instance, the phase-only correlation function was calculated using the waveforms in Figure 2a and Figure 3a, where a time shift of 1 s was intentionally applied to the waveform of Figure 2a, the peak value of the phase-only correlation function was 0.69, compared to a peak value of 0.72 without the time shift (Figure 4a). This result shows that it is also unnecessary to align the first arrivals of waveforms before applying the phase-only correlation as well as other two methods.

A comparison of the peak values from phase-only correlation functions and 2D crosscorrelation coefficients calculated for the similar waveforms shown in Figures 2 and 3 for time moving window lengths from 1.28 to 5.12 s in the calculation of the time-varying representations is shown in Figure 16. The data length for FFT was fixed at 512 points (5.12 s). For all window lengths, the high peak values of the phase-only correlation were better maintained. On the other hand, the peak value of 2D crosscorrelation function decreases with the degradation of time-varying spectral representation. The result suggests that the phase-only correlation function evaluates the similarity of time-varying spectral representations.
Figure 11. Maximum values of the phase-only correlation function, 2D crosscorrelation coefficient, and 1D crosscorrelation coefficient as a function of S/N. (a) Waveform shown in Figure 2 and (b) dissimilar waveforms shown in Figures 2 and 5. The frequency range for analyzes was band-limited to 15–27 Hz.

Figure 12. (a) Maximum value of the phase-only correlation (POC) function versus the maximum value of the 2D crosscorrelation coefficient and (b) the maximum value of the phase-only correlation function versus the maximum value of the 1D crosscorrelation coefficient. The results obtained for the three components of the seismic signal are shown. The value of auto-correlation is not plotted.

Figure 13. (a) Maximum value of the phase-only correlation function versus the maximum value of the 2D crosscorrelation coefficient and (b) the maximum value of phase-only correlation function versus the maximum value of the 1D crosscorrelation coefficient. The mean values of the three signal components shown in Figure 12 were used. The waveforms below show the pair of events, and the numbers in the figures indicates the position of plotted data.

Figure 14. (a) Maximum value of the phase-only correlation function versus the maximum value of the 2D crosscorrelation coefficient and (b) the maximum value of phase-only correlation function versus the maximum value of the 1D crosscorrelation coefficient. A total of 15 seismic events which were selected by referring the source locations are analyzed.

Figure 15. The waveforms of the seismic events which were selected by referring the source locations. The six events with similar waveforms have adjacent source locations and are considered to be the seismic events on the same fault plane. One of the waveforms is the noise.
representations although the time-varying spectral representation is degraded due to the short time window for spectral estimation. It is known that the phase-only correlation function is robust against the degradation of images (Ito et al., 2004).

For visually inspected similar waveforms, the stable and high peak values of phase-only correlation functions applied to time-varying spectral representations of those waveforms are maintained over a wide range of S/N (Figure 11) because analysis in the time-frequency domain decomposes waves into frequency components as a function of time, and the frequency range dominated by noise can be excluded, where the frequency components lower than 15 Hz and higher than 27 Hz are excluded by multiplying the weighting function and the time-varying spectral representations. Thus, application of the phase-only correlation function is robust for waveform data degraded by noise. For visually inspected dissimilar waveforms, the phase-only correlation function and 1D crosscorrelation coefficient produced stable and low peak values over a wide range of S/N (Figure 11b). The 2D crosscorrelation coefficient tends to produce high peaks even for dissimilar waveforms and the values are unstable at lower S/N.

Considering the results, the phase-only correlation function will provide a better estimate of waveform similarity when we search for similar waveforms.

CONCLUSIONS

In this paper, I have proposed phase-only correlation of time-varying spectral representations of waveforms as a new technique to identify similar microseismic events, and I tested its application to earthquake aftershock events. The waveforms have been transformed into time-varying spectral representations to express frequency content in the time-frequency domain, and the phase-only correlation function has been calculated between two time-varying spectral representations. Similarity has been evaluated using the peak value of the phase-only correlation function. The presented method has been applied to arbitrarily selected and visually inspected waveforms from aftershocks of an earthquake. I assessed its ability to identify similar waveforms perturbed by white noise and compared the results with those obtained using both a 2D crosscorrelation function of the time-varying spectral representation and a 1D crosscorrelation function of the raw waveform. It has been shown that the phase-only correlation method shows stable and higher peak values for similar waveforms although the waveforms are polluted by superimposed noise and S/N is low, and that the method shows stable and low peak values over a wide range of S/N for dissimilar waveforms as well as the 1D crosscorrelation coefficient. The phase-only correlation method has also been applied to the aftershock events selected by referring the source locations, and it has been shown that the seismic events with higher peak values of phase-only correlation have similar waveforms and the phase-only correlation function has better performance to distinguish the seismic events when we search for waveforms with relatively high crosscorrelation. Linkage of the phase-only correlation method with cluster analysis will further enhance its application for identification and classification of waveforms. The phase-only correlation may also have applications in other geophysical analyses and interpretations (e.g., facies analysis) that are based on waveform and seismic image data.

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Magnitude estimation for microseismicity induced during the KTB 2004/2005 injection experiment

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ABSTRACT

We determined the magnitudes of 2540 microseismic events measured at one single 3C borehole geophone at the German Deep Drilling Site (KTB) during the injection phase 2004/2005. For this task we developed a three-step approach. First, we estimated local magnitudes of 104 larger events with a standard method based on amplitude measurements at near-surface stations. Second, we investigated a series of parameters to characterize the size of these events using the seismograms of the borehole sensor, and we compared them statistically with the local magnitudes. Third, we extrapolated the regression curve to obtain the magnitudes of 2436 events that were only measured at the borehole geophone. This method improved the magnitude of completeness for the KTB data set by more than one order down to \( M = -2.75 \). The resulting \( b \)-value for all events was 0.78, which is similar to the \( b \)-value obtained from taking only the greater events with standard local magnitude estimation from near-surface stations, \( b = 0.86 \). The more complete magnitude catalog was required to study the magnitude distribution with time and to characterize the seismotectonic state of the KTB injection site. The event distribution with time was consistent with prediction from theory assuming pore pressure diffusion as the underlying mechanism to trigger the events. The value we obtained for the seismogenic index of \( -4 \) suggested that the seismic hazard potential at the KTB site is comparatively low.

INTRODUCTION

Seismic hazard due to fluid injection has become an important issue in recent years. Magnitudes are crucial for the evaluation of seismic hazards. The main objective of this paper is to present a workflow for the magnitude determination with data recorded at only one borehole geophone. We use microseismic data from the hydraulic experiment 2002–2005 at the German Deep Drilling Site (KTB) (Shapiro et al., 2006). The experiment was divided into three phases. It started with a one year extraction phase, in which 22,300 m³ of water were extracted (Graesle et al., 2006). Then, a one year recovery phase followed. Finally, during a 250-day injection period, 84,600 m³ of fresh water was injected at a constant rate of 200 l/min into the open bottom section of the KTB pilot hole at a 4.0 km depth. More than 2,500 microearthquakes were detected by a 3C, 15 Hz borehole geophone, which was installed in the KTB main hole at a horizontal distance of ~250 m from the injection source. The geophone was taken out from the borehole three times during the injection phase. It was clamped to the borehole wall at changing depths of 1950, 3476, 3485, and 3500 m at temperatures up to 120 °C. Due to the absence of calibration shots, the orientation of the horizontal components of the geophone could not be determined. Furthermore, time-variant coupling and instrument gain, presumably due to long-term temperature-dependent alterations of the electronic components of the recording system, did not allow reliable estimates of the seismic moment from the low frequency level of the amplitude spectrum. Also, corner frequencies are above the Nyquist frequency for most of the small events, and a scaling relation between the corner frequency and the seismic moment could not be applied to estimate the seismic moment. All of these limitations preclude the application of conventional techniques for magnitude estimation. Instead, we compared different...
attributes of the waveforms of the borehole geophone to identify a reliable one that is independent of coupling and orientation of the 3C geophone, and that is proportional to the magnitude. Given these parameters, we investigated (a) signal duration similar to the one Real and Teng (1973) used, (b) signal-to-noise spectral ratio (Abercrombie, 1995), (c) signal integral, and (d) the absolute amplitude computed from the three components. Below, we first estimate the local magnitudes with a standard, amplitude based method. Then, these parameters are evaluated. We search the best parameter for extrapolating the local magnitudes to all events that were recorded by the borehole geophone. One objective of magnitude determination is the estimation of the seismic hazard potential. We calculate the seismogenic index (Shapiro et al., 2010), and we analyze the magnitude distribution versus time and test the hypothesis that the magnitude distribution can be explained by pressure-diffusion triggering (Shapiro and Dinske, 2009).

**MAGNITUDE ESTIMATION TECHNIQUE**

**Magnitudes measured at the near-surface seismometers**

We first determined local magnitudes, $M_l$, for the events which were also recorded by a near-surface seismic network. The local magnitude $M_l$ was used instead of the moment magnitude $M_w$ because $M_l$ was calibrated locally by the Bavarian network, and the simultaneous recording of one local event by the Bavarian network and our KTB network provided us with a reference value for the determination of the local magnitude $M_l$. The equation used is a modification of the original formula by Bakun and Joyner (1984)

$$M_l = \log(A) + \log\left(\frac{\Delta}{100}\right) + 0.00301 \left(\Delta - 100\right) + 3.0 \quad (1)$$

(J. Wassermann, personal communication, 2008), where $A$ is the maximum amplitude of the P-phase from zero to peak in $10^{-6}$ m and $\Delta$ the distance in kilometers. The calculated magnitude from the Bavarian local network for the local earthquake is $M_l$ 2.6, and the magnitude we obtained from the KTB network is $M_l$ 2.52, which is in good agreement. We used five stations within a radius of about 3 km from the KTB main hole (Figure 1).

Only 5% of all the detected events were recorded by the near-surface stations. One hundred and four events could be used for the estimation of local magnitudes. The range of the resulting $M_l$ values for the 104 induced events is $-1.8 < M_l < 0.0$.

**Identifying the best magnitude-related parameter for the borehole geophone**

Using a parameter which is more or less independent of the orientation of the borehole 3C geophone is important because

![Figure 1. Location of the KTB site and distribution of the five near-surface stations used in this study. The cross marks the KTB injection point.](image)
the geophone was taken out and put back three times during the experiment. The parameter must, therefore, also be independent of the coupling of single components. We compared seven parameters, which were calculated from the recorded seismograms. To identify the best parameter we used the statistical significance between the local magnitudes of the 104 events measured at the near-surface seismometers and each of the parameters obtained from the borehole seismograms.

**Description of the parameters**

In total we compared seven different parameters from the data of the borehole geophone. The borehole sensor has a sample rate of 1000 Hz.

**Amplitude**

One of the parameters tested is the amplitude of the three components. Each amplitude value is calculated by

\[
S(i) = \sqrt{S_{H1}(i)^2 + S_{H2}(i)^2 + S_Z(i)^2}
\]

where \(S_{H1}\) and \(S_{H2}\) are the amplitudes of the horizontal components of the borehole geophone, and \(S_Z\) is the amplitude of the vertical component. The resulting maximum amplitude of the S-phase is then multiplied by a spherical divergence correction factor. This is done by multiplying the measured amplitude with the distance \(d\). The distance \(d\) was estimated assuming a P-wave velocity \(V_p = 6.0\) km/s and a ratio \(V_p/V_s = 1.72\). These are adequate values for the depth of \(h \approx 3.5\) km at KTB. Thus, we have

\[
t_s - t_p = d \times \left(\frac{1}{V_s} - \frac{1}{V_p}\right) \rightarrow d \approx (t_s - t_p) \times \frac{8}{V_s} \text{ km/s} \tag{2}
\]

**Duration**

Four of the seven parameters tested are different definitions of duration. The duration that correlated best with the local magnitudes is very similar to the one that Real and Teng (1973) used. Real and Teng (1973) introduced a duration definition where the duration is the time interval between the onset of the first arrival and the point at which the seismogram amplitude falls and remains below the background noise level. The duration value that we used is measured on an envelope function that is based on the absolute value of the velocity seismogram, which was built from all three components. The smoothing of this function, to obtain the envelope, was achieved by averaging 70 values. This is short (0.07 s) in comparison to the duration (Figure 2b). The duration is the time difference between the S-phase onset and the time when the envelope drops back to two times the noise level. We used the S-phase because the distance to the events is very short; therefore, the signal of the P-wave in some cases did not fall below two times the noise level before the S-phase (see Figure 2b). Two times the noise level was proven to be more stable compared to the duration definition by Real and Teng (1973), who used just the noise level to mark the end of the duration.

**Spectrum**

We could not use corner frequencies for magnitude calculation because they are above the Nyquist frequency for most of the microearthquakes. From Madariaga (1976) we derive that for the circular fault model with stress drop \(\Delta \sigma = 1\) MPa and S-wave velocity \(\beta = 3000\) m/s, a moment magnitude of \(M_w = 0\) has a corner frequency of about 200 Hz and \(M_w = -1\) has a \(f_c \sim 700\) Hz, which is beyond the Nyquist frequency of our system (500 Hz). Therefore, we calculated the spectrum of the noise as well as the spectrum of the S-phase signal for the same window length of one second (Abercrombie, 1995), at each single component. Then the three components of the signal spectra and the components of the noise spectra were added to a composite signal and a noise spectrum, respectively. Then the integral of the signal spectrum and the integral of the noise spectrum were determined for the frequency range of 60–450 Hz. The difference between those two integrals is the resulting parameter difference of the integral of the amplitude spectra (DIAS). Most of the signal energy content lies below 200 Hz (Figure 3b).

**Time integral**

Another parameter that we compared is the time integral of the absolute values of the S-phase signal (Figure 4). The samples of the three components were added, as mentioned previously, for the amplitudes. The integration was done for the duration time interval. We multiplied the time integral by the spherical divergence correction factor.
Evaluation of the best parameter

To find the best parameter from the borehole sensor of the seismograms, we correlated the different parameters with the local magnitudes that we had obtained for the larger events. Tests showed that the data from the four different installation periods could be integrated to obtain a single relation valid for all recorded microseismic events. The correlation coefficient \( r \) was calculated by the formula of Pearson

\[
 r = \frac{\sum_{i=1}^{N} X_i Y_i - N \bar{X} \bar{Y}}{\sqrt{\left(\sum_{i=1}^{N} X_i^2 - N \bar{X}^2\right) \left(\sum_{i=1}^{N} Y_i^2 - N \bar{Y}^2\right)}} , \tag{3}
\]

where \( \bar{X} \) is the mean of the magnitudes, and \( \bar{Y} \) is the mean of the respective parameter and \( N \) the number of events (\( N = 104 \)).

The statistical significance is affirmed when a certain likelihood of coherence of the two parameters is exceeded. We took the \( \alpha = 0.05 \) likelihood as a useful level. No correlation exists if a critical \( F \)-value of Fisher’s \( F \) distribution is below a certain limit. For about 120 samples in a table like Bortz (1999) a relation with likelihood of 95% cannot be considered significant when the critical \( F \)-value is below 3.07. The parameter \( F_b \) indicates the critical \( F \)-value of the slope \( b \) of a bivariate correlation, \( f(x) = a + bx \).

The logarithm of all our tested parameters exceeds the critical \( F_b \)-value (Table 1), and the parameters correlation with the local magnitudes can be considered significant. The best parameter fitting the local magnitudes is the time integral of the absolute value of the signal. The statistical significance, as well as the correlation coefficient, are highest for this parameter (Table 1). We conclude that the time integral is the most robust parameter, and it is least sensitive to the uncertainties related to the geophone coupling and the long-term negative temperature effects on the instrument gain.

Extrapolation of magnitudes

Using the orthogonal regression, the time integral values of the borehole geophone were fitted logarithmically with the local magnitudes of the near-surface stations (Figure 5). The linear regression curve was then extrapolated to determine the magnitudes of all events that were not detected by the near-surface stations (Figure 5). This allows us to estimate the magnitudes for 2436 events, which were only recorded at the borehole geophone, with help of the parameter “time integral of the absolute value of the signal.”

RESULTS

Magnitude Distribution

The resulting magnitude distribution is shown in Figure 6a. The extrapolated magnitudes are shown as discrete magnitudes in specific intervals, as well as dots showing the number of events greater than a given magnitude. The 104 magnitudes that have been calculated from the maximum amplitudes of the near-surface seismometers are included in Figure 6a. The magnitude of completeness is \( M_c \approx 2.5 \), and the \( b \)-value is \( b \approx 0.78 \), which is similar to the \( b \)-value of the local magnitudes in Figure 6b. The \( b \)-value remained constant throughout the experiment.

Table 1. Overview of correlations \( r \) and coherency \( F_b \) of the logarithm of different attributes of the borehole geophone recordings with local magnitudes from near-surface stations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( r )</th>
<th>( F_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>time integral of signal</td>
<td>0.65</td>
<td>193</td>
</tr>
<tr>
<td>difference of S/N spectrum</td>
<td>0.62</td>
<td>165</td>
</tr>
<tr>
<td>duration</td>
<td>0.53</td>
<td>112</td>
</tr>
<tr>
<td>amplitude</td>
<td>0.44</td>
<td>80</td>
</tr>
</tbody>
</table>
Distribution of magnitudes versus time

The extrapolated magnitudes and $b$-values can be used to test the recently developed model of events against the distribution in time (Shapiro et al., 2007) and to estimate the seismogenic index (Shapiro et al., 2010) at the KTB location. A basic feature of the event distribution versus time is that the logarithm of the cumulative number of induced events greater than a given magnitude rises proportionally with the logarithm of time $t$

$$\log N_{>M}(t) = \log \frac{4\pi p_0 R D t}{F_t} - bM + a \quad (4)$$

in the case of a constant injection pressure $p_0$, where $R$ is the radius of a spherical surface effectively representing hydraulic diffusivity $D$, $F_t = C_{\max}/N_c$ is the tectonic potential of the region, with the maximum possible critical pressure of preexisting cracks $C_{\max}$, the concentration of cracks $N_c$ and the constants $a$ and $b$ of the Gutenberg-Richter-Relation (Shapiro et al., 2007). In a bilogarithmic plot of $N(t)$, the slope is equal to one. A more general formula was introduced by Shapiro and Dinske (2009)

$$N_{>M}(t) = W_{>M} \frac{Q_c(t)}{S F_t}. \quad (5)$$

The cumulative injected volume is $Q_c(t)$, $S$ is a poroelastic compliance related to porosity, and $W_{>M}$ is the probability of an event greater than a given magnitude $M$ (Shapiro and Dinske, 2009). For the KTB2004/2005 experiment, $Q_c(t)$ is proportional to time, $Q_c(t) = Q \cdot t$, with a constant injection rate of $Q = 200$ l/min.

Figure 7 shows the bilogarithmic plot for our KTB case study. The observed relation is linear, in correspondence with the theory (Equation 5). The slope is $m = 1.65$ for events with $M_l > -1.5$. We also compared the observed number of events with the relation predicted from numerical modeling. For this, we constructed a homogenous background model with a diffusivity value of 0.002 m$^2$/s, which is intersected by a layer of increased diffusivity (0.002 m$^2$/s)$\sim$300 m from the injection point. This layer represents the SE2 fault system, which is also the seismically active zone during the injection experiment (Shapiro et al., 2006). In the model, the range of criticality values in the fault system was between $c_{\min} = 0$ MPa and $c_{\max} = 0.8$ MPa, whereas the criticality in the seismically not-activated surrounding area was set higher than the maximum injection pressure ($\sim$12 MPa). The dashed line in Figure 7 is the resulting model with a slope of about 1.74, which is in good agreement with the measured data.

Figure 5. The time integral of the absolute velocity seismogram of the S-phase signal is plotted against the local magnitudes, which have been determined by the near-surface stations. The orthogonal regression curve is $y = (5.2 \pm 0.044) + (0.72 \pm 0.038) \cdot x$. The dashed lines illustrate the 95% confidence intervals for intercept and slope. The event with local magnitude of 2.52 is the regional event that has been recorded; $r$ is the correlation coefficient, $n$ is the number of events, and $F_b$ is the statistical significance.

Figure 6. Shown are the $b$-values from the slope of the cumulative number of events greater than a certain magnitude; bars indicate a discrete number of events within a certain magnitude range. (a) All recorded events. (b) Events recorded only at the near-surface stations.
Seismogenic index

The seismogenic index, introduced by Shapiro et al. (2010), is independent of injection parameters and only depends on the seismotectonic features of the injection area (e.g., $C_{\text{max}}$ and $F_i$). Along with injection parameters one can estimate the occurrence probability of induced events with a given magnitude during the injection phase. The seismogenic index $\Sigma$ can be calculated as following

$$\Sigma = \log(N_M) - \log(Q_c(t)) + bM$$  \hspace{1cm} (6)

where $N_M$ is the number of events greater than a given magnitude $M$, $Q_c$ is the cumulative injected fluid volume and $b$ the value obtained by the Gutenberg-Richter Relation \(\log(N) = a - bM\). Rearranging equation 6 to

$$\log(N_M) = \log(Q_c(t)) - bm + \Sigma$$  \hspace{1cm} (7)

reveals that the seismogenic index can be used to predict the number of events induced during a subsequent experiment at the same injection site, assuming that the $b$ value and $\Sigma$ do not change with time.

Figure 8 shows the resulting seismogenic index for the KTB 2004/2005 injection experiment, as well as for five other injection experiments published by Shapiro et al. (2010). Except the Barnett Shale case study, which is a hydraulic fracture location in a marine shelf deposit, the data shown are from injections into enhanced geothermal systems and brine deposits (Paradox Valley). The seismogenic index at KTB ($\Sigma = -4$) belongs to the range of lowest ones estimated for fluid induced seismicity in hard rock.

CONCLUSION

We tested different parameters to determine the magnitudes of 2540 microevents of the KTB 2004/2005 injection experiment, where most events were detected by a single borehole geophone. The most suitable parameter is the time integral of the absolute value of the seismogram trace. The magnitudes were calibrated against local magnitudes of a subset of 104 larger events, which were determined by a near-surface seismic network. By extrapolation, the magnitude of completeness $M_c$ has been improved by more than one order of magnitude from $Ml \approx -1.5$ to $Ml \approx -2.75$ (compare Figures 6a and 6b). The significantly increased number and spread of the magnitude values allowed us to apply recently developed concepts of magnitude distribution for induced seismicity. We found that the slope in the bilogarithmic plot of the number of events, with a magnitude greater than a given one versus time, is greater than one. This is a consequence of the hydraulic record at KTB (including a long-term extraction phase prior to the injection phase) and the existence of the SE2 fault system nearby. The seismogenic index is low in comparison to other fluid injection experiments (Figure 8), which is consistent with the absence of larger events in spite of the long duration of the injection (almost 1 year). The area of the KTB injection can, therefore, be classified as an area, where the probability to induce significant earthquakes is comparably low.

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Induced seismicity magnitudes at KTB


Magnitudes of induced earthquakes and geometric scales of fluid-stimulated rock volumes

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ABSTRACT

Sometimes rock stimulations by fluid injections into geothermal boreholes are able to trigger perceptible or even potentially damaging earthquakes. This does not seem to be the case for hydraulic fracturing of hydrocarbon reservoirs. Reasons for such a difference and factors defining magnitudes of induced earthquakes (triggered tectonically as well as induced artificially) remain unclear. We analyzed microseismic data obtained by fluid stimulations at different geothermal and hydrocarbon sites. This analysis indicates that a rupture corresponding to a fluid-induced earthquake seems to be only probable along a surface located mainly inside a stimulated rock volume. We approximated the stimulated volume by an ellipsoid, and compared the statistics of induced events with the statistics of randomly distributed thin flat disks modeling rupture surfaces. We found that one of the main factors limiting the probability to induce a large-magnitude event is the minimum principal axis of a fluid-stimulated rock volume. This geometrical scale can be very different in geothermal and hydrocarbon reservoirs. It may control the order of a largest possible magnitude. We quantified an impact of the geometry of a stimulated volume on the Gutenberg-Richter-type frequency-magnitude distribution of induced earthquakes. Our results show that monitoring the spatial growth of seismicity in real time can help to constrain a risk of inducing damaging earthquakes.

INTRODUCTION

Perturbations in pore fluid pressure can induce aftershocks of earthquakes (Nur and Booker, 1972; Jonsson et al., 2003; Miller et al., 2004). Earthquake triggering by fluids results from changing the pore pressure and poroelastic stress (McGarr et al., 2002, Cornet et al., 2007). Pore pressure perturbations can decrease effective normal stresses and cause a slip along interfaces in rocks at locations where a friction coupling closely approaches tangential stresses. The spatial scale of seismicity induced by fluid injections in boreholes is usually of the order of $10^2$ to $10^3$ meters. It can be considered as an analog (a model in situ) of large-scale fluid-related tectonic seismic processes, and thus can be of significance for their understanding. Its better understanding is also of importance for mining of deep geothermic energy (Cornet et al., 2007; Majer et al., 2007; Häring et al., 2008; Giardini, 2009). It is also of significance for CO₂ underground storage and oil and gas production.

Earthquakes of magnitudes three to four occurred at several enhanced geothermal systems (EGS) like those of Basel, Cooper Basin, The Geysers field, and Soultz (Majer et al., 2007; Dyer et al., 2008; Häring et al., 2008; Baisch et al., 2009; Giardini, 2009). In contrast, such events seem to be rarely possible by hydraulic fracturing of oil and gas reservoirs. To our knowledge, they have not yet been observed under such circumstances (Shapiro et al., 2010).

Understanding of this difference in the behavior of the induced seismicity would have importance for constraining a hazard of damaging induced earthquakes. Here we show that the geometry of rock volumes stimulated by fluids is a plausible controlling factor for the occurrence of large events. We quantify this influence of the geometry.

In the next section we analyze seismicity induced by fluid stimulations at different geothermal and hydrocarbon reservoirs. The data indicate that the large-magnitude events have different statistical features than small events. Simple observations suggest that the size of a stimulated volume restricts the occurrence probability of strong events. In the following section we propose a theoretical and numerical model to aid in understanding the role of the geometry. Our consideration is based on computing the statistics of arbitrary-size arbitrary-oriented penny-shaped inclusions (representing potential rupture planes) in a finite ellipsoidal stimulated rock volume. We
show how the geometry of the stimulated volume influences the frequency-magnitude relation of induced seismicity. We conclude that the minimum principal axis of a fluid-stimulated rock volume seems to be a dominant restricting factor. Further we discuss this result and compare it to observations.

**OBSERVATIONS**

During an active fluid injection with nondecreasing injection pressure, the following equation (Shapiro et al., 2007, 2010; Shapiro and Dinske, 2009) can be applied to approximately describe the number \( N_M \) of induced earthquakes with magnitudes larger than \( M \) as function of the injection time \( t \):

\[
\log_{10} N_M(t) = \log_{10} Q(t) - bM + \Sigma, \tag{1}
\]

where \( Q(t) \) is a cumulative volume of the fluid injected until time \( t \), \( b \) is the \( b \)-value of the Gutenberg-Richter frequency-magnitude distribution law (Turcotte et al., 2007) stating that a probability of a magnitude larger than \( M \) is equal to \( 10^{a-bM} \) (the quantity \( a \) is defined so that \( 10^0 \) is a probability of a magnitude larger than zero).

Immediately after the injection termination the curve \( Q(t) \) starts to decrease because of an outflow of the injected water. In a bilogarithmic scale, the theoretical estimate of \( N_M(t) \) (equation 1) is obtained by a time-independent shifting of the injected fluid volume toward the curve of the cumulative event number (equation 1 is applicable for injection periods only). The numerical curve is obtained by a point-like source of a fluid with the injection rate corresponding to the Basel experiment. Then we compare the obtained pore pressure perturbation with randomly distributed critical values of the pore pressure assumed to be necessary for triggering earthquakes at corresponding locations. If the pressure perturbation becomes larger than the critical value at a given location, then we will assume that at this location an earthquake occurs at the corresponding time moment (see Rothert and Shapiro, 2003 for details of such modeling).

In a range of small magnitudes, the theoretical, numerical, and observed curves are in a good agreement. However, the numbers of large-magnitude events are lower than theoretically and numerically predicted ones. This is seen from the fact that curves of \( N_M(t) \) for large magnitudes are not anymore parallel to such curves for small-magnitude ranges. The large-magnitude curves deviate significantly downward (especially for short times elapsed after injection starts; note that corresponding theoretical curves must be just parallel to the \( Q(t) \) curve in the time period of the injection).

We observe the same tendency at other locations. Figure 2 shows examples of distributions \( N_M(t) \) at other sites of fluid injections into rocks. In all examples, an approximate agreement of equation 1 with numbers for small-magnitude events can be demonstrated (see Shapiro et al., 2007, 2010; Shapiro and Dinske, 2009). Indeed, according to equation 1 in a range of not too large magnitudes, lines of \( \log_{10} N_M(t) \) are nearly mutually parallel (theoretically, they all should be parallel to the function \( \log_{10} Q(t) \)), and they are nearly regularly spaced (with an increment given by the product of the
$b$-value with the increment of the magnitude). However, the number of large events is systematically smaller than a regular spacing of the lines $\log_{10} N_M(t)$ would imply.

These observations reveal the following contradiction. Large-magnitude events correspond to large-scale ruptures (i.e., large-scale fractures or faults). They are less common than small-magnitude events (i.e., small-scale ruptures). The statistics of potential rupture surfaces in rocks correspond to a classical Gutenberg-Richter distribution of earthquakes produced on them (e.g., Turcotte et al., 2007, see also our consideration in the next section). Potential rupture surfaces can intersect injection-stimulated volumes of rocks. Any point of a stimulated volume can belong to such an intersection. One could think that large-magnitude events should not be less frequent in respect to the small-magnitude events than large-scale potential rupture surfaces are in respect to small-rupture surfaces. Moreover, for a given rupture surface, a probability to intersect a stimulated volume of rocks will be higher if the rupture surface’s scale is larger. Thus, one could think that in the induced seismicity large-magnitude events should be even more frequent in respect to the small-magnitude events than the potential large-scale rupture surfaces are in respect to small-rupture surfaces. Consequently, this line of reasoning suggests that large-magnitude events should be even overrepresented in comparison to the Gutenberg-Richter statistics for the small-magnitude events. However, in contrary, we observe an underrepresentation of large-magnitude events.

We propose the following explanation. Large-scale events would be overrepresented if hydraulic stimulations of small (point-like) parts of fault surfaces were sufficient to induce these events. The fact that they are underrepresented suggests that, to induce an earthquake on a fault patch, a significant part of this patch must be stimulated. This is in agreement with the Coulomb failure criterion. To enable an earthquake along a given interface, a surface-integrated tangential stress must overcome a total friction force. As soon as a significant part of a potential rupture surface remains unperturbed, the probability of an earthquake remains low. Therefore, to enable an earthquake, the corresponding rupture surface should nearly completely belong to a stimulated volume.

**THEORY**

How does the geometry control the occurrence of large earthquakes? In practice we assume that the stimulated volume is approximately defined by a cloud of induced seismicity (see Figure 3). We consider the following simplified abstract model (see Figure 4). A stimulated volume is an ellipsoid, which can grow with time due to a fluid injection. Rupture surfaces are randomly oriented planar circular disks (penny-shaped inclusions with vanishing thickness). The spatial distribution of centers of the disks is random and statistically homogeneous. Events will be assumed possible if their rupture surfaces (i.e., the disks) occur completely within the stimulated volume. Clearly, these assumptions simplify the reality significantly. However, they will help us to investigate how the finiteness of a stimulated volume influences the magnitude statistics of induced seismicity. Impacts of different deviations of our model from the reality will require additional studies. For example, influences of such factors like nonellipsoidal shapes of stimulated volumes, inhomogeneous distributions of potential rupture surfaces in space, their inhomogeneous orientations, and deviations of their shapes from circular disks can be of significance. They can be numerically studied along the line of the numerical modeling we propose below. Another fact that some events will occur even if small parts of their rupture surfaces are outside of the stimulated volume can also be investigated numerically. To some extent, such potential rupture surfaces can be accounted for by a slight upsampling of a stimulated volume size, which would include such rupture surfaces into the stimulated volume. In reality, the bounds of stimulated volumes are not sharply defined. Some upsampling of the stimulated volume is always possible in the frame of this unsharpness.
Statistics of potential rupture surfaces

We investigate the probability \( W_{\text{vol}}(X) \) that a disk of a diameter \( X \) is completely contained within a given stimulated volume.

We have found an exact expression of the function \( W_{\text{vol}}(X) \) for a spherical stimulated volume (see the Appendix). For this, we consider a disk of a diameter \( X \) of an arbitrary orientation with a center at a given point \( P \) inside of a given stimulated volume. Then we consider a sphere defined by normals of the length \( X/2 \) (the sphere’s radius) at the point \( P \) for all possible orientations of the disk. We calculate then a part of the surface of this sphere (in relation to its complete surface), where a normal can have its end-point under the condition that a corresponding disk still belongs to the stimulated volume. Integration of such parts of the spheres surfaces (defined by allowed positions of end points of disk normals) over all points of the stimulated volume (following by a normalizing to the complete volume) provides us with \( W_{\text{vol}}(X) \). For the case of a spherical stimulated volume of the diameter \( d \) (note that in this case the probability \( W_{\text{vol}} \) can be expressed as a function of \( x = X/d < 1 \))

\[
W_{\text{vol}}(X) = W_{sp}(x) \equiv (1 + x^2/2)(1 - x^3)^{1/2} - (3/4\pi x) + ((3/2x) \arcsin(x)).
\]

This function (the subscript “sp” indicates a spherical stimulated volume) has a satisfactory approximation of a simple form: \( W_{sp}(x) = (1 - x)^2(1 - x/2) \). Note that \( W_{sp}(x) \) and \( W_{sp1}(x) \) are both quickly decreasing with increasing \( x \) (see Figure 5).

We have numerically computed \( W_{\text{vol}}(X) \) for ellipsoidal volumes with principal axes \( L_{\text{min}} < L_{\text{int}} < L_{\text{max}} \). For this, we approximate a disk by a regular polygon with 16 sides (a hexadecagon). The centers of the polygons are distributed in a given stimulated volume with a given bulk concentration. These polygons have random orientations. For a given \( X \) we compute the number of polygons with all vertices located within the stimulated volume (they approximate the disks located completely within the volume). Then, we normalize the result by the number of all centers. This gives an approximation of \( W_{\text{vol}}(X) \).

The approximation can be improved by increasing number of vertexes of the polygons. We found that hexadecagons provide reasonable results (see Figure 5).

Summarizing, the probability \( W_{\text{vol}}(X) \) decreases quickly with increasing \( X \) (see Figure 5). This is also the case for very small \( X(X \ll L_{\text{min}}) \). Therefore, a scale of a stimulated volume should have a significant influence on the statistics of activated rupture surfaces of nearly all sizes. There exists a scale \( Y \) such that for \( X > Y \) the probability \( W_{\text{vol}}(X) \) becomes vanishingly small. The scale \( Y \) is a function of principal axes of a stimulated volume. Precisely, \( Y = L_{\text{int}} \). However, a probability of such a rupture is exactly zero (because it must strictly belong to the plane of the largest ellipsoidal cross section). Numerical simulations show that

\[
Y = [3/(1/L_{\text{min}}^3 + 1/L_{\text{int}}^3 + 1/L_{\text{max}}^3)]^{1/3}
\]

often provides a good estimate of \( Y \). We found also that functions \( W_{sp}(X/Y) \) and \( W_{sp1}(X/Y) \) often provide acceptable approximations for \( W_{\text{vol}}(X) \) (see Figure 5). For \( X > Y \) we observe \( W_{\text{vol}}(X) \ll 0.1 \). If \( L_{\text{min}} \) is sufficiently small, then corresponding to equation 3, it will provide a dominant contribution to \( Y \).

A probability-density to find a given-size potential rupture surface contained completely within the stimulated volume is a product of the probability density \( F(X) \) of such a surface at the injection site with the probability \( W_{\text{vol}}(X) \). Large rupture surfaces are significantly rarer than small ones. Thus, the function \( F(X) \) is strongly decreasing with \( X \). Thus, the product \( F(X) \cdot W_{\text{vol}}(X) \) will be an even more strongly decreasing function. A spatial scale of a rupture surface controls a magnitude of the corresponding earthquake. Thus, the function \( W_{\text{vol}}(X) \) affects a magnitude distribution of induced earthquakes. To investigate this influence numerically we assume a power-law size distribution of potential rupture surfaces in

Figure 3. Two projections of the Soultz ‘93 microseismic cloud and of an approximating ellipsoid of the aspect ratio 43:33:10 (we use such an ellipsoid for numerical illustrations). The left-hand-side projection is a map view. It corresponds approximately to the plane of the intermediate and minimum principal axes (the \( L_{\text{int}}, L_{\text{min}} \)-plane). The right-hand-side projection corresponds approximately to the plane of the maximum and minimum principal axes (the \( L_{\text{max}}, L_{\text{min}} \)-plane). The minimum principal axis of the ellipsoid is nearly parallel to the \( x \)-axis.

Figure 4. A sketch of an ellipsoidal stimulated volume with potential penny-shaped rupture surfaces approximated by regular polygons (we use this approximation for numerical simulations).
an unlimited medium. Power-law size distributions are typical for natural fractal-like sets (Scholz, 1990). Such a type of self-similarity seems also to be a reason of the Gutenberg-Richter frequency-magnitude distribution of earthquakes (Shearer, 1999; Kanamori and Brodsky, 2004; Turcotte et al., 2007).

Magnitude-length scaling and frequency-magnitude distributions

A relationship between a rupture size $X$ and an earthquake magnitude $M$ can be found using a standard relationship for seismic moment magnitudes (Lay and Wallace, 1995; Shearer, 1999; Kanamori and Brodsky, 2004). A seismic moment is given by a product of a rocks rigidity $G$ of a slip displacement $D$ and of an involved rupture surface area $S$. The moment magnitude is given by (Lay and Wallace, 1995; Shearer, 1999; Kanamori and Brodsky, 2004)

$$M = \log_{10}(GDS)/1.5 - 6.07$$

$$= \log_{10}X^2 + [\log \Delta \sigma - \log C]/1.5 - 6.07$$

(4)

for a seismic moments measured in Nm. The last part of the equation is derived in the following way. We accept the conventional assumption that the slip displacement $D$ scales as a characteristic length $X$ of the slipping surface (see also equation 9.26 and Table 9.1 from Lay and Wallace, 1995), and we substitute $(\log_{10}X^2)/1.5 = \log_{10}X^2$. The quantity $\Delta \sigma$ is usually defined as a static stress drop, and $C$ is a geometric constant of the order of one.

A probability density $R(M)$ of events with magnitude $M$ at a given location can be usually well approximated by a Gutenberg-Richter-type relation: $R(M) = 10^{a_d - bM}$, where $a_d$ is the Gutenberg-Richter $a$-value for the magnitude probability density and $b$ is the $b$-value (the same one for probability density, as well as for the cumulative probability and for an event number). Theoretically, the influence of a finite size of a stimulated volume is taken into account by

$$F(X) = X^{-w}$$

for disks of diameter $X$ in an unlimited medium. The corresponding $b$-value of the resulting frequency-magnitude distribution in an unlimited medium is $b = (q - 1)/2$. Then, as described above, we numerically computed numbers of disks completely belonging to stimulated volumes. Figure 6a shows results of such a modeling with $q = 3.5$. This value of $q$ corresponds to $b = 1.25$ and seems

Figure 5. Probability $W_{GR}$ for a spherical and an ellipsoidal (Soultz '93-like) stimulated volumes shown as functions of disk diameters normalized to minimum principal axes of the volumes in a linear (a) and a logarithmic (b) scales. The lines are given for spherical volumes (i.e., the exact functions $W_{GR}$). The lower line was computed for a spherical volume with a diameter equal to $L_{min}$. The upper line was computed for a spherical volume with diameter $\gamma$ (see equation 3). The corresponding numerical results are given as circles and squares, respectively. The crosses denote results of numerical modeling for a Soultz '93-like ellipsoid.

Figure 6. Number $N_M$ of events with magnitudes greater than or equal to $M$. (a) $N_M$ computed numerically and shown as a function of a stimulated volume. The principal axes of the volume were taken in a proportion approximately equal to the one at the injection site at Soultz in 1993. Numerical results show similarity with the behavior of real data (see Figures 1 and 2). (b) Frequency-magnitude distribution for the seismicity induced by the Soultz 1993 injection. Shown are the real data (points), a fitted straight line corresponding to the standard Gutenberg-Richter relation (fitting parameters, $a_{GR}$ and $b_{GR}$ and the observed maximum magnitude $M_{max}$) and a semi-analytical approximation taking into account the finiteness of the stimulated volume (fitting parameters, $a_{num}$, $b_{num}$, and $M_{num}$, an estimate of $M_{GR}$). The deviation of real data and fitted curves in the range of small magnitudes is due to catalog incompleteness (i.e., below the completeness magnitude only a small or even vanishing part of events was recorded).
to be roughly adequate for the Soultz data set addressed below (see also Figure 6b). Here, curves of \( N_M \) are shown as functions of a stimulated volume. The principal axes of the volume were taken in a proportion approximately equal to the one at the injection site at Soultz in 1993 (approximately, 10:33:43). Because the volumetric concentration of centers of potential rupture surfaces was constant, an increasing stimulated volume can be taken as a proxy of the time elapsed after the start of an constant-rate fluid injection. Numerical results show features similar to the behavior of real magnitude distributions (compare Figures 6a and 2). The modeling shows that large events are systematically underrepresented in comparison with what the Gutenberg-Richter distribution would imply (i.e., a regular-spaced extrapolation from the curves describing the behavior of small-magnitude event numbers).

A semianalytical approximation of frequency-magnitude distribution is obtained by a numerical integration of the product \( R(M) \cdot W_{sp}(10^{M-M_{\text{vol}}}/2) \) from a given magnitude \( M \) until the magnitude \( M_Y \). \( R(M) \) is a Gutenberg-Richter probability density of magnitudes in an unlimited medium. We fit the integration results to observed magnitude distributions. This provides estimates of \( M_Y \) and of the Gutenberg-Richter parameters \( a \) and \( b \). Figure 6b shows frequency-magnitude distribution for the seismicity induced by the Soultz 1993 injection. Shown are the real data, a fitted straight line and of the Gutenberg-Richter distribution will quickly drop down.

Discussion of the results

Several forms of a truncated Gutenberg-Richter relation are used in a standard earthquake hazard analysis (see, e.g., Utsu, 2002). Such relations heuristically assume a maximum possible magnitude as a fitting parameter in the frequency-magnitude distribution. Our results above help to quantify factors controlling such a parameter. Our analysis shows that the modification of a frequency-magnitude distribution caused by a finiteness of a stimulated rock volume has a form of a gradual increase of the \( b \)-value in the range of large magnitudes. Due to vanishing of the probability \( W_{\text{vol}} \) for \( X \) larger than a scale \( Y \), there exists a limit for a largest possible magnitude of an induced earthquake \( M_Y \).

One can relate \( M_Y \) to the quantity \( Y \) by the standard magnitude–rupture length scaling used above

\[
M_Y = \log_{10} Y^2 + (\log_{10} \Delta \sigma - \log_{10} C)/1.5 - 6.07. \tag{6}
\]

Our numerical simulations of \( W_{\text{vol}} \) show that the size of the stimulated volume defines the scale \( Y \). In comparison to other geometric parameters, the length \( L_{\text{min}} \) frequently gives a dominant contribution into \( Y \) (see the quantity \( \gamma \) given by equation 3). In reality, the following two factors enhance further the dominance of \( L_{\text{min}} \).

First, a perturbation of an effective stress cannot be restricted to a given surface only. Such a perturbation, if reached on a surface, must also occur in a finite volume surrounding this surface (because of the stress continuity in an elastic homogeneous or heterogeneous medium). In an extreme case, if such a volume were spherical, the probability \( W_{\text{vol}}(X) \) would be equal to a product \((1 - X/L_{\text{min}}) \cdot (1 - X/L_{\text{int}}) \cdot (1 - X/L_{\text{max}})\) and the maximum rupture size \( Y \) would be strictly equal to \( L_{\text{min}} \). In reality (a nonspherical perturbed vicinity of the rupture surface) a maximum rupture size will be between \( L_{\text{min}} \) and \( L_{\text{int}} \) and more probably, close to \( L_{\text{min}} \).

Another factor increasing the dominance of \( L_{\text{min}} \) is a probable anisotropy of orientations of rupture surfaces. Let us assume that such surfaces tend to be inclined by an angle \( \phi \) (defined by the friction coefficient) to a plane of the maximum and intermediate tectonic stresses. Usually this angle is close to 30°. Thus, the rupture surfaces have a larger angle to the minimum stress axis. To simplify the consideration we assume that all potential rupture surfaces have this inclination. Further, we will approximate the stimulated volume by a rectangle parallelepiped with sides \( L_{\text{min}} \), \( L_{\text{int}} \), and \( L_{\text{max}} \), rather than by an ellipsoid. Then the maximum rupture length should be smaller than \( L_{\text{min}} / \sin(\phi) \), if \( \phi \) is not too small, i.e., if \( \tan(\phi) > L_{\text{min}}/L_{\text{max}} \). Note that \( L_{\text{min}} / \sin(\phi) \) is a quantity proportional to \( L_{\text{min}} \). Thus, in such a case, the minimum axis (which is parallel to the minimum stress) will limit the event size. In a contrary case (\( \tan(\phi) < L_{\text{min}}/L_{\text{max}} \)), the maximum rupture length should be smaller than \( L_{\text{int}} \). For example, if \( \phi = 0 \) all potential rupture surfaces will belong to the same plane. Such a geometry is less relevant for seismicity induced by fluid stimulations of rocks. However, it is more adequate for aftershocks of earthquakes in subduction zones.

<table>
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<tr>
<th>Table 1. Magnitudes and minimum principal axis for points shown in Figure 7 of the main text.</th>
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<tr>
<td>( \log_{10}(L_{\text{min}}^2) )</td>
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Our conclusion on a dominant role of $L_{\text{min}}$ in the parameter $Y$ is supported by real data. Let us consider largest observed magnitudes $M_{\text{max}}$ of induced earthquakes for different case studies. We assume $M_{\text{max}}$ to be a proxy of $M_L$. Figure 7 shows the values of $M_{\text{max}}$ as functions of $(L_{\text{min}})^2$, $(L_{\text{int}})^2$ and $(L_{\text{max}})^2$ for all data sets we have in our disposal and several case studies which detailed descriptions we found in the literature. We approximate the stimulated rock volumes by the corresponding clouds of induced seismicity.

The error bars show possible impact of errors in magnitudes and event locations. For magnitudes we assumed the error of the order of 0.5 (this roughly corresponds to possible differences between local and moment magnitudes taken from different literature sources). For the principal axes we assumed the error bars of the order of seismicity location errors: 10 m for hydraulic fracturing sites Barnet Shale (Maxwell et al., 2009) and Cotton Valley (Rutledge and Phillips, 2003), 50 m for geothermal sites Basel (Häring et al., 2008), South, Cooper Basin (Baisch et al., 2009), Fenton Hill (Phillips et al., 1997), Berlin (Bommer et al., 2006), and 100 m for the Paradox Valley (Ake et al., 2005). The red star corresponds to the largest event of the Basel injection. The blue crosses correspond to data we have evaluated ourselves. In addition (red crosses), we included information using literature sources (Phillips et al., 1997; Bommer et al., 2006; Baisch et al., 2009).

We observe a good agreement of the data points with equation 6 for $Y = L_{\text{min}}$. The corresponding value of $\Delta \sigma$ are of the order of 0.0001–1 MPa. Substituting a highest probable limit of stress drops of the order of 10 MPa into equation 6, we obtain a rough estimate of the maximum probable magnitude limit of an induced earthquake for a given location

$$\max\{M_{\text{max}}\} \sim 2 \cdot \log_{10} L_{\text{min}} - 1. \quad (7)$$

This result explains the fact that high-magnitude events are more probable at geothermal sites than by hydraulic fracturing in hydrocarbon reservoirs. In the latter case, $L_{\text{min}}$ is much smaller than in the former one. By hydraulic fracturing, a typical zone of water penetration behind the fracture walls is of the order of 1–10 m (due to localization errors microseismic clouds of hydraulic fractures can have greater $L_{\text{min}}$; examples of a high-precision localization of hydraulic-fracture induced microseismicity can be found, e.g., in Rutledge and Phillips, 2003). Thus, the largest magnitudes are 0–1. By geothermal stimulations, $L_{\text{min}}$ is of the order of 300 m or less. Then, the largest magnitudes there are around four. Because $L_{\text{min}}$ can increase with time, it leads to an enhancement of the large-magnitude probability at the end of injection operations or shortly after it (until the rate of induced seismicity has not yet dropped significantly), which is indeed observed in reality (Majer et al., 2007; Baisch et al., 2009).

Our model and results are based on a hypothesis that a rupture corresponding to a fluid-induced earthquake is only probable along a surface located mainly inside a stimulated volume. We derive this hypothesis from the observational fact (i.e., the hypothesis and the derived formalism are empiric) of underrepresentation of large-magnitude events especially in the range of short injection times. As it was mentioned above, this hypothesis has a simple physical justification based on a balance of the friction and the tangential traction acting along a potential rupture surface. This hypothesis does not contradict the following observations. Sometimes it has been reported that in EGS induced large-magnitude events tend to occur on the edges of the stimulated volume (Majer et al., 2007). Our hypothesis does not exclude a possibility of large-size rupture surfaces close to the surface of the stimulated volume. Probable is a rupture surface that spans from an internal region to a vicinity of the surface of the stimulated volume. Such a rupture surface would be sufficiently perturbed at the moment that the pressure perturbation reaches its farthest from the injection source edge. This domain would correspond to a hypocenter of the corresponding induced event. The rupture would then propagate backward to the injection source. Note, however, that there are also publications reporting a broad spatial distribution of large-magnitude events in microseismic clouds (i.e., an absence of the above-mentioned tendency; see, e.g., Asanuma et al., 2005 and Mukuhira et al., 2008).

**Other factors influencing the magnitude of induced seismicity**

In our consideration, we concentrated mainly on the role of geometric parameters in magnitude statistics of induced seismicity. Other factors, like the injected fluid volume and the local seismotectonic activity (depending on the tectonic stress state), are also of importance. An attempt to quantify such influences is represented by our equation 1 where the role of the cumulative injected volume is seen explicitly and the seismotectonic activity is quantified by the seismogenic index $\Sigma$. This index increases with the increasing bulk concentration of potential rupture surfaces. It increases also with decreasing stress (pressure) perturbations required to produce earthquakes on these surfaces and with increasing probability of magnitudes larger than zero. Shapiro et al. (2010, 2011) computed the seismogenic index for several injection locations. Note that the geometric effects described above modify equation 1 by an additional correction term. This term follows from equation 5, which turns the term $-bM$ of the classical Gutenberg-Richter magnitude-frequency relation into the term $-bM + \Delta(M, M_T, b)$. Substituting this expression into equation 1 instead of $-bM$ gives

$$\log_{10} N_M(t) = \log_{10} Q(t) - bM + \Delta(M, M_T, b) + \Sigma. \quad (8)$$

![Figure 7. Largest observed magnitudes of induced earthquakes as functions of principal axes of corresponding stimulated rock volumes for different case studies. The error bars show possible impact of errors in magnitudes and event locations. The red star corresponds to the largest event of the Basel injection. The blue crosses correspond to data we have in our disposal and evaluated ourselves. The red crosses correspond to case studies that we were able to evaluate using literature sources. Magnitudes, minimum principal axis and corresponding injection sites are given in Table 1.](attachment:image.png)
where the correction term $\Delta(M, M_y, b)$ can be found by numerical simulations analogous to ours reported here. Its possible approximation is given in the explanations of equation 5. Equation 8 is a corrected-for-geometry version of equation 1. It shows that the probability of a high-magnitude event increases with an increasing injected volume $Q(t)$ and an elevated seismicity index $\Sigma$. The most probable maximum magnitude $M_y$ influences this equation through the geometrical correction term $\Delta(M, M_y, b)$.

Our model considers the quantities $\Sigma$ and $M_y$ as independent. This is a possible restriction of the model. We cannot exclude that in reality these quantities may be correlated. For example, an enhanced permeability along a critically stressed fault system (i.e., an elevated $\Sigma$) could lead to an increased size of a stimulated volume along its all principal axes. This would lead to a positive correlation between $\Sigma$ and $M_y$.

Another important factor implicitly influencing equation 8 is the static stress drop $\Delta \sigma$. Recent studies of fluid-induced seismicity indicate that the stress drop can be broadly distributed (e.g., three orders of magnitude variations at a single injection location were shown in results of Goertz-Allmann et al., 2011). The stress drop is positively correlated with $M_y$. An elevated level of the stress drop will also be positively correlated with the seismicity index. The order of a maximum expected stress drop will also influence our equation 7. A possibility of a stress drop of the order of 100 MPa, as well as a geologically or tectonically caused enhanced probability of a rupture along $L_{\text{max}}$ or $L_{\text{sec}}$ directions (e.g., a part of a stimulated volume can be just aseismic in an initial time interval) can significantly restrict its validity domain.

The discussion of the factors above shows that our results further extend a possibility to quantify the magnitude distribution of fluid-induced seismicity. Their application to its hazard requires a further careful statistical analysis of these factors under given geological and seismo-tectonic conditions.

CONCLUSIONS

Fluid injections into boreholes are able to induce potentially damaging earthquakes. With this paper, we contribute to the understanding of factors controlling occurrence of large-magnitude earthquakes in such situations. We infer that finiteness of a fluid-stimulated volume strongly influence the statistics of potential rupture surfaces of induced earthquakes. We provide analytical and numerical results describing this statistics. We show that one of the main factors limiting the probability of large-magnitude events is the minimum principal axis of the fluid-stimulated rock volume. This geometrical scale seems to control the order of a largest possible magnitude. We provide a corresponding relationship (equation 6 with $Y = L_{\text{min}}$) and show that it is in agreement with real observations. We analyze an impact of the stimulated volume geometry on the frequency-magnitude distribution of induced seismicity, where both $a$- and $b$-values of the Gutenberg-Richter distribution are influenced. Our observations and numerical simulations show that large-magnitude earthquakes are systematically underrepresented in comparison with the theoretical expectations derived from frequency-magnitude distributions of the Gutenberg-Richter type. This suggests that a rupture is only probable in cases where sufficiently perturbed effective stress conditions occur over most of the rupture surface (this conclusion is maybe relevant for large-scale tectonic earthquakes also), i.e., a surface located mainly inside a stimulated rock volume. Therefore, monitoring of the spatial growth of seismicity in real time can help to reduce a risk of damaging events during rock stimulations. For example, to mitigate the hazard one could attempt to keep the minimal principal axis of the stimulated volume restricted by terminating the injection if this size achieves a planned critical value. Other geometrical sizes and seismo-tectonic parameters (e.g., the seismicity index and the $b$-value) are also of importance for this task. Numerical simulations like we proposed here, may be helpful for finding critical parameters for such a controlled rock stimulation.

We hypothesize that the above relations between geometric parameters of stimulated rock volumes and the magnitude distributions of induced earthquakes are valid for tectonically and artificially induced seismicity in general. Our results are of significance for further developments of geothermal-energy exploration. They are also of significance for other applications like CO$_2$ underground storage and hydraulic fracturing of shale-gas deposits.

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APPENDIX A

PROBABILITY OF AN ARBITRARILY ORIENTED DISK BELONGING TO A SPHERICAL VOLUME

Here we derive equation 2. We consider a spherical stimulated volume of a radius $R = d/2$ with a center at a point O. We take a disk with a radius $r = X/2$ and a center at a point P inside of the stimulated volume. If the distance $y = |OP|$ is less than $R - r$, then the volume will include the disk independently of its orientation. Thus, the volume $V_1 = 4\pi (R - r)^2/3$ will contribute to a possible location of P completely. Let us consider the spherical shell with the inner radius equal to $R - r$ and the outer radius $R$. If P is located inside of this shell then the disk cannot be arbitrary-oriented. An orientation of the disk is given by the orientation of a normal to its plane at the point P. Let us then consider a sphere $S_1$ of radius $r$ with the center at P. This sphere intersects the surface of the stimulated volume along a circle. We further consider a plane including the straight line OP. Such a plane intersects this circle in two points, A1 and A2, located symmetrically to the line OP. An angle $\alpha$ between the line PA1 (or PA2) and a normal to the line OP crossing this line in the point P and belonging to the same plane (given by points O, P, A1, and A2) is given by the following relation: $\sin \alpha = (R^2 - r^2)/(2yr) - y/(2r)$. To belong to the volume a disk must have a normal located inside of a cone with the symmetry axes coinciding with the line OP and with the limiting angle equal $\alpha$. 
The sphere $S_1$ and this cone define a spherical segment of the height $h = r(1 - \cos \alpha)$ and the surface $2\pi hr$. A probability of a disk to have an orientation necessary for belonging to the stimulated volume is equal to the ratio of this surface to the surface of the half of the sphere $S_1$, i.e., $h/r$. This probability $W_1$ is a function of the variable $y$. The looked-for probability is given by a sum of the contribution of the volume $V_1$ and of the integral of the probability $W_1$ over the spherical shell introduced above:

$$W_{sp}(r/R) = \left(3 \int_{y_{\min}}^{y_{\max}} y^2 \frac{h}{r} dy + (R - r)^3\right)/R^3, \quad (A-1)$$

with $y_{\min} = R - r$ and $y_{\max} = \sqrt{R^2 - r^2}$. This provides equation 2.

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Uncertainty in FPSs from moment-tensor inversion

Jing Du¹ and Norm R. Warpinski¹

ABSTRACT

Although microseismic monitoring of hydraulic fractures has primarily been concerned with the dimensions, complexity, and growth of fractures or fracture systems, there is an ever-increasing desire to extract more information about the hydraulic-fracturing and/or natural fractures from microseismic data. Source mechanism analysis, which is concerned with deducing details of the failure process from the microseismic waveform data, is, therefore, attracting more attention. However, most of the studies focus more on the moment-tensor inversion than on extracting fault-plane solutions (FPSs) from inverted moment tensors. The FPSs can be extracted from the inverted moment-tensor, but there remains a question regarding how errors associated with the inversion of the moment-tensor affect the accuracy of the FPSs. We examine the uncertainties of FPS, given the uncertainties of the amplitude data, by looking into the uncertainty propagation from amplitude data into the moment-tensor and then into the resultant FPS. The uncertainty propagation method will be demonstrated using two synthetic examples.

INTRODUCTION

Microseismic hydraulic-fracture mapping has become a valuable tool for monitoring fracture growth and behavior in unconventional reservoirs and provides information to help improve the stimulation design and achieve optimum recovery from the reservoirs. Basically, microseismic monitoring is the placement of seismic receiver systems in offset wells at a depth relatively close to a hydraulic-fracture, so that small earthquakes induced by the fracture can be detected and located to provide geometric and behavioral information about the hydraulic-fracturing process. When microseismic event locations are the only area of interest, most investigations focus on the impact of the velocity model, array geometry, acquisition parameters, noise, and data quality on location accuracy. With more attention now placed on attempting to extract additional information from those microearthquakes, new interest has focused on source parameters and source mechanisms (Nolen-Hoeksema and Ruff, 2001; Jechmutálová and Eisner, 2008; Rutledge et al., 2004; Šílený et al., 2009), such as the source type, source radius, slip amount, fault-plane solutions (FPSs), slip direction, etc. To solve these problems, investigators have been adopting technologies used in traditional seismology, such as moment-tensor inversion, spectral analysis, etc. The primary information derived from moment-tensor inversion is the slippage plane orientation, the slip direction and the moment. Extracting FPSs from the moment-tensor can be used as a reservoir characterization tool. However, although these advanced analyses have been performed on microseismic data, there has been little investigation about how accurate these estimates can be given the uncertainties in the data.

It is known that for any geophysical inverse problem, it is very important to quantify the uncertainties of the inverted parameters. As Julian and Foulger (2009) point out, most moment-tensor analyses of earthquakes to date have lacked any kind of error analysis. They use linear programming and invert the amplitude ratios and polarities of P and S-waves to establish confidence regions for computed moment tensors. After the best-fit moment-tensor was found, a new objective function was formulated so that it could be maximized to obtain the confidence regions of the derived moment-tensor. Usually, the least square solution is implemented for moment-tensor inversion, which allows for standard covariance matrix methods (Menke, 1989) to quantify the uncertainties of the inverted moment-tensor. However, propagating the uncertainties of the moment-tensor into the FPSs is difficult because it involves solving the eigenvalues and eigenvectors of the moment-tensor to obtain the orientations of the fault-plane, which is a nonlinear problem. A statistical method, called bootstrap analysis (Efron and Tibshirani, 1986), has been used to obtain the uncertainties.

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of the principal axes (T, P, and B axes) (Manthei, 2005). Another method is to perturb each of the inverted moment tensors by the amount of the inverted moment-tensor uncertainties (Kolář, 2007) and then estimate orientation uncertainties from the total of 64 combinations (i.e., $2^6$) of moment tensors.

In this paper, a first step is taken to try to quantify the uncertainties of the FPSs. There are several causes for the uncertainties, such as the accuracy of the velocity model and event location, errors in traveltime picks, noise in the raw data (waveforms), etc. This paper only focuses on the uncertainties caused by the noise in the raw data and investigates how it induces uncertainties in the moment-tensor, and then how uncertainties in the moment-tensor propagate into the FPS. The methods described here could be applied to other cases, too, as long as the other uncertainties induced by the velocity model, event locations, and traveltime picks, etc. are characterized and propagated into the moment-tensor.

First, an introduction to moment tensors, moment-tensor decomposition, and FPSs is provided, then moment-tensor inversion is briefly described, and finally two methods (an extended analytical method and the Monte Carlo method) are presented to account for the uncertainty propagation. Results from two synthetic cases are presented, followed by some discussion and conclusions.

**MOMENT-TENSOR DECOMPOSITION AND FPS**

The moment-tensor is a real and symmetrical tensor that provides the general description of the seismic source. It depends on the source strength and fault orientation, which can be deduced from observed waveforms with a longer wavelength than the linear dimension of the source (Aki and Richards, 2002). The general expression of a symmetric moment-tensor is represented as (Dufumier and Rivera, 1997)

$$ MT = \lambda A[u](s \cdot n)I + \mu A[u](sT + ns^T), $$

where $\lambda$ and $\mu$ are Lame’s constants, $A$ is the area, the scalar, $|u|$, is the displacement discontinuity, $n$ is the normal vector, and $s$ is the slip vector. The normal and slip vectors can be interchanged for any data set and, thus, represent the normal directions of two possible fault planes. They are called FPSs in this paper.

The isotropic part of the moment-tensor is

$$ \text{Iso}MT = \frac{1}{3} \text{Trace}(MT)\delta_{ij}. $$

The deviatoric part of the moment-tensor is

$$ \text{Dev}MT = MT - \text{Iso}MT. $$

The eigenvalues and eigenvectors of the deviatoric part are (Dufumier and Rivera, 1997)

$$ \nu_1' = \left(\frac{1}{3}(n \cdot s) + 1\right)\mu A[u] $$

$$ \nu_2' = -\frac{2}{3}(n \cdot s)\mu A[u] $$

$$ \nu_3' = \left(\frac{1}{3}(n \cdot s) - 1\right)\mu A[u] $$

\[ (\nu_1' \geq \nu_2' \geq \nu_3') \quad (\nu_1' > 0, \nu_3' < 0); \]

$$ t = \frac{n + s}{|n + s|} \quad b = n \otimes s \quad p = \frac{n - s}{|n - s|}. $$

The $t$ and $p$ vectors represent tension (T) and pressure (P) axes, and the $b$ vector represents the mutually orthogonal B axis. It can be found that (Dufumier and Rivera, 1997):

$$ n \cdot s = \frac{-3\nu_3'}{\nu_1' - \nu_3'} = \frac{3}{2} \frac{\nu_1' + \nu_3'}{\nu_1' - \nu_3'} \quad A[u] = \frac{\nu_1' - \nu_3'}{2\mu}. $$

The angle ($\alpha$), which is defined as the angle between the slip direction and fault-plane (Figure 1), is calculated as

$$ \sin \alpha = n \cdot s = \frac{3}{2} \frac{\nu_1' + \nu_3'}{\nu_1' - \nu_3'}. $$

The unit vectors (normal direction $n$ and slip direction $s$) are calculated from the eigenvectors (P and T axes) as the following

$$ n = \frac{1}{\sqrt{2}} \left((n + s) + (n - s)\right) $$

$$ = \frac{1}{\sqrt{2}} \left(\sqrt{1 + (n \cdot s)t + \sqrt{1 - (n \cdot s)p}}\right) $$

$$ s = \frac{1}{\sqrt{2}} \left((n + s) - (n - s)\right) $$

$$ = \frac{1}{\sqrt{2}} \left(\sqrt{1 + (n \cdot s)t - \sqrt{1 - (n \cdot s)p}}\right). $$

Equations (1) to (7) are used to determine the moment-tensor. The angle ($\alpha$) is calculated from these equations, and the fault-plane solution (FPS) is provided.

**Figure 1. Angles and axes for pure shear source (left) and non-pure shear source (right) (after Vavryčuk, 2001).**
The relationship between the normal/slip directions and P/T axes are shown in Figure 1 for different source types. The left figure is for a pure shear source in which there are 45° between the normal and the P/T axes. For a source with some volumetric component, as shown in the right figure, the angles between the normal direction and T/P axis are β and γ, correspondingly. The relations between β, γ, and α are shown in the following equation:

\[
\begin{align*}
\cos \beta &= \frac{1 + \sin \alpha}{\sqrt{2}} \\
\sin \beta &= \frac{1 - \sin \alpha}{\sqrt{2}} \\
\beta &= \frac{\pi}{4} \pm \frac{\alpha}{2} \\
\gamma &= \frac{\pi}{4} \pm \frac{\alpha}{2}.
\end{align*}
\]

(9)

The moment-tensor can be decomposed into three components (Vavryčuk, 2001):

\[ MT = M_{T}\text{ISO} + M_{T}\text{CLVD} + M_{T}\text{DC}. \]

(10)

The percentages of each component (volumetric (ISO), compensated linear vector dipole (CLVD), and double-couple (DC) are calculated as

\[ e_{\text{ISO}} = \frac{1}{3} \frac{M_{\text{max}}}{|M|_{\text{max}}} \]

\[ e_{\text{CLVD}} = 2(1 - |e_{\text{ISO}}|) \]

\[ e_{\text{DC}} = 1 - |e_{\text{ISO}}| - |e_{\text{CLVD}}|, \]

(11)

where \( M_{\text{max}} \) is the eigenvalue of the moment-tensor and \( MT \) (which has the maximum absolute-value) and \( e \) (which measures the size of CLVD relative to DC) are defined as

\[ e = -\frac{M_{\text{*}}}{|M|_{\text{min}}}, \]

(12)

where \( M_{\text{max}} \) and \( M_{\text{min}} \) are the eigenvalues of the deviatoric moment-tensor (DevMT) with the maximum and minimum absolute values.

**MOMENT-TENSOR INVERSION**

Moment-tensor inversion is a method used in traditional seismology to invert the amplitudes of P- and S-waves for earthquake source mechanisms, and most typically for cases where the seismometers are located on the surface. For oil-and-gas applications, the same methodology is being applied to microseismic monitoring of hydraulic-fracturing treatments where geophone arrays are installed in an observation well. This receiver geometry has very limited focal coverage, which presents a challenge to investigators working on this problem. Vavryčuk (2007) showed that the complete moment tensors of seismic sources in homogeneous or vertically inhomogeneous isotropic structures cannot be retrieved using receivers deployed in one vertical borehole. They can be retrieved from amplitudes of P-waves, provided that receivers are deployed in at least three boreholes. Using amplitudes of P- and S-waves, two boreholes are, in principle, sufficient.

In general, the least-squares method is adopted as in the standard seismological moment-tensor inversion, which is formulated as \( Gm = d \) (see Appendix A) and specifies how the amplitude data vector, \( d \), is linked to the moment-tensor through the formation Green’s function. However, there are different formulations regarding how to form the discrete Green’s function (\( G \)). The method used in this paper (Sokos and Zahradník, 2008; Warpinski and Du, 2010) is to construct the Green’s function using six basic focal mechanisms. In this formulation, the sixth focal mechanism is the volumetric component, which simplifies the deviatoric moment-tensor inversion by setting the sixth elementary mechanism to be zero. This is the most common technique to regularize the ill-posed inverse problem for retrieval of the moment-tensor from a single monitor well. It is among the best possible choices to reduce the number of unknowns from six to five (Dufumier and Rivera, 1997).

**METHODS OF UNCERTAINTY PROPAGATION IN MOMENT-TENSOR INVERSION FOR FPS**

After the best-fit moment-tensor is calculated using the least-square method, a standard covariance matrix method (Menke, 1989) can be used to quantify the uncertainties of the inverted moment-tensor when the uncertainties in the amplitude data are given. Obtaining the FPS from the inverted moment-tensor is a highly nonlinear problem that involves solving a nonlinear problem to obtain the eigenvalues and eigenvectors (or eigenparameters) of the moment-tensor and rotating the eigenvectors to obtain the normal and slip directions of the fault-plane. In addition, the angle between the fault-plane normal direction and slip direction, which is not necessarily orthogonal, can be obtained from the eigenvalues of the deviatoric moment-tensor. This nonorthogonality can be described by angle \( \alpha \), which is the deviation of the slip direction from the fault-plane (Figure 1).

This paper will examine two methods that propagate the uncertainties obtained from the covariance matrix of the moment-tensor into the FPS. The first is an analytical uncertainty propagation method extended from the method presented by Han et al. (2007). The second is the Monte Carlo method.

**Extended analytical uncertainty propagation method**

The analytical uncertainty propagation method for the FPS is extended from Han et al.’s (2007) method, in which an analytical expression of the covariance matrix of the eigenvalues and eigenvectors of an n-D symmetric tensor is derived using linear algebra and differential calculus. The authors also showed that it is very important to consider the parameter dependencies between the eigenparameters.

The full moment-tensor is the 3D symmetrical tensor, where only six out of the nine total elements are independent. For the full moment-tensor, the method presented by Han et al. (2007) is adopted here to propagate the uncertainties of the moment-tensor into the eigenparameters (eigenvalues and eigenvectors). For the deviatoric moment-tensor, only five out of the total of nine elements are independent, as the trace of the diagonals must be zero. The formulation is derived following Han et al.’s (2007) method by considering the extra constraint for the deviatoric moment-tensor; that is, the summation of the three eigenvalues must be zero. The
detailed derivation of the propagation from the uncertainties of deviatoric moment-tensor to its eigenparameters is shown in Appendix A.

After the uncertainties of the eigenparameters are obtained, the uncertainties of the FPS are calculated using differential calculus, because the FPS (the normal and slip directions of the fault) are functions of the eigenvectors and eigenvalues of the deviatoric moment-tensor. The details are shown in Appendix A.

Monte Carlo method

The Monte Carlo method is a computational algorithm that relies on repeated random sampling to compute the statistics of the variables, especially for solution techniques that are nonlinear and/or algorithmic, and for which there is typically no simple way to propagate the data uncertainties to the uncertainties of the model parameters (Aster et al., 2005). Starting with the covariance matrix of the moment-tensor, which is obtained from the least-squares method, it is assumed that the moment-tensor elements follow a multivariate normal distribution in the studies in this paper. In general Monte Carlo methods, any type of statistical distributions could be adopted as long as they are known. The multivariate normal distribution is parameterized with a mean vector $\mu$ and a covariance matrix $\text{Cov}_{MT}$. The mean vector is essentially the array of moment-tensor elements here. The probability distribution for $n$-variables is represented as the following

$$f(x_1, x_2, x_3, \ldots, x_n) = \frac{1}{\sqrt{(2\pi)^n|\text{Cov}_{MT}|}} e^{-\frac{1}{2}(X-\mu)^t \text{Cov}_{MT}^{-1} (X-\mu)},$$

where $n$ is equal to six for a full moment-tensor or five for a deviatoric moment-tensor, and $\text{Cov}_{MT}$ is the covariance matrix of the moment-tensor, which is calculated from the least-squares method (Equation A-3).

In this study, a series of random moment-tensor arrays are generated. They follow a multivariate normal distribution with mean as the inverted moment-tensor and covariance matrix output from the least-squares method. Then, the eigen-decomposition analysis is performed, and eigenvalues and eigenvectors are obtained. With these eigenparameters, the FPSs are obtained for each realization. In the end, circular statistics (Berens, 2009) are used to separate the two sets of FPSs, and the mean and standard deviations of the angles are obtained for each FPS.

SYNTHETIC DATA EXAMPLES

Two synthetic data examples are generated to demonstrate the uncertainty propagation methods adopted in this paper. The first example is the case of two observation wells, which allows for a full moment-tensor inversion. Two vertical tool strings with 12 sensors each are placed at (0, 0) and (1000, 0) ft. The first string is in the center of a ring of 22 microseismic events, as shown in Figure 2, and the events are ordered based on the event azimuth angle (angle from the north, azimuth from the first observation well to the event position) starting from the one on the north. The sensors are located at depths from 5000 to 5440 ft, and all microseismic events are at a depth of 5200 ft and a distance of 500 ft, spaced 15° apart. All the microseisms have the same source type, which is a vertical plane oriented north/south with the slip direction away from the plane by $\alpha$ degree, as shown in Figure 2. With this geometry set up, two FPSs are vertical; one FPS is the original vertical fault oriented in north-south direction, and the second is the plane that has the slip direction of the first FPS as its normal direction. The second example is a single-well case where all event locations are the same as the first example and only the amplitude data from the observation well surrounded by events are perturbed by noises and used for moment-tensor inversion. Also, only a pure shear source ($\alpha = 0^\circ$) is adopted in the second example.

The fault-plane normal and slip directions are used to construct the moment-tensor (Equation 1), which is used to compute P- and S-wave amplitude data given the geometry described above and shown in Figure 2. These synthetic amplitude data are perturbed by adding the random noise. The noise is assumed to be white noise with zero mean and standard deviation set as a fraction of the maximum P-wave amplitude in all the sensors, which is called the noise level. In this example, two sets of noise levels (0.1 and 0.2) are run for different angles $\alpha$ ($0$, $15$, and $30^\circ$) to show how different noise levels and angles $\alpha$ affect the uncertainties.
The results from both analyses show (Figure 3 and Figure 4) that the uncertainties in angles ($\alpha$, strike, and dip) follow the trend of the condition number, which is defined as the ratio of the largest to smallest singular values of the discrete Green’s function ($G$), and reflect the instability of the moment-tensor solution (Aster et al., 2005). The larger the condition number, the bigger the resultant angle uncertainties are likely to be. This behavior reflects the nature of the condition number, which represents how the data uncertainties propagate into the model uncertainties. The results also show that the uncertainties in $\alpha$ are larger than the strike uncertainties, and the strike uncertainties are larger than the dip uncertainties. The reason for the low dip uncertainties is caused by the geometry setup for this study, where both FPSs are vertical, and the tool strings are straddling the zone where the events are located, as well as the pure vertical fault planes. The uncertainties comparisons between $\alpha$ and the two strike angles are shown in Figure 5, where it is observed that the uncertainties of $\alpha$ are approximately twice the uncertainties of the strike angle for the same noise level.

Looking at the three sets of curves ($\alpha$ of 0, 15, and 30°) in Figure 6, it can be seen that the uncertainty of $\alpha$ increases with its value for the same noise level for both uncertainties propagation methods. Also, for the same $\alpha$ value and noise level, it is observed that the uncertainty of $\alpha$ from the extended analytical propagation method is roughly three times those of the Monte Carlo method for this specific case study. The effect of $\alpha$ on the uncertainties of strike and dip angles is shown in Figure 7. From three different curves ($\alpha$ of 0, 15, and 30°), we can see that, in general, the larger the $\alpha$ is, the bigger the uncertainties of strike and dip are. The larger the condition numbers are (such as for events with azimuth between 240° and 300°), the magnification effect due to angle $\alpha$ becomes greater.
The uncertainties in each component percentage (double-couple (DC), isotropic (ISO), and compensated linear vector dipole (CLVD)) are examined in Figure 8. For different $\alpha$ values (0, 15, and 30°), it can be seen that the percentage uncertainties of each component (DC, ISO, and CLVD) are very similar, with the uncertainties all less than 10% for the lower noise level of 10%. However, by looking at the absolute percentages of the components for the input moment-tensor shown in Figure 8d, it can be seen that the DC percentage drops with increasing $\alpha$, while the ISO and CLVD percentages increase. So, although the absolute percentage uncertainties for all components are similar, the relative magnitudes of the percentage uncertainties increase for the DC component and decrease for the ISO and CLVD components, with increasing $\alpha$.

For the single-well case, only one noise level ($f = 0.1$) is run for the pure shear source ($\alpha = 0°$). The results are shown in Figure 9, where it can be seen that the uncertainties of $\alpha$, strike, and dip angles from these two uncertainty propagation methods are very similar; the uncertainty of $\alpha$ is slightly larger than the strike uncertainty, and the strike uncertainty is larger than the dip uncertainty. The up-and-down trend of the uncertainties for different event azimuths contradicts the trend of the condition numbers, which vary in a small range. This is caused by the way the noises are added because the added noises are normalized according to the maximum P-wave amplitude for each event. For different events, the maximum P-wave amplitude can be different, as shown in Figure 10b, so the noises added are different. For the events with slightly smaller condition numbers but larger uncertainties, the added noises have a larger absolute magnitude for the same noise level used for all events. The effect of a larger noise magnitude surpasses the effect of condition numbers, which only vary over a small range.

The comparison of $\alpha$ determination between the two-well and one-well case is shown in Figure 10. The results show that the $\alpha$...
is better resolved using two-well data (Figure 10a); most of the events in the two-well case have the condition numbers around five, except two events with azimuths of 255° and 285° that have condition numbers of approximately 14. For the single-well case, all the events have condition numbers between eight and 11.

**DISCUSSION**

From the synthetic data example, it can be seen that the uncertainty of $\alpha$ is usually larger than the strike and dip. The reason could be that $\alpha$ is a function of the eigenvalues of the deviatoric moment-tensor, as shown in equation 7, where the absolute values of each moment-tensor element matter, whereas the strike and dip angles of the FPS (equation 8) are a function of the angle $\alpha$ and the P and T axes, which are the eigenvectors of the moment-tensor, where only the relative magnitudes of moment-tensor elements count. Furthermore, for the cases shown in the paper, the angle $\alpha$ is relatively small and its effect on the normal and slip vectors is small. This is similar to cases where amplitude ratios of P- and S-waves are used to retrieve the FPSs, as Jechumtálová and Sileny (2005) point out, showing that the amplitude ratios are less vulnerable to the mismeasuring of the earth structure than the amplitudes.

The extended analytical uncertainty propagation method for the FPS is extended from Han et al.’s (2007) method, which propagates the covariance of eigenparameters of the general n-D symmetrical tensor. It provides more reasonable estimates of uncertainties, whereas the traditional Monte Carlo method implemented in the case study where the moment-tensor elements are assumed to follow a multivariate normal distribution which may not be true underestimates the uncertainties.

It is important to consider the dependency between the variables in any uncertainty propagation. This is similar to the case of the

---

**Figure 7.** The effect of value of $\alpha$ on the standard deviations of strike and dip angles for noise level $\alpha = 0.1$. (a) And (c) for standard deviation of strike angles. (b) And (d) for standard deviation of dip angles in two-well case. Extended analytical uncertainty propagation is used in (a) and (b). Monte Carlo method is used in (c) and (d).

**Figure 8.** The uncertainties in DC (a) ISO (b) and CLVD and (c) percentages in the two-well case (noise level $\alpha = 0.1$; Monte Carlo method used in all cases); (d) DC, ISO, and CLVD percentages of the input moment-tensor (Poisson ratio = 0.25).
calculation of the total differential of a function with respect to the individual variable, where these variables have to be independent and cannot be a function of other variables. Han et al. (2007) demonstrate that it would provide more reasonable uncertainty estimates compared to the one without considering the dependencies between variables. It is shown in Appendix A for the uncertainty propagation in the deviatoric moment-tensor inversion case, where the dependency of the third diagonal element on the first two diagonal elements has to be eliminated, first in the propagation process in equation A-9, as the summation of the three diagonal elements in the deviatoric moment-tensor has to be zero.

Regarding the uncertainties of the percentages of each component (DC, ISO, and CLVD), only the results from the Monte Carlo method are provided because of the limitation of the extended analytical uncertainty propagation method. From equation 11 it can be seen that all the percentage calculations involve the absolute-value function, which means that the function of each percentage is no longer differentiable at the point zero and the total differentiation used in the extended analytical uncertainty propagation method is no longer valid.

Mathematically performing the deviatoric moment-tensor inversion is enough to extract the FPS, because the angle $\alpha$ is only a function of the eigenvalues of the deviatoric moment-tensor, and fault normal and slip directions are a function of the angle $\alpha$ and the eigenvectors of the moment-tensor, which are the same as the eigenvectors of the deviatoric moment-tensor. This means that the FPS could be extracted using the deviatoric moment-tensor inversion from single-well data, even for a nonpure shear source type. However, there are sampling errors that might be inherent in the single-well case caused by the limited coverage of the focal sphere that could potentially prevent using single-well data for a nonpure shear source type.

The comparison between the two-well case and single-well case shows that multiwell analyses will generally give better FPS results and are less affected by noise because the geometry is usually more conducive to the extraction of these parameters, which is reflected in the condition number.

**CONCLUSION**

This paper presents an extended analytical uncertainty propagation method for estimating the uncertainties of FPS caused by noises in the amplitude data. The method is general so that it can be applied to the uncertainty estimates caused by other uncertainty sources as long as the model covariance matrix of the moment-tensor is provided. This extended analytical uncertainty propagation method is also compared with the traditional Monte Carlo method. The study shows that the uncertainty of $\alpha$, which is defined as the angle between the slip direction and fault-plane, is larger than the uncertainties of the strike and dip angles. The uncertainties of $\alpha$, strike, and dip angles, as well as the relative magnitude of percentage of DC component, all increase with increasing $\alpha$, except the relative magnitude of percentages of ISO and CLVD components. The Monte Carlo method underestimates the uncertainties compared to the extended analytical uncertainty propagation method.

**APPENDIX A**

**EXTENDED ANALYTICAL UNCERTAINTY PROPAGATION**

Obtaining the uncertainties of the FPS requires three steps:

**Step 1: Moment-tensor uncertainties estimation**

For the moment-tensor inversion, a geophysical inverse problem needs to be solved:

$$Gm = d,$$  

(A-1)

where $G$ is the discrete Green’s function, which links the model parameter $m$, which is a six-element column vector (or a
five-element for deviatoric moment-tensor) consisting of the moment-tensor elements to the amplitude data vector $d$. The best-fit moment-tensor vector could be obtained using the least-squares method for each event:

$$m^\text{est} = [G^TG]^{-1}G^Td = G^{-g}d, \quad (A-2)$$

where the matrix $G^{-g}$ is called the generalized inverse matrix. The covariance of the model parameter could be calculated if the data covariance is known as

$$\text{cov}(m) = G^{-g}\text{cov}(d)(G^{-g})^T. \quad (A-3)$$

Assuming the amplitude data is uncorrelated with equal variance of $\sigma_d^2$, the covariance matrix of the moment-tensor is

$$\text{cov}(m) = \sigma_d^2[G^TG]^{-1}. \quad (A-4)$$

If the inversion is constrained to zero trace (or zero volumetric component, also called deviatoric moment-tensor inversion) from the formulation of the discrete Green’s function adopted in this paper, it means that the sixth basic mechanism is restricted to zero. Now the inversion with this constraint becomes the following

$$G\text{reduced}m\text{reduced} = d$$

$$G\text{reduced} = G(:,1:5)$$

$$m\text{reduced} = (a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5)^T$$

$$m\text{reduced} = [G\text{reduced}G\text{reduced}]^{-1}$$

$$G^\text{reduced}d = G^{-g}\text{reduced}d, \quad (A-5)$$

where $G^{-g}\text{reduced}$ is the reduced generalized inverse matrix. The covariance of the model parameter could be calculated if the data covariance is known as

$$\text{cov}(m\text{reduced}) = G^{-g}\text{reduced}\text{cov}(d)(G^{-g}\text{reduced})^T. \quad (A-6)$$

For the deviatoric moment-tensor inversion, the uncertainties of the inverted moment-tensor could be estimated similarly to the full moment-tensor inversion:

$$\text{cov}(m\text{reduced}) = \sigma_d^2[G\text{reduced}G\text{reduced}]^{-1}. \quad (A-7)$$

**Step 2: Uncertainties of eigenvalues and eigenvectors of moment-tensor estimation**

For a full moment-tensor, the covariance of the eigenvalues and eigenvectors could be calculated as the following

$$M = S^T AS$$

$$\tilde{F}_{(6,6)} = D_E \left( S^T \Theta S \right)^. \quad (D_5 + I)(I \otimes M)D_\Omega$$

$$K_{(9,6)} = \begin{pmatrix} I & 0 \\ 0 & -(I \otimes S)D_\Omega \end{pmatrix} \tilde{F}_{(6,6)}^{-1}$$

$$\sum_{\{d, \text{vec}(S)\}} = \delta_{k} K^{T} \quad (A-8)$$

where $M$ is the moment matrix, and Matrix $S$ is formed by the eigenvectors of matrix $M$. Matrix $A$ is a diagonal matrix with eigenvalues of matrix $M$, which are also represented in a vector format $\{\lambda_1, \lambda_2, \lambda_3\}$. The $\Theta$ denotes the Khatri-Rao product defined by $A\Theta B = [A_1 \otimes B_1, \ldots, A_p \otimes B_p]$ if $A_i$ and $B_i$ $(i = 1, 2, \ldots, p)$ are (column) portioned matrices of $A$ and $B$ (Rao & Mitra, 1971), and $\otimes$ denotes Kronecker Product defined by $A \otimes B = [a_{ij}B]$ if $A = [a_{ij}]$. $I$ is an identity matrix. The constant matrices $D_E$, $D_5$, and $D_\Omega$ could be written out explicitly (equation 3.1.2, 3.1.3, and 3.1.4 in Han et al., 2007). Additionally, $\tilde{F}_{(6,6)}$ and $K$ are matrices calculated from equation A8, and $\sum_{\{d, \text{vec}(S)\}}$ is the covariance matrix of moment-tensor elements $(M_{ij})$, and $\sum_{\{d, \text{vec}(S)\}}$ is the covariance matrix of the eigenvalues and eigenvectors of moment-tensor $M$.

For a deviatoric moment-tensor, the covariance of the eigenvalues and eigenvectors could be calculated as the following equation:

$$M = \begin{pmatrix} M_{11} & M_{21} & M_{31} \\ M_{21} & M_{22} & M_{32} \\ M_{31} & M_{32} & -M_{11} - M_{22} \end{pmatrix} \quad (A-8)$$

$$\tilde{F}_{(5,6)}^* = D_E \left( S^T \Theta S \right)^. \quad (D_5 + I)(I \otimes M)D_\Omega$$

$$H_{(6,5)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\tilde{H}_{(5,5)} = \tilde{F}_{(5,6)}^* H_{(6,5)}^{-1}$$

$$K_{(11,5)} = \begin{pmatrix} I_{(2,2)} & 0 \\ 0 & -(I \otimes S)D_\Omega^{(9,3)} \end{pmatrix} \tilde{H}_{(5,5)}^{-1}$$

$$\sum_{\{d, \text{vec}(S)\}} = \delta_{k} K^{T} \quad (A-9)$$

For the deviatoric moment-tensor case, the three eigenvalues are no longer independent because they have to satisfy the condition where the summation of three eigenvalues is equal to zero. So the $\lambda^*$ is a 2D vector $\{\lambda_1^*, \lambda_2^*\}$, which represents the two independent
The covariance matrix of deviatoric moment-tensor elements. The matrices $\hat{\mathbf{F}}_{[5,6]}$ and $\mathbf{K}$ are calculated from equation A.9. Additionally, $\sum_{[\nu]}$ is the covariance matrix of deviatoric moment-tensor elements ($\mathbf{M}_{[\nu]}$), and $\sum_{[\nu,vec(Si)]}$ is the covariance matrix of the eigenvalues and eigenvectors of deviatoric moment-tensor $\mathbf{M}$. Lastly, $\mathbf{D}_E$ and $\mathbf{D}_O$ remain the same, as in the full moment-tensor case (Han et al., 2007), in which $\mathbf{D}_E$ is changed into

$$
\mathbf{D}_{E[5,9]} = \begin{bmatrix}
2/3 & 0 & 0 & 0 & -1/3 & 0 & 0 & 0 & -1/3 \\
-1/3 & 0 & 0 & 0 & 2/3 & 0 & 0 & 0 & -1/3 \\
0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\
\end{bmatrix} .
$$

(A-10)

**Step 3: Uncertainties of fault-plane angles estimation**

The angle between slip direction and fault-plane is defined as $\alpha$, so:

$$
\sin \alpha = \frac{-3\nu'_1}{\nu'_1 - \nu'_3} = 3 \frac{\nu'_1 + \nu'_3}{\nu'_1 - \nu'_3} \\
d\alpha = \frac{3}{(\nu'_1 - \nu'_3) \cos \alpha} \left( \frac{1 - \sin \frac{\alpha}{3}}{1 + \sin \frac{\alpha}{3}} \right) \left( d\nu'_1 \right) \\
F_a = \frac{3}{(\nu'_1 - \nu'_3) \cos \alpha} \left( \frac{1 - \sin \frac{\alpha}{3}}{1 + \sin \frac{\alpha}{3}} \right) .
$$

(A-11)

where $\nu'_1$, $\nu'_2$, and $\nu'_3$ are eigenvalues of the deviatoric moment-tensor. To calculate the uncertainties of the other angles (strike and dip) the total differentiation of the unit vectors first shown in equation 12 needs to be obtained. The unit vectors (normal direction $\mathbf{n}$ and simple slip $\mathbf{s}$ away from the plane) are calculated from the eigenvectors (P and T axes), as in equation 8. The following equations are arrived at for calculating the covariance of normal and slip unit vectors:

$$
dn = C_n dt + C_p dp + F_n,dn \\
ds = C_n dt - C_p dp + F_n,dn \\
C_n = \frac{\sqrt{1 + \sin \alpha}}{\sqrt{2}} \\
C_p = \frac{\sqrt{1 - \sin \alpha}}{\sqrt{2}} \\
F_{n(3,2)} = \frac{\cos \alpha}{2\sqrt{2}} \left( \frac{1}{\sqrt{1 + \sin \alpha}} - \frac{1}{\sqrt{1 - \sin \alpha}} \right) \\
F_n = \frac{\cos \alpha}{4} \left( \frac{1}{C_t} - \frac{1}{C_p} \right) F_a \\
F_{t(3,2)} = \frac{\cos \alpha}{2\sqrt{2}} \left( \frac{1}{\sqrt{1 + \sin \alpha}} + \frac{1}{\sqrt{1 - \sin \alpha}} \right) \\
F_a = \frac{\cos \alpha}{4} \left( \frac{1}{C_t} + \frac{1}{C_p} \right) F_a .
$$

(A-12)

where $\text{Cov}(\mathbf{a}, \mathbf{b})$ means the covariance of two vectors $\mathbf{a}$ and $\mathbf{b}$. The next is the strike ($\theta$ angle from north) and dip ($\varphi$) angles of a plane calculated from unit normal direction $(n_1, n_2, n_3)$ in (east, north, up) coordinate system as the following ($\theta_n$ is azimuth angle of the normal direction):

$$
\cos \theta_n = \frac{n_2}{\sqrt{n_1^2 + n_2^2}} \\
d\theta_n = \left( \frac{n_2}{n_1^2 + n_2^2} \right) (d\theta_n) = \frac{F_{\theta} d\theta_n}{d\theta_n} \\
\text{Cov}(\theta, \theta) = \text{Cov}(\theta_n, \theta_n) = F_{\theta} \text{Cov}(n_1, n_2) F_{\theta}^T \\
\cos \varphi = n_3 \\
d(\cos \varphi) = -\sin \varphi d\varphi = dn_3 \\
\text{Cov}(\varphi, \varphi) = \text{Cov}(n_3, n_3)/\sin^2 \varphi .
$$

(A-13)

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Solid angles and the impact of receiver-array geometry on microseismic moment-tensor inversion

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ABSTRACT

Seismic moment tensors provide a concise mathematical representation of point sources that can be used to characterize microseismic focal mechanisms. After correction for propagation effects, the six independent components of a moment tensor can be found by least-squares inversion based on P- and/or S-waveform (or spectral) amplitudes observed at different directions from the source. Using synthetic waveform data, we investigated geometrical factors that affect the reliability of such inversions. We demonstrated that the solid angle subtended by the receiver array, as viewed from the source location, plays a fundamental role in the stability of the inversion. In particular, the condition number of the generalized inverse scales approximately inversely with the solid angle, implying that for a solid angle of zero (as is the case for a single vertical borehole) the inversion is ill-conditioned. The presence of random noise also has a significant effect on the inversion results; our results showed that the signal-to-noise ratio (S/N) for reliable inversion scales approximately as the square root of the condition number. Taken together with geometrical considerations, we found that a S/N > 10 is generally needed to obtain reliable inversion results for the full moment tensor under certain microseismic acquisition scenarios that include dual observation wells or surface star pattern. Our numerical tests indicated that least-squares moment-tensor solutions obtained under nonideal conditions are biased toward limited regions of the full parameter space. In particular, random noise introduces a bias toward volumetric source types, whereas ill-conditioned inversions may exhibit bias toward poorly resolved eigenvector(s) of the inversion matrix. Possible strategies to improve the reliability of moment-tensor inversion include ensuring a nonzero solid-angle aperture by using multiple observation wells, and/or incorporating other types of data such as a priori knowledge of fracture orientation.

INTRODUCTION

Seismic moment tensors provide an idealized mathematical representation of rupture processes within the earth. In global seismology, they have been in routine use for decades, providing valuable insight into earthquake mechanisms (Jost and Herrman, 1989). They are traditionally presented as focal mechanism (a.k.a. beach ball) diagrams, which depict the amplitude radiation pattern for P-waves, revealing the type of fault and possible orientations of the fault plane.

Monitoring microseismicity induced by geothermal field development, geologic sequestration of carbon dioxide, and hydraulic fracture treatments has the potential to improve our understanding of fracture processes and to optimize reservoir production (Lumley, 2001; Baig and Urbancic, 2010; Eaton and Forouhideh, 2010). For example, moment tensors can provide information about the inducing mechanisms, size and orientation of fractures and the connectivity of fracture systems (Maxwell and Urbancic, 2001; Foulger et al., 2004), and whether the fracture is opening or closing (Kirkpatrick et al., 1996). Fracture processes that cause microseismicity may include tensile opening of a fracture (mode I), or slip on a preexisting fracture surface (modes II and III). Accurate estimation of moment tensors using microseismic data, however, remains a significant challenge due to typically low S/N recording conditions and small-aperture acquisition geometry.

In this paper, we consider certain limitations of moment-tensor inversion in a microseismic monitoring environment. After briefly reviewing moment-tensor theory, we use a synthetic-modeling approach to assess fundamental geometrical considerations for inversion of moment tensors. We consider the case of inversion for the full...
moment tensor (six independent parameters) and show that the solid angle subtended by the microseismic array, viewed from the source location, is a key consideration for stability of the inversion. Building on this concept, we then consider the question of whether any bias exists for moment tensors obtained by least-squares inversion of data from noisy data and/or a single vertical observation well.

**MOMENT-TENSOR RADIATION PATTERNS**

Seismic moment tensors can be represented as a $3 \times 3$ array, normalized to unit amplitude (e.g., Lay and Wallace, 1995),

$$
M = M_0 \begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{bmatrix},
$$

where $M_0$ is the seismic moment and each element $M_{ij}$ represents a force couple composed of opposing forces of equal amplitude pointing in the $±i$-direction, separated by an infinitesimal distance in the $j$-direction. $M$ is symmetric, reducing the number of independent moment-tensor elements to six in any coordinate system. For a homogeneous region around a source located at the origin, the moment-tensor elements to six in any coordinate system. For a homogeneous region around a source located at the origin, the moment-tensor elements to six in any coordinate system. For a homogeneous region around a source located at the origin, the moment-tensor elements to six in any coordinate system. For a homogeneous region around a source located at the origin, the moment-tensor elements to six in any coordinate system.

$$
\tau_\rho = \text{traveltime to the receiver, the dot notion signifies time derivative and the summation convention for repeated indices is used. Terms within the braces \{} denote the P-wave radiation pattern. Similarly, the radiated S-wave may be written as}
$$
$$
where $\beta$ is shear-wave velocity and $\delta$ is the Kronecker delta.

Plots of 3D radiation patterns for P- and S-waves (Figure 1) provide a useful tool to visualize the directionality of seismic energy radiated from a moment-tensor source. These diagrams depict a geometrical surface whose radius, for any given ray direction, is given by the terms within the braces \{} in equations 1 and 2. A particularly simple moment tensor is a purely volumetric source (explosion or implosion), for which the moment tensor is represented by the normalized identity matrix. These sources radiate P-waves with uniform amplitude in all directions, with no associated S-waves. Another simple moment tensor is the linear dipole source, which is closely related to a tensile crack opening and is represented by a moment tensor with a single nonzero element along the diagonal. P-waves are radiated preferentially in the axial direction of the linear dipole, and S-waves are radiated preferentially at 45° from the dipole axis (Figure 1). The compensated linear vector dipole (CLVD) has a double-strength force couple along one axis, and unit-strength force couples in the directions of the two perpendicular directions (Lay and Wallace, 1995). CLVD sources have been used to approximate tensile events that occur in hydraulic fracturing (Nolen-Hoeksema and Ruff, 2001). It should be noted that, unlike a tensile crack, a CLVD source does not involve any volumetric change, because strain along one axis is compensated by contraction or expansion along the other axes (Baig and Urbancic, 2010). A double-couple source has two nonzero elements of the moment tensor and provides a representation of slip on a fault. In the case of a double-couple source, both the P-wave radiation pattern and S-wave radiation pattern exhibit fourfold symmetry (Figure 1).

Source-type diagrams (Figure 2) were introduced by Hudson et al. (1989) and provide a convenient graphical representation of seismic moment tensors. This diagram is constructed using a particular transformation of the three real eigenvalues (principal moments) of
a moment tensor, \( M_1, M_2, \) and \( M_3 \). The isotropic component of the moment tensor \( M \) is defined by the average of the principal moments

\[
M = \frac{M_1 + M_2 + M_3}{3}.
\]  

The three deviatoric moments are given by

\[
M'_j = M_j - M, \quad j = 1, 2, 3,
\]  

where \( |M'_1| \geq |M'_2| \geq |M'_3| \). From these basic moments, two parameters \( T \) and \( k \) are defined that characterize the type of shear component in the source and the volumetric component, respectively. These parameters vary between \(-1\) and \(1\) and are given by (Hudson et al., 1989):

\[
T = -2M'_1/M'_2, \quad M'_1 > 0,
\quad T = 0, \quad M'_1 = 0,
\quad T = 2M'_1/M'_2, \quad M'_1 > 0,
\]  

and

\[
k = M |M|^{-1/2} M'_3 \geq 0,
\quad k = M |M|^{1/2} M'_3 \leq 0.
\]  

A crossplot of these parameters naturally separates events on the basis of source type. For example, depending on the sign, \( |k| = 1 \) defines a source with a volume increase (positive) or a volume decrease (negative). Similarly, \( |T| = 1 \) defines a pure CLVD source and double-couple sources correspond to \( k = T = 0 \). The source diagram is constructed based on an equal area projection of \( k \) and \( T \) parameters chosen such that the a priori probability density of the moment ratios is uniform throughout the plot (Hudson et al., 1989).

By separating double-couple, explosive, CLVD, and linear-dipole sources, this diagram emphasizes the type of source mechanism represented by the moment tensor, rather than its absolute magnitude or orientation.

To illustrate synthetic seismograms for various moment-tensor sources, Figure 3 shows several representative microseismic recording geometries for which the source is located at the origin of a Cartesian coordinate system. The first recording geometry comprises several 21-element 3C downhole receiver arrays, representing a dual-well monitoring scenario. For each array, the deepest receiver is located 400-m from the source and at the same depth as the source, with the shallowest receiver located 200 m above the source. In the other microseismic recording geometry, a simple surface array is considered, consisting of 82 3C receivers arranged in an X-pattern with arms that are 2-km in length. The surface array is centered 3 km above the source point.

Synthetic seismograms for a homogeneous background medium are presented in Figure 4 for several types of sources. These simple numerical examples illustrate some of the variability in P- and S-wave amplitudes and polarities that may be expected for double-couple and CLVD sources. For each receiver, the synthetic seismogram was constructed by: (1) calculating direct-arrival times \( \tau_p \) and \( \tau_s \); (2) computing the corresponding amplitudes based on equations 2 and 3, above; (3) convolving impulsive sources, given by \( \delta(t - \tau_p) \) and \( \delta(t - \tau_s) \), with a Gaussian wavelet; (4) multiplying the resulting waveforms by the direct-arrival amplitudes; and (5) summing the P-wave and S-wave results. The capability to distinguish different source types, or to carry out moment-tensor inversion, derives from observed waveform differences such as those illustrated in Figure 4.

**LEAST-SQUARES INVERSION**

Here, we use a simple method for retrieving the seismic moment tensor that is based on a generalized least-squares inversion of the 3C amplitudes of P- and S-wave direct arrivals. This approach that relies on high fidelity of the recorded signal. Considering equations 2 and 3, the relationship between observed data and moment-tensor elements can be written in matrix form:

\[
d = Am + n,
\]  

where \( d = (a_P^1, a_P^2, a_P^3, a_S^1, a_S^2, a_S^3)^T \) defines the observed amplitudes values of the direct arrivals, the subscript of \( a \) denotes the Cartesian component \( \{1, 2, 3\} \) and the superscript of \( a \) denotes the wave type. In addition, \( m = (M_{11}, M_{22}, M_{33}, M_{12}, M_{13}, M_{23})^T \) defines the six independent components of the seismic moment tensor and \( n \) denotes noise. For a single receiver, the \( 6 \times 6 \) matrix \( A \) can be easily surmised from equations 2 and 3. For \( n \) receivers, the \( 6n \times 6 \) system can be formed by appending additional rows, as required, onto \( d \) and \( A \). A least-squares solution to this over-determined system can be obtained using (Menke, 1989)

\[
m = A^{-1} d = (A^T A)^{-1} A^T d,
\]  

where the notation \( A^{-1} \) denotes the generalized inverse, as defined by the term on the right.
The inverse is constructed from 3C amplitudes of both P- and S-waves. As this study focuses primarily on the geometrical aspects of microseismic array design, the problem of robust and accurate estimation of observed P- and S-wave amplitudes from microseismic observations (e.g., Urbancic et al., 1993) is not considered here. Furthermore, uncertainties for the moment-tensor elements that arise from incorrect hypocenter locations and/or inaccurate background velocity model, although very important for moment-tensor inversion (e.g., Vavrycuk, 2007), are beyond the scope of the current study. Instead, we focus here on geometrical aspects of the recording array.

To investigate the influence of array design for computation of the generalized inverse, we consider the stability of the matrix inversion for \( B = A^T A \). In general, this can be characterized by the condition number for \( B \) (Cheney and Kincaid, 2008):

\[
\kappa(B) = \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}}.
\]

where \( \lambda_{\text{max}} \) and \( \lambda_{\text{min}} \) are maximum and minimum (by moduli) eigenvalues of \( B \), respectively. A matrix with a large condition number is more prone to numerical instability in the presence of noise (e.g., Cheney and Kincaid, 2008). We therefore use the condition number for \( A^T A \) as a proxy for the degree of instability for the generalized inverse. We also investigate, explicitly, the effects of random noise on the solution.

**SOLID ANGLE**

Although the effects of network geometry are well known in earthquake seismology (e.g., Bondar et al., 2004), the recording

![Figure 4](image_url)

**Figure 4.** Examples of synthetic seismograms for two different source types (double couple and CLVD) computed using equations 2 and 3, using the two recording geometries in Figure 3. The number in the top left of each diagram denotes a multiplicative factor applied to the P-wave data prior to summation with the S-wave data. Note differences in polarity and relative amplitudes of the arrivals, which form the basis for distinguishing between different source types.
geometry of earthquake networks differs markedly from the microseismic case in having a significantly wider observational aperture.

To explore the relationship of recording geometry to the inverse problem for a narrow aperture, consider the canonical scenario depicted in Figure 5. A set of 20 receivers is randomly distributed within a triangular patch on the unit sphere, with three additional receivers at the vertices of the patch. Numerical computations show that as the area of the receiver patch increases the condition number of $A^T A$ decreases (Table 1), implying that the generalized inverse becomes more stable with increasing patch size. Furthermore, tests computed using only the three vertices of the triangular patch have lower condition number than those computed using interior receivers as well as the vertices (Figure 6). This occurs because the inclusion of interior receivers increases the number of observations (and thus the dimensions of the matrix) without changing the aperture of the array. We conclude that, for the purposes of stable inversion for seismic moment tensors, receivers located on the perimeter of the array are the most important.

Numerical tests undertaken on spherical surfaces of different radii reveal that the area of the receiver patch is not, of itself, a sufficient indicator; i.e., for two receiver patches of equal area on spheres of different radius, the patch that is closer to the source has a lower condition number (more stable inverse). Here, we test the conjecture that a more suitable parameterization for this problem is the solid angle (Eaton, 2009), given by

$$\Omega = \int_S \frac{\mathbf{r} \cdot \mathbf{\hat{n}}}{r^3} \, dS,$$

where $r = |\mathbf{r}|$ is radial distance from the source to the receiver and $\mathbf{\hat{n}}$ is the outward-pointing surface normal to the receiver patch. The solid angle has a range from zero to $4\pi$ steradians (sr), and represents the surface area for the projection of the receiver patch onto the unit sphere. The solid angle ($\Omega$) is the 3D equivalent of the more familiar angle subtended by two intersecting line segments.

As illustrated in Figure 6, the condition number for the generalized inverse decreases with increasing solid angle. We propose that the solid angle subtended by the receiver array, as viewed from the source, represents an important design parameter for microseismic arrays. Survey-design applications of the solid angle are considered in the sections below.

### Table 1. Results of canonical tests for solid angle.

<table>
<thead>
<tr>
<th>Solid angle ($\Omega$) of triangular patch (sr)</th>
<th>Condition number (vertices only)</th>
<th>Condition number (random receivers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0004</td>
<td>2911</td>
<td>12,586</td>
</tr>
<tr>
<td>0.0017</td>
<td>831</td>
<td>2026</td>
</tr>
<tr>
<td>0.0038</td>
<td>454</td>
<td>973</td>
</tr>
<tr>
<td>0.0067</td>
<td>326</td>
<td>689</td>
</tr>
<tr>
<td>0.0104</td>
<td>267</td>
<td>498</td>
</tr>
<tr>
<td>0.0149</td>
<td>233</td>
<td>394</td>
</tr>
<tr>
<td>0.0202</td>
<td>211</td>
<td>368</td>
</tr>
<tr>
<td>0.0262</td>
<td>194</td>
<td>305</td>
</tr>
<tr>
<td>0.0329</td>
<td>180</td>
<td>269</td>
</tr>
<tr>
<td>0.0404</td>
<td>168</td>
<td>247</td>
</tr>
<tr>
<td>0.0485</td>
<td>156</td>
<td>230</td>
</tr>
</tbody>
</table>

$^a$The solid angle (units, steradians or sr) is numerically equal to the area of the patch on a unit sphere. Representative triangular patches are shown in Figure 5.

$^b$Tests conducted using three receivers located at the vertices of a triangular patch on the unit sphere.

$^c$Tests conducted using 20 receivers randomly distributed within a triangular patch on the unit sphere, with three additional receivers at the vertices.

---

### EFFECTS OF RANDOM NOISE

The presence of random noise is unavoidable for microseismic field observations and will generally lead to artifacts that contaminate the retrieved seismic moment tensors. The susceptibility of a matrix inversion to noise is related to the condition number (Cheney and Kincaid, 2008). Here, we have undertaken a series of tests to

---

**Figure 5.** Canonical tests of random receivers within a triangular patch on the unit sphere. Patch areas are summarized in Table 1.

**Figure 6.** Graph of condition number versus solid angle subtended by the receiver array (Figure 5). Top curve shows results for a full set of 20 random receivers plus three receivers at the vertices of the triangular patch. Lower curve shows the vertices only. In general, condition number decreases with increasing solid angle. The vertices appear to contribute the most to stability of the inversion. See text for details.
characterize the effects of noise in relation to solid angle. Figure 7a presents a series of tests computed using three receiver arrays with solid angles from .0004 sr to 0.0485 sr, with varying levels of S/N, which is defined here as

\[ S/N = \frac{\max (A \cdot m)}{\max (n)}, \]  

(12)

where \( A \cdot m \) represents the modeled 3C amplitude components of direct P- and S-wave arrivals and \( n \) is additive random white noise. The normalized error is given by

\[ E = \left( \frac{1}{9} \sum_{i=1}^{3} \sum_{j=1}^{3} (\hat{M}_{ij} - M_{ij})^2 \right)^{1/2}, \]  

(13)

where \( M_{ij} \) are the moment-tensor elements used as input and \( \hat{M}_{ij} \) are the corresponding values estimated from the noisy data using the generalized inverse. As expected, normalized error decreases with increasing S/N. In addition, normalized error decreases with increasing solid angle \( \Omega \).

To investigate the relationship of solid angle to S/N, we define an error threshold to be \( E_{\text{thresh}} = 1/e \sim 0.3679 \). This arbitrary value was selected for purposes of comparison, to represent a maximum acceptable error in the retrieved moment tensor. We then compute the corresponding value of “acceptable” S/N (denoted \( \xi \)). Our calculations (Figure 7b) show that although \( \xi \) generally decreases with increasing \( \Omega \), for many of the models considered a S/N of approximately 10 or better is required to obtain satisfactory inversion results.

Figures 8 and 9 show P- and S-wave radiation patterns that depict the effects of random noise. These figures were obtained using an arbitrary input moment tensor (a single realization of the random moment tensors described below) given by

Figure 8. P-wave and S-wave radiation patterns corresponding to an input moment-tensor source and the moment tensor obtained by inversion for a S/N of 10 and 0.5. Calculations were performed using the surface recording geometry (Figure 3).

Figure 9. P-wave and S-wave radiation patterns corresponding to an input moment-tensor source and the moment tensor obtained by inversion for a S/N of 10 and 0.5. Calculations were performed using the downhole geometry (Figure 3), which has dual borehole arrays.
based on the surface recording array in Figure 3 with S/N of 10 and 0.5, respectively. For reference, the surface and dual monitoring well recording geometries considered here subtend nearly identical solid angles of 0.034 and 0.033 sr; the corresponding inversion condition numbers are 157 and 128, respectively. For the surface recording geometry (Figure 8) with a S/N of 10, the inverted radiation patterns closely resemble those for the input source mechanism. For a S/N of 0.5, however, the radiation pattern is significantly different from the input source. Figure 9 shows similar results obtained using the dual borehole array (Figure 3). These examples demonstrate the strong effect that random noise has on the moment-tensor inversion results. We remark that, in practice, the noise characteristics of surface and borehole recordings are likely to be much different from each other, with significantly higher S/N expected in a borehole environment.

STOCHASTIC MODELING

Figures 10 and 11 present inversion results for surface and dual borehole arrays, respectively, based on 1000 input moment-tensor sources, in which each element of the moment tensor is treated as a random variable. The source locations are fixed for all of these synthetic tests. For ease of comparison, the results are displayed using a source-type diagram as in Figure 2. For each row, the left panel shows a scatter plot and the right panel shows a density plot (the number of sources within each indicated bin). The random input event distribution is different for the two recording scenarios, but each represents a similar distribution that provides a quasi-uniform sampling of the source-type parameter space.

In the case of S/N = 10, we find that the distribution of source types after inversion also appears to span the source-type parameter space for both recording geometries (Figures 10 and 11). Although there are slight differences between inverted and input source points, consistent with results presented in Figures 8 and 9, there is no apparent bias in the overall source-type distribution. In the case of S/N = 0.5; however, both recording geometries exhibit an inversion bias that manifests as a concentration of source mechanisms in the vicinity of an explosive (or implosive) regime. This bias emerges as a result of random noise being added to the P- and S-wave amplitudes because random noise has no preference in radiation and thus manifests as an isotropic source. For this reason, the volumetric component of seismic moment tensors are often discarded or set to zero (e.g., Vavrycuk, 2007). It should be noted, however, that real microseismic data can contain coherent noise that does include directional characteristics.

A final numerical test is presented in Figure 12. In this case, least-squares inversion for moment-tensor elements is attempted using modeled data for a single vertical downhole array. Because the solid angle subtended by a vertical array of geophones is identically zero, the condition number for the matrix $A^T A$ tends toward infinity.

Figure 10. Stochastic modeling of moment-tensor inversion using the surface recording geometry (Figure 3). Left panels show source-type scatter plots based on 1000 random sources. Right panels show density of sources within discrete bins. Inversion performed with S/N = 0.5 appears to produce a significant source-type bias.
this case the shallower receiver level has been offset slightly (10 cm) to ensure a large but finite condition number \((1.4 \times 10^{19})\). As a consequence of the ill-conditioned nature of the inversion, the numerical solution obtained contains a strong inversion bias despite the high S/N of 10. For the particular choice of receiver geometry considered in these simulations, the inversion results appear to cluster around the tensile crack opening (or closing) domain. Insights into the origin of this spurious effect can be obtained by considering the singular-value decomposition of \(A^TA\), which may be written as (e.g., Lay and Wallace, 1995)
where $V$ is a matrix of eigenvectors and $\Lambda$ is a diagonal matrix containing the corresponding eigenvalues. These eigenvectors collectively span the model space, and those associated with numerically small eigenvalues represent the least well-resolved components of the model (i.e., they are close to the null space of the inverse problem; Menke, 1989). In the numerical example considered here, one of the eigenvalues is negligibly small compared to the remaining five. The corresponding eigenvector (indicated by the diamonds in Figure 12) plots close to the dipole source type; ill-conditioned solutions cluster in the region between this poorly resolved source component and the volumetric end-member associated with random noise (see above). Although this example is deliberately ill-conditioned for purposes of demonstrating the concept, it is clear that, in general, caution should be exercised in the interpretation of moment-tensor inversion results in cases where the receiver geometry is inadequate.

**CONCLUSIONS**

Seismic moment tensors provide a useful mathematical representation of microseismic sources as a set of force couples acting at a point, with no net torque. Moment tensors can be visualized using P- and S-wave radiation patterns, which depict the outward radiation of seismic wave energy within a homogeneous region around the source. Robust determination of moment tensors from microseismic observations can provide a basis for distinguishing different modes of fracturing. Here, the moment-tensor inversion problem is cast as a generalized inversion using three-component P- and S-wave direct-arrival amplitudes. As a design consideration for microseismic surveys, we test a conjecture that the solid angle subtended by the receiver array is related to the condition number of the generalized inverse. Numerical tests presented here suggest that the moment-tensor inversion is more stable (i.e., the condition number of the generalized inverse decreases) with increasing solid angle. Furthermore, receivers located on the perimeter of the receiver array make the greatest contribution to the stability of the inversion. These factors should be considered in the design of microseismic arrays for which the recovery of moment tensors is deemed important. In particular, a single vertical observation well subtends a solid angle of zero; hence, in the absence of a priori constraints on the solution, this type of array is not suitable for moment-tensor determination because the inverse problem is ill-conditioned.

Random noise introduces artifacts into moment-tensor inversion, the severity of which depends on the condition number and thus the solid angle ($\Omega$) subtended by the receiver array. For most of the tests considered here, a S/N of approximately 10 is required to produce acceptable inversion results (i.e., normalized error $<1/\epsilon$). Stochastic tests suggest that if the S/N is insufficient, inversion results are biased toward explosive (or implosive) source types. Furthermore, if the inversion is ill-conditioned due to poor recording geometry, derived moment tensors are generally biased toward poorly resolved eigenvector(s) of the inversion matrix.

We note that our results incorporate only the effects of random noise and there are several important factors in the determination of moment tensors using real data that are not considered here. These include the effects of hypocenter location errors and/or errors in the background velocity model. We have also neglected the effects of anelastic attenuation on seismic wave amplitudes, which are likely to be more significant for a surface recording geometry than for borehole microseismic recordings. Further research is needed to quantify the sensitivity of moment-tensor inversion to these other factors for typical small-aperture microseismic surveys used for hydraulic-fracture monitoring.

**ACKNOWLEDGMENTS**

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Focal mechanism determination of induced microearthquakes in an oil field using full waveforms from shallow and deep seismic networks

Junlun Li¹, H. Sadi Kuleli¹, Haijiang Zhang¹, and M. Nafi Toksöz¹

ABSTRACT

A new, relatively high frequency, full waveform matching method was used to study the focal mechanisms of small, local earthquakes induced in an oil field, which are monitored by a sparse near-surface network and a deep borehole network. The determined source properties are helpful for understanding the local stress regime in this field. During the waveform inversion, we maximize both the phase and amplitude matching between the observed and modeled waveforms. We also use the polarities of the first P-wave arrivals and the average S/P amplitude ratios to better constrain the matching. An objective function is constructed to include all four criteria. For different hypocenters and source types, comprehensive synthetic tests showed that our method is robust enough to determine the focal mechanisms under the current array geometries, even when there is considerable velocity inaccuracy. The application to several tens of induced microseismic events showed satisfactory waveform matching between modeled and observed seismograms. Most of the events have a strike direction parallel with the major northeast-southwest faults in the region, and some events trend parallel with the northwest-southeast conjugate faults. The results are consistent with the in situ well breakout measurements and the current knowledge on the stress direction of this region. The source mechanisms of the studied events, together with the hypocenter distribution, indicate that the microearthquakes are caused by the reactivation of preexisting faults. We observed that the faulting mechanism varies with depth, from strike-slip dominance at shallower depth to normal faulting dominance at greater depth.

INTRODUCTION

Induced seismicity is a common phenomenon in oil/gas reservoirs accompanying changes in internal stress due to water injection or water/oil/gas extraction, etc. (e.g., Suckale, 2010; Maxwell et al., 2010). For example, the gas/oil extraction can cause reservoir compaction and reactivate preexisting faults and induce microearthquakes (e.g., Chan and Zoback, 2007; Miyazawa et al., 2008; Sarkar et al., 2008), or injection of water can cause the decrease of effective stress and slippage along preexisting faults (Grasso, 1992). The reactivation of preexisting faults is very likely responsible for the sheared casings of production wells in some fields (Maury et al., 1992) or is a serious source of wellbore instability during drillings (Willson et al., 1998; Zoback and Zinke, 2002). The hydraulic fracturing activities in an enhanced geothermal system or in shale gas extraction can also result in crack openings and closures and induce microseismicity (Baig and Urbancic, 2010). Through the studying of locations and source characteristics (e.g., focal mechanism) of the induced seismicity over an extended time period, temporal and spatial changes of the stress in the fields may be reconstructed; this can help to understand the intrinsic response of geological formations to the stress disturbance.

Microearthquakes usually have small magnitudes and are generally recorded at sparse local stations. As a result, it is difficult to obtain enough seismic waveforms with high signal-to-noise ratio for picking the polarity information of first P-wave arrivals. Therefore, it is challenging to use only the P-wave polarity information (even when adding S/P amplitude ratios) as used in conventional methods to constrain the focal mechanisms of the induced earthquakes (e.g., Hardebeck and Shearer, 2002, 2003), especially when there are only a limited number of stations. Waveform matching has been used to determine earthquake focal mechanisms on a regional
and global scale using low frequency waveform information (e.g., Štlený et al., 1992; Zhao and Helmberger, 1994; Tan and Helmberger, 2007). Štlený et al. (1992) use waveform matching to determine the best-fit focal mechanism, source time function and source depth. Zhao and Helmberger (1994) allowed time-shift in the synthetic seismograms to account for the imperfect Green’s functions when matching the synthetic with observed seismograms. Tan and Helmberger (2007) match the direct P-arrival phases (the first whole cycle after initial P-arrival) between synthetic and observed seismograms to determine the focal mechanisms. However, in the case of induced seismicity, waveforms usually have higher frequencies. There have been many studies on determining the focal mechanism of the induced seismicity in the cases of enhanced geothermal system development, mining, and hydraulic fracturing. Godano et al. (2011) use the direct amplitudes of P, SV, and SH to study the focal mechanisms of induced microearthquakes in a geothermal site using full-space homogeneous velocity models. Nolen-Hoeksema et al. (2001) use the first half cycle after the first arrivals from the observed seismograms and synthetics from full-space Green’s functions to determine the focal mechanisms of several hydraulic fracture events. Julian et al. (2007) use first arrival polarities and amplitude ratios from 16 three-component borehole stations and 14 three-component surface stations to determine the full moment tensors of the induced events and studied the volume change accompanying the geothermal process. High frequency waveform matching, in addition to polarity information, has been used to determine the focal mechanisms of induced earthquakes in a mine with a dense network of 20 stations (Julia and Nyblade, 2009). Julia and Nyblade (2009) use a full-space homogeneous model to calculate the Green’s functions, and they performed the focal mechanism inversion in the frequency domain without phase information in a least-squares sense between the synthetic and filtered observed data generally below 10 Hz. The simplification to the full-space homogeneous model is valid when the receivers are deployed deep in the subsurface and close to the induced events, such as deploying borehole monitoring sensors in the vicinity of the hydraulic well, or when complexities in rock structure are not large compared to the frequencies recorded.

To retrieve reliable solutions, we developed a method to use high-frequency, full waveform information (both P and S) to determine the focal mechanisms of small earthquakes (Li et al., 2011). Using the known velocity model (one-dimensional layered model in this study), we calculate the Green’s functions for all moment tensor components of the source at each location (hypocenter) and then the synthetic seismograms by convolving them with the source time function. To find the best match between the observed and synthetic seismograms, we formulate an objective function that incorporates information from different attributes in the waveforms: the cross-correlation values between the modeled waveforms and the data, the $L_2$ norms of the waveform differences, and the polarities of the first P arrivals and the S/P average amplitude ratios. Compared to previous studies, our method uses more attributes of seismograms to better determine the focal mechanisms of induced seismicity. The “high frequency” referred to in our study (several hertz for the shallow network and tens of hertz for the deep network) is a relative term: it is much higher than the frequency band (0.05–0.5 Hz) often used in the study of large earthquakes (e.g., Tan and Helmberger, 2007), but it is lower than the frequency band often used for exploration seismic imaging (e.g., Etgen et al., 2009). Essentially, the frequency bands used in our study include a considerable portion of the energy radiated from the source; thus, the waveforms have good signal-to-noise ratio (S/N) and can reflect the characteristics of the source rupture.

Compared with full waveform tomography or migration techniques, which focus on improving the knowledge of the subsurface structures illuminated by simple active sources with known signatures (e.g., explosion or vibration source with known location and origin time; similar frequency, amplitude, radiation pattern, etc., are expected for all shots), the source mechanism determination method assumes the velocity model input, and focuses on determining the complicated source signature associated with the events. For induced seismicity in oil and gas fields, the velocity model is generally known from seisms and well logs. Comprehensive synthetic tests with random velocity perturbations are also performed to examine the robustness of our algorithm in the presence of the velocity uncertainties.

Previously, we tested our newly developed focal mechanism determination method on induced microearthquakes monitored by a five-station surface network at an oil field in Oman (Li et al., 2011). The field, operated by Petroleum Development Oman (PDO), was discovered in 1962 and put into production in 1969. An official program to monitor induced seismicity using a surface station network in the field commenced in 1999, and a borehole network was installed in February of 2002. The primary objective of this passive seismic monitoring program was to locate the events and to correlate them with production and injection activities to understand and monitor the cause of induced seismicity in the field. In this paper, we apply the newly developed focal mechanism determination method to data from the borehole network. The source mechanisms determined using the borehole network are compared to those determined using the surface network. The robustness of the method is tested extensively on synthetic data sets generated for both the surface and borehole networks using a randomly perturbed velocity model.

### INDUCED MICROEARTHQUAKE DATA SET

The petroleum field discussed in this paper is a large anticline created by deep-seated salt movement (Sarkar, 2008). The dome is about 15 x 20 km in size with a northeast-southwest axial elongation that is probably a result of regional deformation. The structure is dominated by a major central graben and two systems of faulting with two preferred directions (southeast-northwest and northeast-southwest) that affect the trapping mechanism in the oil reservoir. The northeast-southwest major network of faults and fractures partially connects all parts of the fields (Figures 1, 2). The main oil production is from the Lower Cretaceous Shuaiba chalk overlain unconformably by Nahr Umr shale, while gas is produced from the shallower Natih Formation overlain by the Fiqa shale Formation (Sarkar, 2008; Zhang et al., 2009).

Since 1996, increasing seismic activity has been reported by the staff working in the field. Significant surface subsidence in the center of the field has also been observed by InSAR, GPS, and leveling surveys, and has been attributed to compaction of the Natih formation (Bourne et al., 2006). To monitor the induced seismicity in the field, PDO first deployed a surface array of monitoring stations in 1999 (Figure 1). The stations are instrumented with SM-6B geophones with a natural frequency ($f_n$) of 4.5 Hz. In 2002, another network, independent of the shallow network, was installed in...
the field as part of a Shell/PDO collaborative study (Figure 2). Unlike the surface array/shallow network, this network had borehole installations of seismic sensors (SM-7m, $f_n = 30$ Hz) at multiple levels, roughly ranging from depths of 750 to 1250 m. The instrumentation for this network was much deeper than that of the surface network, and therefore, this monitoring network is referred to as the "borehole network." A schematic diagram of the wells and sensor positions is shown in Figure 2. The borehole network consisted of five closely spaced monitoring wells in the most seismically active part of the reservoir and covered a much smaller area than the surface network. Due to sensor positions at depths, the ability to acquire data at much higher frequencies and the proximity to the two producing units (Natih gas and Shuaiba oil), the deep network recorded much smaller magnitude events than the shallow network, resulting in a greatly increased detectability of induced seismicity (roughly about 25 times more induced events per day) compared to the shallow network. The borehole network was operational for about 18 months starting in February 2002; however, only microseismic data from the last 11 months (October 2002–August 2003) were available for this study. During that 11-month monitoring period, about 15,800 events were identified with an average rate of ~47/day, out of which we analyzed and located about 5,400 events (Sarkar, 2008). Attempts were made to select common events detected during this period by both (deep and shallow) networks for a joint location analysis; however, due to clock synchronization problems and difference in sensor frequency bands between the two networks, the common events could not be identified, and hence the task could not be accomplished. Some research indicated that by carefully identifying the largest events in different networks, synchronization between networks sometimes can be achieved by shifting the origin times in one network with a constant time (Eisner et al., 2010). A similar strategy will be adopted in the future.

During the period of 1999 to 2007, over 1500 induced earthquakes were recorded by the surface network, and their occurrence frequency was found to be correlated with the amount of gas production (Sarkar, 2008). The distribution of induced events in the field recorded by the surface network is shown in Figure 1 (Sarkar, 2008; Sarkar et al., 2008; Zhang et al., 2009). All the events have a residual traveltime of less than 30 ms, indicating they are well located. Figure 2 shows the microearthquake locations determined using the deep borehole network and the double-difference tomography method (Zhang et al., 2009). The root-mean-square

Figure 1. Distributions of near-surface stations and located events. (a) Map view of the studied field. The blue hexagons (E1, E2, and E3) are the epicenters of synthetic events and the green triangles (VA11, VA21, VA31, VA41, and VA51) are the five near-surface stations. These stations are located in shallow boreholes, 150 m below the surface, to increase the signal-to-noise ratio (S/N). The black lines are the identified faults. (b) Side view of the studied field. Most of the induced microearthquakes are localized around 1 km below the surface. A few shallow events have the largest traveltime residues among all events.

Figure 2. (a) Map view of the borehole network and the microearthquakes located by this network. The yellow diamonds (E4, E5) are the epicenters of synthetic events. The green circles are the surface locations of the five wellbores where receivers are installed. (b) Side view of the borehole network and located microearthquakes. The green triangles indicate the borehole stations. The vertical distance between two consecutive receivers in a monitoring well ranges from ~20 to ~70 m.
traveltime residual is around 10 ms (Zhang et al., 2009). In the map view, the earthquakes can be found mainly distributed along the mapped two northeast-southwest fault systems. This earthquake distribution suggests that most of the earthquakes are induced by the reactivation of the existing faults in the field. Figure 3 and Figure 4 show typical events and their spectrograms recorded by the surface network and borehole network, respectively. Because of the proximity of the earthquake source to the deep borehole network, the frequency content of the recorded waveform by the borehole network is much higher than by the surface network. For the waveforms recorded by the surface network, there is a considerable amount of energy in the frequency range of 3 to 9 Hz (Figure 3). For the deep borehole network, the recorded waveforms contain significant energy between 15 to 35 Hz (Figure 4).

Figure 3. The vertical components of seismograms of a typical event recorded by the surface network and the corresponding spectrograms. The filtered seismograms (3 ~ 9 Hz) are in the left column; the original seismograms are in the middle; the spectrograms of the original seismograms are at the right. The zero time is the origin time of the event.

Figure 4. The vertical components of seismograms of a typical event recorded by the borehole network. The filtered seismograms (15 ~ 35 Hz) are in the left column; the original seismograms are in the middle; the spectrograms of the original seismograms are at the right. The zero time is the origin time of the event. It should be noted that the borehole data are dispersive, i.e., higher frequency contents arrive later as the energy is trapped within layers and propagates as guided waves.

FOCAL MECHANISM DETERMINATION METHOD

A detailed description of the method can be found in Li et al. (2011). Here, the method is briefly explained. The focal mechanism can be represented by a three-by-three second order moment tensor with six independent components (Aki and Richards, 2002). Here, we assume the focal mechanism of the small induced events can be represented by pure double couples (Rutledge and Phillips, 2002), though it is possible that a volume change or compensated linear vector dipoles (CLVD) part may also exist, especially in hydraulic fracturing cases, and the non-double-couple components are informative for understanding the rock failure under high-pressure fluid (Ross and Foulger, 1996; Jechmutálová and Eisner, 2008; Šílený et al., 2009; Song and Toksöz, 2010). The constraining of focal
mechanism as double couple (DC) can eliminate the spurious non-DC components in the inversion raised by modeling the wave propagation in anisotropic medium with isotropic Green’s functions or inaccuracy of the velocity model (Šliený and Vavryčuk, 2002; Godano et al., 2011). However, if strong non-DC components actually exist in the source rupture process, the determined fault plane may be biased (e.g., Jechumlálová and Šliený, 2001, 2005). In our analysis, we describe the DC focal mechanism of seismic source in terms of its strike (Φ), dip (δ), and rake (ξ), and determine double couple components from these three parameters. The simplification of the source is supported by the observation that almost all the detected microearthquakes occurred along preexisting faults, i.e., reactivated faults slipping along preexisting weak zones would not cause significant volumetric or CLVD components (Julian et al., 1998). For each component of a moment tensor, we use the discrete wavenumber method (DWN) (Bouchon, 1981, 2003) to calculate its Green’s functions \( G_{ij,k}(t) \) for the horizontally layered medium. Appendix A gives the modified reflectivity matrix for computing the seismograms when the receiver is deeper than the source, such as in the borehole monitoring case. It should be noted that if the full moment tensor needs to be determined, e.g., in the hydraulic fracturing cases, the seismic source should be described with six independent tensor components, which will increase the cost in searching for the best solution. The structure between the earthquake and the station is represented as a 1D horizontally layered medium, which can be built from (1) averaging borehole sonic logs across this region, or (2) extracting the velocity structure between the source and the receiver from the 3D velocity model from double-difference seismic tomography for passive seismic events (Zhang et al., 2009).

The modeled waveform from a certain combination of strike, dip, and rake is expressed as a linear combination of weighted Green’s functions:

\[
V_i^m = \sum_{j=1}^{3} \sum_{k=1}^{3} m_{jk} G_{ij,k}^m(t) * s(t),
\]

where \( V_i^m \) is the modeled \( i \)th (north, east, or vertical) component at station \( n \); \( m_{jk} \) is the moment tensor component and is determined by the data from all stations; \( G_{ij,k}^m(t) \) is the \( i \)th component of the Green’s functions for the \( (j,k) \) entry at station \( n \), and \( s(t) \) is the source time function. In this study, a smooth ramp is used for \( s(t) \), the duration of which can be estimated from the spectra of the recorded seismograms (Bouchon, 1981). The source time functions are found to be insensitive to the waveform fitting, as both the synthetic and observed seismograms are low-pass filtered before comparisons (Zhao et al., 2006). Using reciprocity by straining Green’s tensors can improve the efficiency of calculating the Green’s functions, especially when the sources greatly outnumber the stations (Eisner and Clayton, 2001; Zhao et al., 2006). For instance, only one numerical simulation with reciprocity (e.g., finite difference method), by setting a source at a station, is needed to calculate the Green’s functions for all six components of the moment tensor between anywhere in the field and one component at the station in a 3D heterogeneous medium.

Earthquake locations are usually provided by the traveltime location method. However, due to uncertainties in velocity model and arrival times, the seismic event locations may have errors, especially in focal depth determined from the surface network. While searching the modeled and observed waveforms, we also search for an improved location \((x, y, z)\) around the catalog location.

To determine the best solution, we construct an objective function that characterizes the similarity between the modeled and observed waveforms. We use the following objective function, which evaluates four different aspects of the waveform information:

\[
\text{maximize} \left( J(x, y, z, \Phi, \delta, \lambda, t_s) \right) = \sum_{n=1}^{N} \sum_{j=1}^{3} \left\{ a_1 \max \left( \tilde{d}_n^j \otimes \tilde{v}_n^j \right) - a_2 \| \tilde{d}_n^j - \tilde{v}_n^j \|_2 
+ a_3 f(\text{pol}(\tilde{d}_n^j), \text{pol}(\tilde{v}_n^j)) + a_4 h \left( \text{rat} \left( S(\tilde{d}_n^j), P(\tilde{d}_n^j) \right), \text{rat} \left( S(\tilde{v}_n^j), P(\tilde{v}_n^j) \right) \right) \right\}.
\]

(2)

Here \( \tilde{d}_n^j \) is the normalized data and \( \tilde{v}_n^j \) is the normalized modeled waveform; \( x, y, \) and \( z \) are the event hypocenter that will be redetemined by waveform matching; \( t_s \) is the time shift that gives the largest crosscorrelation value between the observed and synthetic seismograms (first term). Because it is difficult to obtain accurate absolute amplitudes due to site effects in many situations, we normalize the filtered, observed, and modeled waveforms before comparison. The normalization used here is the energy normalization, such that the energy of the normalized wavetrain within a time window adds to unity. Compared to peak amplitude normalization, energy normalization is less affected by site effects, which may cause abnormally large peaks due to focusing and other factors. In a concise form, this normalization can be written as

\[
\tilde{d}_n^j = \frac{d_n^j}{\sqrt{\int_{t_1}^{t_2} (d_n^j)^2 dt}}
\]

(3)

where \( t_1 \) and \( t_2 \) are the boundaries of the time window.

The objective function \( J \) in equation 2 consists of four terms. \( \alpha_i \) through \( \alpha_i \) are the weights for each term. Each weight is a positive scalar number and is optimally chosen in a way such that no single term will overdominate the objective function. We used \( \alpha_1 = 3, \alpha_2 = 3, \alpha_3 = 1 \) and \( \alpha_4 = 0.5 \) for the synthetic tests and real events. The first term in equation 2 evaluates the maximum crosscorrelation between the normalized data \( \tilde{d}_n^j \) and the normalized modeled waveforms \( \tilde{v}_n^j \). From the crosscorrelation, we find the time-shift \( t_s \) to align the modeled waveform with the observed waveform. The second term evaluates the L2 norm of the direct differences between the aligned modeled and observed waveforms (note the minus sign of the second term to minimize the amplitude differences). The first two terms are not independent of each other, however, they have different sensitivities at different frequency bands and by combining them together the waveform similarity can be better characterized. The third term evaluates whether the polarities of the first P-wave arrivals as observed in the data are consistent with those in the modeled waveforms. The pol is a weighted sign function which can be \( \{\beta, -\beta, 0\} \), where \( \beta \) is a weight reflecting our confidence in picking the polarities of the first P-wave arrivals in the observed data. Zero (0) means undetermined polarity; \( f \) is a function that penalizes the polarity sign inconsistency in such a way that the polarity consistency gives a positive value, while polarity inconsistency gives a negative value. The matching of the first P-wave polarities between
modeled and observed waveforms is an important condition for determining the focal mechanism, when the polarities can be clearly identified. Polarity consistency at some stations can be violated if the polarity is not confidently identified (small β) and the other three terms favor a certain focal mechanism. Therefore, the polarity information is integrated into our objective function in a flexible way. By summing over the waveforms in a narrow window around the arrival time and checking the sign of the summation, we determine the polarities robustly for the modeled data. For the observed data, we determine the P-wave polarities manually.

The fourth term in the objective function is to evaluate the consistency of the average S/P amplitude ratios in the observed and modeled waveforms (Hardebeck & Shearer, 2003). The “rat” is the ratio evaluation function and it can be written as

$$\text{rat} = \frac{\int_{T_1}^{T_2} |r^p(t)| \, dt}{\int_{T_1}^{T_2} |r^s(t)| \, dt},$$

where $[T_1, T_2]$ and $[T_3, T_4]$ define the time window of P- and S-waves, respectively, and $r^p$ or $r^s$ denotes either $d^p$ or $v^s$. The term $h$ is a function that penalizes the ratio differences so that the better matching gives a higher value. Note that here we use the unnormalized waveforms $d^p$ and $v^s$.

In general, the amplitudes of P-waves are much smaller than those of S-waves. To balance the contribution between P- and S-waves, we need to fit P- and S-waves separately using the first two terms in equation 2. Also, by separating S- from P-waves and allowing an independent time-shift in comparing observed data with modeled waveforms, it is helpful to deal with incorrect phase arrival time due to incorrect $V_P/V_S$ ratios (Zhu and Helmberger, 1996). Here, we allow independent shifts for different stations as well as for P- and S-waves. We calculate both the first P- and S-arrival times by the finite difference eikonal solver (Podvin and Lecomte, 1991). The wavelet is then separated into two parts at the beginning of the S-wave. The window for the P-wave comparison is from the first arrival to the beginning of the S-wave, and the window for the S-wave comparison is proportional to the epicenter distance. It should be noted that the full wavelet is not included as later arrivals, usually due to scattering from heterogeneous media, cause larger inaccuracies in waveform modeling.

In some cases, when we have more confidence in some stations, e.g., stations with short epicenter distance, or stations deployed on known simpler velocity structure, we can give more weight to those stations by multiplying $\alpha_1-\alpha_4$ with an additional station weight factor.

The comparison algorithm (equation 2) is optimized such that it can be performed on a multicore desktop machine usually within 30 minutes, even when tens of millions of synthetic traces are compared with the data. The computation of the Green’s function library using DWN takes more time, but it only needs to be computed once.

The passive seismic tomography only provides a detailed 3D velocity model close to the central area of the field due to the earthquake-station geometry (Zhang et al., 2009). Therefore, for the focal mechanism determination through the surface network, of which most stations are not placed within the central area (Figure 1), we use the 1D layered velocity model from the averaged sonic logs (Sarkar, 2008; Zhang et al., 2009). Considering that we use a frequency band of 3–9 Hz (Figure 3) in our waveform matching for this surface network, corresponding to a dominant P-wave wavelength of 800 m and S-wave wavelength of 400 m, the velocity model should satisfy our modeling requirement. The deep network consists of five boreholes with eight levels of receivers at different depths in each borehole (Figure 2). Due to the proximity of borehole receivers to the seismicity, we were able to record the seismograms of very small induced seismicity. Waveforms between 15 and 35 Hz are used to determine the focal mechanisms (Figure 4). To better model the waveforms, we replaced part of the 1D average layered velocity model with the extracted P- and S-wave velocities from the 3D tomographic model between 0.7 km and 1.2 km in depth, where it has the highest resolution and reliability. Note that the updated 1D velocity model between the earthquake and each station becomes different for the deep borehole network.

SYNTHETIC TESTS FOR THE SURFACE AND DEEP BOREHOLE NETWORKS

In Li et al. (2011), we tested the robustness of the method on the surface network. To account for the uncertainty of the 1D velocity model, a 5% random perturbation was applied. Here, we consider a greater uncertainty in the velocity model — up to 8% — and test more cases for different focal mechanisms and event locations. We first use the station configuration of the surface network in our test because it provides a considerable challenge due to the large epicenter distance and the relative inaccuracy in the computation of Green’s functions by using the 1D averaged velocity model from several sonic logs. We choose three different epicenters (E1, E2, and E3), and for each epicenter we choose three different depths (D1 = 1000 m, D2 = 1200 m, and D3 = 1700 m), corresponding to shallow, medium, and deep events in this field, respectively. At each depth, we test three different focal mechanisms, which yield 27 different synthetic tests in total. The different focal mechanisms and widely distributed hypocenters in the synthetic test give a comprehensive robustness test for the focal mechanism determination in this region. The station configuration and the hypocenter distribution are shown in Figure 1. At each hypocenter, three distinct mechanisms are tested, namely M1: Φ = 210°, δ = 50°, λ = −40°; M2: Φ = 50°, δ = 60°, λ = −70°; and M3: Φ = 130°, δ = 80°, λ = 80° (Table 1). Three or four first P-arrival polarities are used in each synthetic test, resembling the measurements we have for real data for this surface network. In real cases, as inevitable differences exist between the derived velocity model and the true velocity model, we need to examine the robustness of our method under such circumstances. We add up to 8% of the layer’s velocity as the random velocity perturbation to the reference velocity model in each layer (Figure 5) and use the perturbed velocity models to generate synthetic data. The perturbation is independent for five stations, i.e., the velocity model is path-dependent and varies among different event-station pairs to reflect the 3D velocity heterogeneities in the field. Also, the perturbation is independent for the P-wave and S-wave velocities in a specific velocity model for an event-station pair. The Green’s functions (modeled data) are generated with the reference velocity model. Figure 6 shows the modeled seismograms with offset using the reference velocity model. The predicted traveltimes by the eikonal equation and the first arrivals in the waveforms are matched well. It should also be noted that the P-wave and S-wave velocity perturbation from one station to another can reach up to 800 m/s in some layers. Considering that this reservoir consists mainly of sedimentary rocks, the magnitude of the random lateral velocity perturbation should reflect the upper bounds of the local
lateral velocity inhomogeneity. The density is not perturbed in this test, as the velocity perturbation is dominant in determining the characteristics of the waveforms. The test results are summarized in Table 1. Although the perturbation can change the waveform characteristics to a very large extent, the synthetic test shows that our method can still find a solution very close to the correct one by including information from different aspects of the waveforms, even when only records from five vertical components are used. Figure 7 shows a waveform match between the synthetic data and the modeled data. The best solution found is (230°, 60°, −40°), close to the correct solution (210°, 50°, −40°) in comparison. The synthetic event is at 1220 m in depth.

In general, the focal mechanisms are reliably recovered (Table 1). To quantify the recoverability, we define the mean recovery error for the focal parameters:

$$\Delta \phi^e_m = \frac{\sum_{d=1}^{3} |\phi^e_{m,d} - \phi_m|}{3}$$

where $\phi^e_{m,d}$ is the recovered strike, dip, or rake for epicenter $e$, with mechanism $m$ at depth $d$, where $e,m,d \in \{1,2,3\}$, and $\phi_m$ is the reference (true) focal parameter for mechanism $m$. It is found that $\Delta \phi$ is only a weak function of epicenter, with marginally smaller

| Table 1. Recovered focal mechanisms in the synthetic tests for different hypocenters and faulting types. The true focal mechanisms are listed in the row indicated by REF. Rows D1, D2, and D3 list the events at 1000 m, 1200 m, and 1700 m in depth, respectively. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | E1              |                | E2              |                | E3              |                |
|                | M1              | M2              | M3              | M1              | M2              | M3              | M1              | M2              | M3              |
| REF            |                 |                 |                 |                 |                 |                 |
| D1             |                 |                 |                 |                 |                 |                 |
| D2             |                 |                 |                 |                 |                 |                 |
| D3             |                 |                 |                 |                 |                 |                 |
| $\Delta \phi^o$| 16              | 23              | 6               | 10              | 20              | 26              | 6               | 14              | 20              |
| $\Delta \delta^o$| 20             | 3               | 3               | 13              | 6               | 6               | 6               | 10              | 3               |
| $\Delta \lambda^o$| 3             | 10              | 13              | 3               | 27              | 10              | 16              | 8               | 18              |

Figure 5. P- (right) and S-wave (left) velocity perturbations for the synthetic tests. The reference velocities, plotted with the bold black line, are used for calculating the Green’s functions. The perturbed velocities (colored lines) are used to generate the synthetic data for each station.
value for E1 than for E2 or E3, in general. Also, we found that for each individual depth $\Delta \phi (d = 1, 2, 3)$ is marginally smaller for shallower earthquakes (D1 and D2) than for deeper earthquakes (D3) (results not tabulated). Due to our use of only vertical components, we found that the uncertainty in strike is slightly larger than that in dip or rake. In general, no distinct variation of $\Delta \phi$ is found against the hypocenter or faulting type. Therefore, we conclude that our method is not very sensitive to the faulting type, to the azimuthal coverage of the stations, or to the hypocenter position within a reasonable range for the array geometries studied.

For the borehole network, we perform a similar synthetic test to check the reliability of our method for the deep network configuration. As we have shown that the reliability of our method is not very sensitive to the azimuthal coverage of the stations or to the depth of the event in a reasonable range, we only perform synthetic experiments at two hypocenters with three different mechanisms, respectively, for the deep borehole network (Table 2). Nine to eleven receivers are used for each case. The frequency band is the same as we used for the real data set (15–35 Hz). A typical waveform comparison for the synthetic test is shown in Figure 8. It is also found that the method is robust with the borehole receiver configuration using higher frequency seismograms.

**APPLICATION TO FIELD DATA**

We applied this method to study 40 microearthquakes using surface and deep borehole networks. The instrumental responses have been removed before processing. An attenuation model with $Q$ value increasing with depth (Table 3) was used for the waveform.
modeling. In general, we consider the attenuation larger (smaller $Q$) close to the surface due to weathering, and the attenuation for S-waves larger than for P-waves at the same depth. The attenuation model is built from empirical knowledge of the local geology, and we also tested that reasonable deviation from our $Q$ model (50%) causes only small changes in our synthetic waveforms. Figure 9 shows the beachballs of the nine best solutions out of millions of trials for a typical event recorded by the surface network. Our best solution (the one at the bottom right, reverse strike-slip) has a strike of 325°, which is quite close to the best known orientation 320° of the northwest-southeast conjugate fault (Figure 1). Figure 10 shows the comparison between the modeled and the observed data for this event. The waveform similarity between the modeled and observed data is good. Typically, the crosscorrelation coefficient is greater than 0.7. Additionally, the S/P waveform amplitude ratios in the modeled and observed data are quite close, and the first P arrival polarities are identical in the modeled and observed data for each station. In this example, all four criteria in equation 2 are evaluated, and they are consistent between the modeled and observed data.

For the deep borehole network, we use the frequency band 15 ~ 35 Hz, which includes enough energy in the spectra to provide good S/N, for determining the focal mechanisms of these small magnitude earthquakes from the borehole network data (Figure 4). The lower frequency here is limited by the bandwidth of the borehole instrumentation ($f_c = 20$ Hz), and the frequency contents below the corner frequency $f_c$ may suffer from an increased noise level. As there is also uncertainty in the orientations of the horizontal components, we use only the vertical components of the 4C sensors configured in a proprietary tetrahedral shape for each level (Jones et al., 2004). Although there are, in total 40, vertical receivers, we often only use about 10 seismograms in determining each event due to the following reasons:

- Some receivers are only separated by ~30 m vertically and therefore do not provide much additional information for determining the source mechanism.
- Some traces show peculiar, unexplainable characteristics in seismograms and are, therefore, discarded. The S/N for some traces is also very poor.

In our selection of seismograms, we try to include data from different wells to provide a better azimuthal coverage, as well as from different depths spanning a large vertical range, providing waveform samplings at various radiation directions of the source.

Figure 11 shows the comparison between the observed and modeled seismograms for a typical event recorded by the deep borehole network. Eleven receivers from four boreholes are used in this determination. Among the eleven seismograms, five first P-wave arrival polarities are identified and then used in this determination. The waveform similarities, average S/P amplitude ratio, and consistency in the P-wave arrival polarities are satisfactory. Comparing Figure 11 with Figure 10, we found the fewer matched cycles in the deep borehole case. Similar comparison can also be found between the shallow and deep borehole synthetic tests (Figures 7 and 8), where focal mechanisms close to the correct solutions were still found in both synthetic cases.

Using this method, we have studied 40 earthquakes distributed across this oil field from both the surface network and the borehole network. Among these studied events, 22 events are recorded by the surface network, 18 events are from the borehole network. Figure 12 shows that most of the events primarily have the normal faulting mechanism, some have the strike-slip mechanism, and some have a reverse faulting mechanism. The strike directions of most events are found to be approximately parallel with the northeast trending fault, suggesting the correlation of these events with the northeast trending fault. However, some events also have their strikes in the direction of the conjugate northwest trending fault, suggesting that the reactivation also occurred on the conjugate faults. Although the number of studied events is small compared to the total recorded events, their mechanisms still provide us with some insights on the fault reactivation in this field: (1) The hypocenter distribution and the determined source mechanisms (e.g., strikes) indicate that the reactivation of preexisting faults is the main cause of the induced microearthquakes in this field, and both the northeast trending fault and its conjugate fault trending in the northwest direction are still active. Interestingly, we note that the strike directions of the normal faulting events (red) are slightly rotated counterclockwise with respect to the mapped fault traces from the 3D active seismic data and are consistent with the trend of the located earthquake locations (Figure 1). (2) The counterclockwise rotation may be due to the nonplanar geometry of the fault, i.e., the strike of the shallow part of the fault as delineated by the surface seismic survey does not need to be the same as the deeper part of the fault, where most induced seismicity is located. Most strike-slip events (cyan) are shallow, suggesting that the maximum horizontal stress ($S_{H_{max}}$) is still larger than the vertical stress ($S_V$) at this depth range. However, deeper events (e.g., red, blue) mainly have a normal faulting mechanism, suggesting $S_V$ exceeds $S_{H_{max}}$ when depth increases beyond ~1 km in this region. The dominance of normal faulting is consistent with the study by Zoback and Zinke (2002) on the Valhall and Ekofisk oil fields, where reservoir depletion induced normal

<table>
<thead>
<tr>
<th>Table 2. Recovered focal mechanisms in the synthetic tests for different faulting types using the deep borehole network. The true focal mechanisms are listed in the row indicated by REF. The synthetic events at two different hypocenters are tested (Figure 2).</th>
</tr>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>REF</td>
</tr>
<tr>
<td>$\Delta\Phi^\circ$</td>
</tr>
<tr>
<td>$\Delta\delta^\circ$</td>
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<tr>
<td>$\Delta\lambda^\circ$</td>
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</tbody>
</table>
faulting in and above the productive horizon. In this oil field, most induced earthquakes occurred above the oil layer, which is located around 1.5 km below the surface. (3) Assuming $S_{\text{Hmax}}$ is parallel with the strike of normal faulting events, perpendicular to the strike of reverse events, and bisects the two fault planes of the strike-slip events (Zoback, 2007), most of the determined events then suggest a $S_{\text{Hmax}}$ trending northeast or north-northeast, which is consistent with the well breakout measurement and local tectonic stress analysis in the region (Al-Anboori, 2005). The observations indicate that the regional preexisting horizontal stress and the vertical stress played an important role in the reactivation of these preexisting faults.

**DISCUSSION**

Although we only applied our method to a particular oil/gas field, the method is applicable to any microseismic monitoring case, especially to cases when the monitoring stations are sparse. We only used the vertical components in our study, but the waveform comparison can be easily expanded to include three components. Considering each component at a station contains different information in the radiation pattern (Aki and Richards, 2002), the incorporation of multicomponent observations should further reduce the solution uncertainty.

The attenuation needs to be taken into account in the synthetic waveform modeling. Not only is the amplitude changed, but frequency-dependent phase-shift also occurs as the phase velocity becomes dependent on frequency due to the attenuation effect (Aki and Richards, 2002). It should be noted that the attenuation-induced phase-shift is in addition to any phase-shift related to the wave propagation, e.g., guided wave effect. In our waveform modeling, compared to the pure elastic case we have observed notable waveform change in the frequency band of observation when moderate attenuation is included. Attenuation tomography (e.g., Quan and Harris, 1997) should be considered to construct an attenuation model if receivers are not located in the vicinity of the microseismic events.

Our synthetic test indicates that when there are errors in the velocity model, the inverted mechanisms are affected and can deviate from the true ones. Therefore, it is difficult to tell whether the oscillation in the inverted strike, dip, and rake is true or if it is caused by our limited observations and errors in the velocity model.

In general, we find the inversion results are not sensitive to the weighting parameters that are within reasonable ranges. The rule of thumb is to choose a parameter set that balances the contribution from each term in the objective function. The weighting parameters used in our study may not be optimal in other fields and need to be determined for individual data sets.

Although tens of millions of synthetic seismograms are usually compared with observed seismograms in the global grid search, some manipulation in the crosscorrelation and filtering (Li et al., 2011) can be used to greatly reduce the time consumption.

Figure 8. Comparisons between modeled waveforms (red) and synthetic data (blue) at nine borehole stations with the perturbed velocity model. In this test, nine vertical components in borehole YA, YB, YC, and YD are used. The waveforms are filtered between 15 and 35 Hz. The true mechanism is $(210^\circ, 50^\circ, -40^\circ)$, and the best recovered one is $(240^\circ, 60^\circ, -10^\circ)$ in comparison.
Additionally, our inversion algorithm can be easily parallelized. Our experience is that twenty million synthetic seismograms from different source mechanism and hypocenter combinations can be searched through on an eight-core workstation in about 20 minutes. Therefore, our algorithm can be easily extended to monitoring cases where many more stations and components are available.

Our methodology can also be applied to solve for the full moment tensor. In that case, we will have six independent moment tensor components $m_{ijk}$ associated with the source mechanism in our objective function. The increase in the degree of freedom will require more search time. In addition, it is more challenging to resolve the six independent moment tensor components because velocity model error, anisotropy or even the inconsistency in the source time function in different moment tensor components become the hindrance.

### Table 3. One-dimensional attenuation model used for the DWN waveform modeling. The attenuation affects the waveform amplitudes and causes waveform dispersion.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$Q_P$</th>
<th>$Q_S$</th>
</tr>
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<tbody>
<tr>
<td>0–60</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>60–110</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>110–160</td>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td>160–264</td>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td>264–470</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>470–1090</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>1090–bot.</td>
<td>300</td>
<td>150</td>
</tr>
</tbody>
</table>

Figure 9. Focal mechanism solutions for a typical event determined by the shallow network. The one at the bottom right (#1) is the best solution with maximum objective function value. The epicenter is shifted northward ($y$) by about 750 m, eastward ($x$) by about 300 m and the depth is shifted 50 m deeper compared to the original hypocenter. The shift in epicenter may be biased by inaccuracy in the velocity model and by only using the vertical components. The shift can compensate the phase difference between the modeled seismograms and the real seismograms.
Figure 10. Comparison between the modeled waveforms (red) and the real data (blue) at five surface network stations for a typical event. For P-waves, zero time means the origin time, and for S-waves, zero time means the S-wave arrival time predicted by the calculated traveltime.

Figure 11. Comparison between the modeled waveforms (red) and the real data (blue) from the borehole network. Eleven stations and five first P-wave arrival polarities which can be clearly decided in the observed waveforms are used in this determination. For P-waves, zero time means the origin time, and for S-waves, zero time means the S-wave arrival time predicted by the calculated traveltime.
CONCLUSIONS

In this study, we used our recently developed high-frequency waveform matching method to determine the microearthquakes in an oil field with the surface and borehole network data. This method is especially applicable to the study of microearthquakes recorded by a small number of stations, even when some first P arrival polarities are not identifiable due to noise contamination, or only the vertical components are usable. The objective function, formulated to include matching phase and amplitude information, first arrival P polarities and S/P amplitude ratios between the modeled and observed waveforms, yields reliable solutions. We also performed systematic synthetic tests to verify the stability of our method.

For the 40 studied events, we found that the hypocenters and strikes of the events are correlated with preexisting faults, indicating that the microearthquakes occur primarily by reactivation of the preexisting faults. We also found that the maximum horizontal stress derived from the source mechanisms trends in the northeast or north-northeast direction; this is consistent with the direction of the maximum horizontal stress obtained from well breakout measurements and local tectonic stress analysis. Our investigation shows that the study of the source mechanisms of the induced microearthquakes can provide insights into the local stress heterogeneity and help to better understand the induced microearthquakes by oil or gas production.

ACKNOWLEDGMENTS

The work presented here was partly supported by the Department of Energy under grant DE-FG3608GO18190. J. Li is grateful for the financial support by the MIT-Kuwait center, and the authors want to thank Petroleum Development Oman (PDO) for providing the data and support for this study. The authors also want to thank four anonymous reviewers for their helpful suggestions to improve the manuscript.

APPENDIX A

GREEN’S FUNCTIONS CALCULATION FOR THE DEEP BOREHOLE NETWORK

The reflectivity method used in the discrete wavenumber waveform modeling of Bouchon (2003) was originally developed in global seismology where sources are located underground and
receivers are at the surface or near the surface. For the surveys using borehole receivers, however, the receivers can be located deeper than the source; thus the original reflectivity method needs to be revised and calculations in the reflectivity method need to be modified for this configuration. We followed the symbols and definitions used in the paper by Muller (1985) on the reflectivity method and only show the key modified equations. Figure A-1 shows the diagram for borehole receiver configuration.

The source and receivers are required to be located at the interface between two identical layers in the implementation (Bouchon, 2003). The position of the source and receiver can be anywhere within a layer; however, an artificial splitting of the layer is applied at the depth of the receiver or the source, i.e., splitting the layer into two identical layers with an interface at the depth of the source or receiver. The reflectivity method is easier to apply in this way. After the splitting, the source is located at the bottom of layer j, and the receiver is located at the top of layer m for the shallower-source-deeper-receiver situation.

In the following derivation, we use the P-SV system. For the SH system, the matrices and vectors are replaced with scalars. The overall amplitude vector $V_{1,2}^D$ for the down-going waves at the source depth is

$$
V_{1,2}^D = \begin{pmatrix} A_{1,2} \\ C_{1,2} \end{pmatrix}
$$

$$
= (S_{1,2}' + R^* R^- S_{1,2}' + R^* R^- R^* R^- S_{1,2}' + \cdots)
$$

$$
+ (I - R^* R^-)^{-1} (S_{1,2}' + R^- S_{1,2}'),
$$

(A-1)

where $R^*$ and $R^-$ are the reflectivities illustrated in Figure A-1; $S_{1,2}'$ and $S_{1,2}\prime$ are the source amplitude vectors; $I$ is the identity matrix. $V_{1,2}^D$ takes all the reflections from the lower layers (first bracket) and the upper layers (second bracket) into consideration and, therefore, is the amplitudes of the overall down-going P- and SV-waves at the source depth. After the overall down-going amplitudes are obtained at the source level, we need to propagate them down through the layers between the source and receiver by the overall down-going transmissivity matrix,

$$
TT^D = F_{m-1} F_{m-2} \cdots F_{j+1} F_j,
$$

(A-2)

where $F_k$ characterizes the amplitude change through layer $k$ and through the bottom interface of layer $k$. Note that for layer $j$ there is no phase shifting through the phase matrix $E_j$ in $F_j$, as the source is already located at the bottom of layer $j$ after the artificial splitting.

The overall down-going amplitudes at the receiver then are

$$
V_{1,2}^{D,R} = \begin{pmatrix} A_{1,2} \\ C_{1,2} \end{pmatrix} = TT^D V_{1,2}^D,
$$

(A-3)

and the overall amplitudes of the up-going waves at the receiver are related to the amplitudes of the downgoing waves by

$$
V_{1,2}^{U,R} = \begin{pmatrix} B_{1,2} \\ D_{1,2} \end{pmatrix} = MT_m V_{1,2}^{D,R},
$$

(A-4)

where $MT_m$ is the local reflectivity matrix at the top of layer $m$. Combining the amplitudes $V_{1,2}^{D,R}$ and $V_{1,2}^{U,R}$ with the Green’s functions calculated by the discrete wavenumber method (Bouchon, 2003) and integrating in the wavenumber and frequency domain, we can then obtain the analytic solution in a stratified medium where the receiver is deeper than the source.

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Full-waveform based complete moment tensor inversion and source parameter estimation from downhole microseismic data for hydrofracture monitoring

Fuxian Song¹ and M. Nafi Toksöz¹

ABSTRACT

Downhole microseismic monitoring is a valuable tool in understanding the efficacy of hydraulic fracturing. Inverting for the moment tensor has gained increasing popularity in recent years as a way to understand the fracturing process. Previous studies utilize only part of the information in the waveforms, such as direct P- and S-wave amplitudes, and make far-field assumptions to determine the source mechanisms. The method is hindered in downhole monitoring, when only limited azimuthal coverage is available. In this study, we develop an approach to invert for complete moment tensor using full-waveform data recorded at a vertical borehole. We use the discrete wavenumber integration method to calculate full wavefields in the layered medium. By using synthetic data, we find that, at the near-field range, a stable, complete moment tensor can be retrieved by matching the waveforms without additional constraints. At the far-field range, we discover that the off-plane moment tensor component is poorly constrained by waveforms recorded at one well. Therefore, additional constraints must be introduced to retrieve the complete moment tensor. We study the inversion with three different types of constraints. For each constraint, we investigate the influence of velocity model errors, event mislocations, and data noise on the extracted source parameters by a Monte Carlo study. We test our method using a single well microseismic data set obtained during the hydraulic fracturing of the Bonner sands in East Texas. By imposing constraints on the fracture strike and dip range, we are able to retrieve the complete moment tensor for events in the far-field. Field results suggest that most events have a dominant double-couple component. The results also indicate the existence of a volumetric component in the moment tensor. The derived fracture plane orientation generally agrees with that derived from the multiple event location.

INTRODUCTION

Downhole microseismic monitoring is a valuable tool for hydrofracture mapping. The locations of microseismic events, with sufficient resolution, provide information on fracture geometry and properties (Warpinski et al., 1998; Phillips et al., 2002). In addition to location, the seismic moment tensor is derived to understand the microseismic source mechanisms and stress state (Nolen-Hoeksema and Ruff, 2001; Baig and Urbancic, 2010). The complete moment tensor of the general source mechanism consists of six independent elements (Aki and Richards, 2002). Some researchers (Phillips et al., 1998; Warpinski, 1997) observed high S/P-wave amplitude ratios which “could not be explained by tensile opening” (Pearson, 1981) and concluded that the induced events are shear failure along preexisting joints in rocks that surround hydraulic fractures and are caused by elevated pore pressure. Thus, most studies have focused on double-couple mechanisms (Rutledge and Phillips, 2003). However, recent studies have shown the existence of nondouble-couple mechanisms for some hydrofracture events (Šílený et al., 2009; Warpinski and Du, 2010). The knowledge of nondouble-couple components, especially the volumetric component, is essential to understand the fracturing process. Moreover, Vavryčuk (2007)
showed that, for shear faulting on nonplanar faults, or for tensile faulting, the deviatoric source assumption is no longer valid and can severely distort the retrieved moment tensor and bias the fault-plane solution. Therefore, the complete moment tensor inversion is crucial not only to the retrieval of the volumetric component but also to the correct estimation of the fault-plane solution.

Currently, most moment tensor inversion methods rely only on far-field direct P- and S-wave amplitudes (Nolen-Hoeksema and Ruff, 2001; Vavryčuk, 2007; Jechumtálová and Eisner, 2008; Warpinski and Du, 2010). Vavryčuk (2007) used the far-field approximation of the P- and S-wave Green’s function in homogeneous isotropic and anisotropic media to show that a single-azimuth data set recorded in one vertical well cannot resolve the dipole perpendicular to the plane of geophones and the hypocenter. Thus, the complete moment tensor of the general source mechanism is underdetermined with data from one well. To overcome this problem, previous studies have proposed using data recorded in multiple monitoring wells at different azimuths (Vavryčuk, 2007). For events far from the monitoring data sets are frequently limited to a single array of geophones and one vertical well. Therefore, the issue of complete moment tensor inversion from one-well data remains unsolved.

In this paper, we try to address this problem from the standpoint of full-waveform inversion. We propose a full-waveform approach for moment tensor inversion using data from one monitoring well. It uses the discrete wavenumber integration method to calculate elastic wavefields in the layered medium. By matching the waveforms across the geophone array, we show that, when the events are close to the monitoring well, the inversion can be stabilized so that the complete moment tensor can be retrieved from data recorded in a single borehole without making additional source assumptions. We quantify the closeness of events by studying the condition number of the sensitivity matrix. For events far from the monitoring well, we demonstrate that additional constraints must be introduced to retrieve the off-plane dipole component (also pointed out by Vavryčuk [2007] and Jechumtálová and Eisner [2008]). Three types of constraints have been studied in this paper to invert the complete moment tensor for events at far-field. Furthermore, we investigate the influence of velocity model errors, source mislocations, and data noise on the extracted source parameters using synthetic data. Finally, we describe the application of the constrained inversion to a field data set from East Texas. By applying the constraint on the fracture strike and dip range, we show that a reliable, complete moment tensor solution and source parameters can be obtained for each event.

**METHODOLOGY**

**Full-waveform based complete moment tensor inversion**

The complete moment tensor of a microseismic event is characterized by the six independent elements of the 3 by 3 symmetric moment tensor matrix $m_{jk}$. To improve the complete moment tensor inversion with a single borehole, we use all phases that are embedded in the full-waveform data. Our approach starts from fast, full elastic waveform modeling in a layered medium with the discrete wavenumber integration method (DWN; Bouchon, 2003). The $i$th component (north, east, down) of the observed waveform at geophone $n$ is modeled as

$$v_i(x_n^i,x_r,t) = \sum_{j=1}^{3} \sum_{k=1}^{3} m_{jk}G_{ij}(x_n^i,x_r,k) \ast s(t).$$  \hspace{1cm} (1)

where $*$ denotes the convolution operation (same hereinafter); $G_{ij}(x_n^i,x_r,k)$, the spatial derivative of the Green’s function, is the $i$th component of the elementary seismograms at the $jth$ geophone $x_n^i$ due to a point moment tensor source $m_{jk}$ at $x_r$; $s(t)$ is the source time function. In this study, a smooth ramp function with a center frequency of 550 Hz is used as $s(t)$, according to the spectral analysis of field data. The sampling frequency is 4 kHz in both synthetic and field study. Considering that the moment tensor matrix $m_{jk}$ has only six independent elements, equation 1 can be written as

$$\sum_{j=1}^{6} A_{ij}(x_n^i,x_r) M_l(x_r) = v_i(x_n^i,x_r,t),$$ \hspace{1cm} (2)

where $M_l$ is the $l$th moment tensor element: $M_1 = m_{11}, M_2 = m_{22}, M_3 = m_{12}, M_4 = m_{13}, M_5 = m_{12}, M_6 = m_{23}$, while $A_{ij}$ denotes the $i$th component of the elementary seismograms at geophone $x_n^i$ due to a point moment tensor source $M_l$ at $x_r$. In matrix form, equation 2 becomes

$$A M = v.$$ \hspace{1cm} (3)

Here the sensitivity matrix $A$ (i.e., data kernel) is composed of six columns, with each column consisting of the elementary seismograms from a point moment tensor source $M_l$. The six element vector $M$ represents the complete moment tensor:

$$M = [M_1, M_2, M_3, M_4, M_5, M_6]^T.$$ \hspace{1cm} (4)

Data column vector $D$ is comprised of all available components recorded at all geophones ranging from time $t_{l_0}$ to $(t_{l_0} + T_n)$, where $t_{l_0}$ and $T_n$ are the starting time and the duration of recorded data used in the inversion from geophone $n$, respectively. In this study, we choose $T_n$ to include both P- and S-wave trains and keep it fixed for all $N$ geophones. The starting time $t_{l_0}$ is determined from the event origin time $t_0$ and the P-wave traveltime from the event to geophone $n$. Event origin time is obtained by a grid search around its initial estimate within the dominant signal period. The initial estimate of the origin time can be found by cross-correlating the synthetic and observed waveforms.

To reduce the influence from errors in source locations, during the inversion, we also perform a grid search around the initial location. The spatial search range and grid size are selected based on the location uncertainty. The uncertainty in locations from a vertical array is estimated from the standard deviations of P- and S-wave arrival times and P-wave polarization angles (Eisner et al., 2010). For the field data, we calculate standard deviations and obtain 3.0 m (10 ft) in the radial direction, 7.6 m (25 ft) in the vertical direction, and 5° in P-wave derived back-azimuths. We further determine the location uncertainty in the horizontal directions (north, east) from the standard deviations of the radial distances and P-wave derived back-azimuths for a monitoring array at a typical distance of 100.6 m (330 ft). The standard deviation is estimated to be 9.1 m (30 ft). Therefore, in this study, we use a spatial grid of 5 ft and a spatial search cube with measurements of $15 \times 15 \times 11$ grids (north, east, down). The best solution of the event location
\[ J(x_s, t_0, M) = \sum_{n=1}^{N} \sum_{i=1}^{N_e} \sum_{k=1}^{N} (d_i(x^n_s, x_s, k\Delta t) - v_i(x^n_s, x_s, k\Delta t))^2, \]

where \( N \) is the number of geophones, \( N_e \) is the number of time points, and \( N_i \) is the number of components used in the inversion. The sampling interval of the recorded data is \( \Delta t \).

To further stabilize the inversion, both synthetic data and observed data are band-pass filtered. Based on the spectral analysis of the signal and pre-event noise from the field data example, a band-pass filter of [200, 900] Hz is used in this paper. For \( N \) geophones, the sensitivity matrix \( A \) has a size of \( N \times N_t \), where \( N_t \) is the number of components used in the inversion. Therefore, in this study \( N_t = 2 \). However, the method itself is not limited to two components. If matrix \( A \) is well-conditioned, a least-squares solution to the overdetermined system can be obtained using the generalized inverse:

\[ M = (A^T A)^{-1} A^T D. \]  

The condition number of matrix \( A \) will be discussed in the synthetic study.

The processing steps can be summarized as follows:

1) generate a Green’s function library, calculate the elementary seismograms, and apply the band-pass filter to the elementary seismograms for each possible event location;
2) apply the same band-pass filter to the recorded waveforms;
3) estimate the initial event origin time at every possible event location;
4) carry out a cascaded grid search around the initial estimated event origin time and location. For each event location, conduct a grid search on event origin time. For each origin time and location, find the least-squares solution \( M(x_s, t_0) \) according to equation 6, and evaluate the L-2 waveform fitting error according to equation 5;
5) determine the best solution of moment tensor, event location, and origin time with the least amount of waveform fitting error.

**Source parameter estimation**

The complete moment tensor can be decomposed into the isotropic (ISO), compensated linear vector dipole (CLVD), and double-couple (DC) components. In this paper, we use the decomposition of a moment tensor proposed by Vavryčuk (2001). The symmetric moment tensor matrix \( m_{jk} \) can be diagonalized and represented as the sum of the deviatoric moment \( M^{\text{dev}} \) (i.e., the moment tensor with zero volumetric component), and the isotropic moment \( M^{\text{iso}} \). Parameter \( \varepsilon \) is introduced to measure the size of CLVD relative to DC:

\[ \varepsilon = \frac{\lambda_{\text{dev}}^{\text{max}} - \lambda_{\text{dev}}^{\text{min}}}{\lambda_{\text{dev}}^{\text{max}}}, \]

where \( \lambda_{\text{dev}}^{\text{min}} \) and \( \lambda_{\text{dev}}^{\text{max}} \) are the minimum and maximum absolute eigenvalues of the deviatoric moment, respectively. For a pure DC \( \varepsilon = 0 \), and for a pure CLVD \( \varepsilon = \pm 0.5 \). Parameter \( \varepsilon \) is positive for tensile sources and negative for compressive sources. The percentages of each component (ISO, CLVD, DC) can be calculated as

\[ c^{\text{ISO}} = \frac{1}{3} \text{trace}(m_{j,k}), \]

\[ c^{\text{CLVD}} = 2\varepsilon(1 - |c^{\text{ISO}}|), \]

\[ c^{\text{DC}} = 1 - |c^{\text{ISO}}| - |c^{\text{CLVD}}|, \]

where \( M_0 \) is the seismic moment in N \( \cdot \) m, defined as the largest absolute eigenvalue of the moment tensor matrix \( m_{jk} \)

\[ M_0 = \max_{(j)} |\lambda_j|. \]

The moment magnitude is calculated as

\[ M_w = \frac{2}{3} \log_{10}(M_0) - 6.607. \]

According to Jost and Hermann (1989), the eigenvector \( b \) of the moment tensor matrix \( m_{jk} \) corresponding to the intermediate eigenvalue gives the null axis, while the eigenvectors \( t \) and \( p \) corresponding to the maximum and minimum eigenvalue give the tension and compression axis, respectively. The fracture plane normal \( v \) and the slip vector \( u \) can be derived from the \( t \) and \( p \) axes as

\[ u = \frac{1}{\sqrt{2}}(t + p), \quad v = \frac{1}{\sqrt{2}}(t - p). \]

The fracture plane solutions, including strike \( \phi \), dip \( \delta \), and rake \( \lambda \), can be further derived from the fracture plane normal \( v \) and the slip vector \( u \) (Jost and Hermann, 1989).

**SYNTHETIC STUDY**

**Condition number of the sensitivity matrix in full-waveform inversion**

In this section, we study the influence of borehole azimuthal coverage and the source-receiver distance on the condition number of the sensitivity matrix and discuss its implications in complete moment tensor inversion using synthetic data from a single well.

Figure 1 gives the source-receiver configuration. In this experiment, we fix the microseismic event at \( (0, 0, 3946 \text{ m}) \). An array of six-level three-component (3C) geophones is deployed in each vertical well at the same depth range as the field setup from 3912 m (12,835 ft) to 3944 m (12,940 ft). The horizontal location of the well is adjusted so that the mean source-receiver distance falls into the range between \( 4\lambda_s \) and \( 36\lambda_s \), where \( \lambda_s \) is the dominant S-wave wavelength. For each mean source-receiver distance, we calculate the elementary seismograms and apply the [200, 900] Hz band-pass filter to obtain the filtered elementary seismograms and form the sensitivity matrix \( A \). Figure 2 shows the one-dimensional (1D) P- and S-wave velocity models derived from the field study. We use this velocity model to generate elementary seismograms for the condition number study.
Figure 3a shows the condition number of the sensitivity matrix $A$ as a function of both borehole azimuthal coverage and the mean source-receiver distance when all 3C data are used in the inversion. Three observations are clearly seen on Figure 3a. First, the condition number increases dramatically with the increased mean source-receiver distance for the one-well case. This signifies that the resolvability of complete moment tensors deteriorates at far-field when only one-well data are used in moment tensor inversion. In addition, the eigenvector corresponding to the minimum eigenvalue gives the least resolvable moment tensor element. In the case of well B1 at the azimuth of $0^\circ$, the off-plane element $m_{22}$ is the least resolvable moment tensor element. This is consistent with the far-field study in the homogeneous media. Second, the condition number for the multiple-well cases is significantly lower than that of the one-well case at large source-receiver distances, while the condition number is low for all cases at small source-receiver distances. This indicates that complete moment tensor inversion is possible, even with one-well data, when the receivers are at the near-field range. There is no clear distinction between near-field and far-field. At a noise level of

Figure 1. (a) Horizontal plane view of the source and receiver array distribution in the condition number study. The microseismic event, labeled as the plus sign, lies in the center, with eight monitoring wells, B1 to B8, evenly spreading from the north direction to the north-west direction. The azimuthal separation between two adjacent wells is $45^\circ$. (b) 3D view of the single well configuration used in the inversion study (B1 well, at the azimuth of $N0^\circ E$). The gray star denotes the hypocenter location of the microseismic event, while the six receivers, deployed in the well, are shown as black triangles. (North: x, East: y, and Down: z)

Figure 2. One-dimensional P- and S-wave velocity model derived from field study.

Figure 3. The condition number of the waveform sensitivity matrix $A$, plotted as a function of the mean source-receiver distance, shown in multiples of the dominant S-wave wavelength. The matrix $A$ is formed using: (a) three-component full-waveforms under different well configurations; (b) full-waveforms of three components or two horizontal components from the six-receiver array in the B1 well at the azimuth of $0^\circ$. Well azimuth is defined as east of north.
10%, as is the case in the following synthetic study, a rule of thumb is that at a mean source-receiver distance that is less than five times the S-wave wavelength, a stable complete moment tensor solution can be determined from the one-well data. Finally, the condition number of the two-well case is similar to that of the eight-well case. This seems to imply that, with two wells separated at 45°, the resolvability of complete moment tensor is comparable to that of eight wells, although, for a more complex scenario, such as a laterally heterogeneous medium, eight wells can bring additional benefits in enhancing the source azimuthal coverage and improving S/Ns of recorded events. The condition number of the two-well case barely increases with increased source-receiver distances. This indicates that the complete moment tensor inversion is feasible for both near-field and far-field with two-well data. Figure 3b compares the condition number of the sensitivity matrix of the one-well case using all 3Cs and only two horizontal components. The result suggests that two horizontal components have a similar capability of constraining the moment tensor as three components.

Complete moment tensor inversion of events in the near-field

As we see in the previous section, for events that are close to the monitoring well, it is possible to invert the complete moment tensor from one-well data. Figure 4a shows the total wavefields of the two horizontal components recorded in the well B1 at an azimuth of 0°. The synthetic data are generated with the reference velocity model plotted in Figure 2. Without losing generality, a nondouble-couple microseismic source with 74% of DC, 15% of CLVD, and 11% of ISO component is used in the simulation. The microseismic source has a strike of 108°, a dip of 80°, and a rake of 43°. The distance from the source to six receivers ranges from one to six dominant S-wave wavelengths. At a distance of one to two dominant S-wave wavelengths, complex waveforms are seen on geophones 5 and 6, due to the near-field effects. At a distance larger than three S-wave wavelengths, distinct P and S phases are observed on geophones 1 to 4. Figure 4b gives the near-field terms of the two horizontal components. It is seen on Figure 4b that the near-field terms decrease quickly as the source-geophone distance increases. To quantify the contribution of near-field information, we calculate the peak amplitude ratio of the near-field term to the total wavefields for each component on each geophone. The average peak amplitude ratios of the two horizontal components are 9%, 11%, 14%, 18%, 22%, and 60% for geophones 1 to 6, respectively. Therefore, the major contribution of near-field information to the inversion comes from geophones 5 and 6, which are close to the microseismic source.

Figure 5a shows the noisy seismograms by adding zero-mean Gaussian noise with a standard deviation reaching 10% of the average absolute maximum amplitude of the two components across all six geophones. Figure 5b gives the band-pass filtered data used to invert for the complete moment tensor.

The P- and S-wave velocity models are randomly perturbed up to half of the velocity difference between adjacent layers so that the sign of the velocity difference between adjacent layers does not change. The perturbation is independent between different layers and P- and S-wave velocities are independently perturbed. The perturbed velocity model is used as the approximate velocity model for moment tensor inversion throughout the paper. As mentioned in the methodology section, to mimic the field example, the event location is randomly perturbed up to 9.1 m (30 ft) in the north and east directions and 7.6 m (25 ft) in the vertical direction. In the inversion, a grid search is carried out around the randomly perturbed event location. The moment tensor solution corresponding to the minimum L-2 waveform fitting error is selected as the inversion result. Figure 6 gives the best waveform fitting for one Gaussian noise realization. A good agreement between modeled data in black and band-pass filtered synthetic data in red is seen on both components.

The source parameters are then estimated from the inverted complete moment tensor. To obtain statistically relevant results, we perform 100 moment tensor inversions and source parameter estimations, each with a different noise realization. Figure 7 shows...
the histograms of the ISO, CLVD, DC, seismic moment, strike, dip, rake errors for the nondouble-couple event. The average absolute errors in the percentages of the ISO, CLVD, and DC components are about 4%, 4%, and 6%, respectively, while the average absolute relative error in seismic moment is around 6%. The average absolute error in the strike, dip, and rake is less than 2°. Moreover, the complete moment tensor inversion, using the horizontal component data from geophones 5 and 6, gives comparable results in the inverted source parameters. This indicates that the near-field information contributed to the retrieval of $m_{22}(m_{ve})$ mainly comes from geophones 5 and 6. Considering the inaccuracies in the source location and velocity model together with 10% Gaussian noise, the inverted source parameters agree well with the true values. This demonstrates that for events in the near-field (i.e., at a mean source-receiver distance less than five times the S-wave wavelength), the complete moment tensor inversion is feasible with one-well data using only two horizontal components. The retrieval of $m_{22}$ with one-well data at near-field is further illustrated in Appendix A.

**Complete moment tensor inversion of events in the far-field**

As we see in the condition number study, for events that are far from the monitoring well (i.e., at a mean source-receiver distance greater than five times the S-wave wavelength), the condition number of the sensitivity matrix using one-well data is high compared to that of near-field events. In the case of well B1 at the azimuth of 0°, the off-plane element $m_{22}$ is the least resolvable moment tensor element from full-waveform inversion.

Figure 8 shows the condition number of the sensitivity matrix when inverting for all six moment tensor elements and five moment tensor elements, except $m_{22}$, with only two horizontal components. It is observed that at far-field in the layered medium, when $m_{22}$ is excluded from the inversion, the condition number of the sensitivity matrix is reduced to the level of complete moment tensor inversion at near-field. This shows that the full waveforms are mainly sensitive to the five moment tensor elements, except $m_{22}$. Therefore, for events in the far-field, additional constraints must be introduced to retrieve $m_{22}$.

The basic idea of the constrained inversion is to invert for the rest of the five moment tensor elements using waveforms, assuming a known value of $m_{22}$. The source parameters are then estimated from the complete moment tensor as a function of $m_{22}$. As suggested by Jechutňalová and Eisner (2008), we test the $m_{22}$ value between $-10M_5$ and $10M_5$, where $M_5$ is the maximum absolute value of the five inverted elements. By using a priori source information (for example, fracture orientations) as constraints $m_{22}$ can be determined. Finally, the complete moment tensor and the source parameters are derived.

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**Figure 5.** Synthetic data from the nondouble-couple microseismic source. (a) After adding 10% Gaussian noise to the horizontal component data shown in Figure 4. (b) After applying the [200, 900] Hz band-pass filter to the noise contaminated data in (a). The north component is plotted in red, while the east component is shown in blue. The scaling factor is 30.

**Figure 6.** Comparison between the modeled data in black and the band-pass filtered synthetic data in red for the nondouble-couple source in Figure 4. The modeled data are generated from the inverted microseismic moment tensor matrix (six independent elements). The unconstrained inversion is performed with the band-pass filtered horizontal components in Figure 5b). (a) North component plot. (b) East component plot. The scaling factor is 30. All the inversions in this study are performed with only horizontal components from well B1 and use the approximate velocity model and the mislocated source (see text).
It is also seen from Figure 8 that in the layered medium, the condition number is not a monotonous function of mean source-receiver distance for the case of constrained inversion, while the condition number in the homogeneous medium is a monotonous function of mean source-receiver distance. This can be explained by the difference in the take-off angle coverage at the source between the homogeneous medium and the layered medium.

Eaton (2009) pointed out that in the homogeneous medium the condition number is inversely proportional to the solid angle at the source subtended by the geophone array. In the homogeneous medium only direct rays are available, and, therefore, the take-off angle coverage at the source is fully characterized by the solid angle. However, in the layered medium, as is the case in this study, not only direct but also reflected and refracted rays exist, even if the source and geophone array are situated in the same layer. Therefore, the take-off angle coverage at the source has been increased in the layered medium than in the homogeneous medium scenario, considering the additional reflected and refracted rays.

The increase in the take-off angle coverage at the source produces a decreased condition number. Hence, in the layered medium, the condition number is controlled by the geometry of the receiver array relative to not only the source but also the velocity model. An increase in the mean source-receiver distance will reduce the take-off angle coverage of the direct rays. It may, however, increase the take-off angle coverage from reflected and refracted rays. There is also a critical distance for the refracted rays to occur. Thus, the nonmonotonous behavior for the constrained inversion case in the layered medium is probably due to the complex interaction of the increased take-off angle coverage from the reflected and refracted rays and the decreased take-off angle coverage of direct rays.

Several types of constraints may be applied in the constrained inversion. In this paper, we study three types of constraints. In type I constraint, the range of the strike and dip is assumed to be known. This will give a permissible range of $m_{22}$ values. We further assume that the source mechanism is mostly double-couple, and therefore

![Figure 7](image-url) Image 1

**Figure 7.** The histograms of errors in the inverted source parameters. The microseismic source is nondouble-couple. The true moment tensor and source-receiver locations are described in Figure 4. The unconstrained inversion is performed with the band-pass filtered horizontal components from well B1.

![Figure 8](image-url) Image 2

**Figure 8.** The condition number of the waveform sensitivity matrix $A$, plotted as a function of the mean source-receiver distance, shown in multiples of the dominant S-wave wavelength. The matrix $A$ is formed using full waveforms of two horizontal components recorded by the six-receiver array in the monitoring well B1. The condition number of the unconstrained inversion in the layered medium for all six independent moment tensor elements is plotted in red, while the condition numbers of the constrained inversion in the layered and homogeneous medium for five independent moment tensor elements, except $m_{22}$, are shown in black and blue, respectively.
we determine the $m_{22}$ value by maximizing the DC percentage within that permissible range. Figure 9 gives an example of applying type I constraint. In this example, we use the same nondouble-couple source and source-receiver configuration as the previous near-field case, shown in Figure 4. The mean source-receiver distance increases to 91.4 m (17.5λs). In Figure 9 we invert for the five moment tensor elements, except $m_{22}$, from the band-pass filtered noise-free horizontal component data recorded in well B1. Assuming that the strike and dip range is known to be $\pm15^\circ$ around the true values, the cyan strip gives the permissible range of $m_{22}$ values. The vertical line in green denotes the determined $m_{22}$ value by maximizing the DC percentage within that permissible range.

In type II constraint, we assume that the exact strike value is known so that the $m_{22}$ value is determined directly. In type III constraint, the fracture plane solution is unknown; instead, we assume the event is predominantly double-couple. This suggests that the $m_{22}$ value is obtained by maximizing the DC percentage among all possible values.

Table 1 compares the nondouble-couple source inversion results under three different constraints using noise-free horizontal component data from well B1. For each constraint, it shows the deviation of the inverted source parameters from the original input source parameters. Two observations are seen in Table 1. First, in this case, type I constraint gives the same result as type III constraint; this indicates the strike and dip range from type I constraint may be too large to bring additional information in constraining $m_{22}$ for this noise-free data set. Second, among all three constraints, type II gives the least error in the inverted source parameters. This is because maximizing the DC percentage, as in type I and III, is not a good assumption about the actual source (the true moment tensor is nondouble-couple, with 74% of the DC, 15% of the CLVD, and 11% of the ISO component). Moreover, knowing the strike value not only helps constrain the fracture plane geometry of the strike, dip, and rake values, but also enables the recovery of $m_{22}$ and, eventually, moment component percentages.

Next, we add 10% Gaussian noise into the synthetic horizontal component data and perform 100 moment tensor inversions on the band-pass filtered noisy data, each with a different noise realization. The histograms of the inverted source parameters are plotted in Figure 10 for this nondouble-couple source.

Table 2 summarizes the statistics of the histograms in Figure 10. It gives the mean absolute errors in the inverted source parameters under three different inversion constraints. With data noise, we observe that mean absolute errors in the strike, dip, and rake of the type I constraint are less than those of the type III constraint; this implies that even a rough knowledge of the strike and dip range helps reduce the uncertainty of $m_{22}$ and, eventually, the fracture plane solution (strike, dip, and rake). The errors in strike and dip estimates are also bounded, as specified in the type I constraint ($\pm15^\circ$ for Table 2).

Knowing the exact strike value, as in the type II constraint, greatly reduces the errors in the estimated fracture plane solution and seismic moment. However, the mean absolute errors in the CLVD, DC percentages seem to be slightly higher than those of the type I constraint. This may indicate a trade-off in errors between the fracture plane solution and the moment component percentages for the noisy data scenario. Furthermore, a comparison between the noise-free case (Table 1) and the 10% Gaussian noise case (Table 2) shows that random noise does not cause a serious distortion in the inverted source parameters; the closeness of the applied constraints to the true source model probably plays a bigger role in the constrained moment tensor inversion for events at far-field.

Similar to Figure 10, we conduct a Monte Carlo study of the constrained moment tensor inversion for a double-couple source.
with the same strike, dip, and rake values as the previous nondouble-couple case. The histograms of the inverted source parameters are given in Figure 11.

Table 3 summarizes the double-couple source inversion results under three different constraints. We see that maximizing DC percentage, as in the type III constraint, gives the smallest mean absolute errors in component percentage estimates, while knowing strike value, as in the type II constraint, helps reduce the errors in the fracture plane solution. In general, from Tables 2 and 3, we see that, with a reasonable amount of data noise and errors in velocity model and source location, the complete moment tensor can be inverted from one-well data at far-field by imposing additional constraints, such as the fracture plane orientation.

It is worth noting that the synthetic study conducted here is not a complete test on the influence of velocity model errors because only one random perturbation of the velocity model is used in the inversion. Furthermore, one should be aware that the influence of velocity model errors can be more serious when the source and the geophone array are situated in two different velocity layers.

FIELD STUDY

Field setup

A microseismic survey was conducted during the hydraulic fracturing treatment of the Bonner sands in the Bossier play at a depth from ≈3956 m (12,980 ft) to ≈3981 m (13,060 ft). The

Table 1. Summary of microseismic source inversion with one-well data under different constraints. The inversion is performed with noise-free data and uses the approximate velocity model and the mislocated source. The average source-receiver distance is 91.4 m (300 ft). The true moment tensor of this nondouble-couple source is described in Figure 4.

<table>
<thead>
<tr>
<th>Type of inversion constraints</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Errors in the inverted source parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isotropic component percentage (%)</td>
<td>−28</td>
<td>−12</td>
<td>−28</td>
</tr>
<tr>
<td>CLVD component percentage (%)</td>
<td>−15</td>
<td>4</td>
<td>−15</td>
</tr>
<tr>
<td>DC component percentage (%)</td>
<td>9</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Seismic moment (%)</td>
<td>24</td>
<td>−16</td>
<td>24</td>
</tr>
<tr>
<td>Strike (°)</td>
<td>14</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Dip (°)</td>
<td>−9</td>
<td>1</td>
<td>−9</td>
</tr>
<tr>
<td>Rake (°)</td>
<td>−8</td>
<td>−4</td>
<td>−8</td>
</tr>
</tbody>
</table>

Figure 10. The histograms of errors in the inverted source parameters (nondouble-couple source). The true moment tensor and the source-receiver configuration are described in Figure 9. The constrained inversion is performed with 10% Gaussian noise contaminated data. Left column: inversion with the type I constraint. Middle column: inversion with the type II constraint. Right column: inversion with the type III constraint. See main text for details on different constraint types.
microseismic data were collected using a twelve-level, three-component geophone array deployed in the vertical monitoring well at a depth from 3874 m (12,710 ft) to 3944 m (12,940 ft). The treatment well is approximately 151 m (495 ft) away from the monitoring well. The recorded data were analyzed and located for hydraulic fracturing mapping as outlined by Griffin et al. (2003) and Sharma et al. (2004). The velocity model for location, shown in Figure 2, was derived from the well logging data and calibrated using perforation shots (Warpinski et al., 2003). The information on local geology was also considered when building the velocity model.

In this study, we test our method on several located microseismic events to invert for the complete moment tensor and estimate source parameters. The microseismic data from the bottom six geophones at a depth from 3912 m (12,835 ft) to 3944 m (12,940 ft) are selected due to their higher S/Ns. The P-waves on the upper six geophones are barely identifiable due to the greater distance from the events. The average S-wave S/N on the upper six geophones is also 10 dB lower than that on the bottom six geophones. Moreover, due to the poor clamping of vertical component geophones, the average S/N of the band-pass filtered vertical component data is at least 10 dB lower than that of the band-pass filtered horizontal component data. On the other hand, from Figure 3b, it is observed that two horizontal components have a similar capability in resolving the moment tensor as three components. Therefore, only the

Table 2. Statistics of nondouble-couple microseismic source inversion with one-well data under different constraints (refer to Figure 10). The inversion is performed with 10% Gaussian noise contaminated data and uses the approximate velocity model and the mislocated source. The average source-receiver distance is 91.4 m (300 ft). The true moment tensor is described in Figure 4.

<table>
<thead>
<tr>
<th>Mean absolute errors in the inverted source parameters</th>
<th>Type of inversion constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Isotropic component percentage (%)</td>
<td>23</td>
</tr>
<tr>
<td>CLVD component percentage (%)</td>
<td>11</td>
</tr>
<tr>
<td>DC component percentage (%)</td>
<td>10</td>
</tr>
<tr>
<td>Seismic moment (%)</td>
<td>25</td>
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<tr>
<td>Strike (°)</td>
<td>12</td>
</tr>
<tr>
<td>Dip (°)</td>
<td>9</td>
</tr>
<tr>
<td>Rake (°)</td>
<td>9</td>
</tr>
</tbody>
</table>

Figure 11. The histograms of errors in the inverted source parameters (double-couple source). The source has a strike of 108°, a dip of 80°, and a rake of 43°. The source-receiver configuration is described in Figure 9. The rest of the figure description is analogous to Figure 10.
two horizontal components from the bottom six geophones are used in the following moment tensor inversion.

Figure 12 illustrates the horizontal plane view of the located events, with a monitoring well at the origin. The average fracture trend is seen along the N87°E or N-93°E direction (Sharma et al., 2004). Seven events at a depth from 3975 m to 3993 m are selected and plotted as red circles. The mean source-receiver distance for the selected events is around 152 L (106.7 m). The average noise level as a percentage of maximum absolute signal amplitude is about 7% for the selected events, which is lower than the 10% noise level used in the synthetic study.

In the following section, we will begin with one event, named “test event 1,” to demonstrate the procedure of the constrained moment tensor inversion and source parameter estimation using full-waveforms. After that, we will present and discuss the results from all seven chosen events.

Moment tensor inversion and source parameter estimation

As discussed in the synthetic study, for events that have a mean source-receiver distance greater than 5L, the complete moment tensor can be inverted from full-waveforms by imposing additional constraints. Warpinski and Du (2010) used direct P- and S-wave amplitudes from this one-well data set and applied a zero-trace (deviatoric source) constraint to invert for the source mechanisms and reported a large amount of scatter in the inferred strike and dip values.

In this study, instead of the deviatoric source constraint, a more realistic constraint on the fracture geometry is applied in the inversion. A conservative strike range of ±60° around the average fracture trend and a dip range of 60° ∼ 90° is used as the type I constraint in this field example. The source parameters, including the fracture plane solution, seismic moment, and component percentages are estimated from the inverted complete moment tensor.

Figure 13 shows the constrained inversion for test event 1 with the type I constraint. The cyan strip gives the permissible range of m22 values. The m22 value is determined by the green vertical line representing the maximum DC percentage within the allowed strike and dip range. Thus, the complete moment tensor is obtained.

Figure 14a and b gives the waveform fitting for test event 1 between modeled and observed data. A good agreement of dominant P- and S-wave trains is seen in Figure 14a and b. This gives confidence in the event location and 1D velocity models. The unmodeled wave packages are probably due to random noise and unmodeled lateral heterogeneities.

The source parameters of test event 1 estimated from the complete moment tensor are listed in Table 4. The seismic moment for event 1 is around 1.8 × 108 N · m, suggesting a moment magnitude around 3.22. The two strike values estimated from the double-couple component correspond to the orientation of the fracture plane and the auxiliary plane, respectively. It is hard to distinguish the two planes with only one event. The estimated strike, dip, and rake values for all test events are listed in Table 4. The first set of values agrees well with the average fracture trend of N87°E or N-93°E observed by Sharma et al. (2004) and is chosen as the fracture strike. Although the constraint used in the inversion assumes a strike range of ±60° around the average fracture trend, the actual inverted strike values for the six out of seven events have a maximum deviation from the average fracture trend of less than ±35°. In other words, additional information brought by the constrained inversion improves our a priori knowledge of source parameters, specifically the fracture strike. The difference between the inverted strike values and the average fracture trend comes from the fact that the orientation of small local fractures described by individual events differs from the average fracture orientation given by multiple event locations (Rutledge and Phillips, 2003). Noise contamination may also contribute to the difference.

Table 4 also summarizes the estimated component percentages. The results indicate a dominant double-couple component for most events. However, even considering the errors in the component percentage estimates as discussed in the synthetic study, a nonnegligible volumetric component is observed for some events, such as test events 3 and 6.

<table>
<thead>
<tr>
<th>Type of inversion constraints</th>
<th>( m_{22} ) values</th>
<th>( m_{33} ) values</th>
</tr>
</thead>
<tbody>
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<td>I</td>
<td>13</td>
<td>6</td>
</tr>
<tr>
<td>II</td>
<td>6</td>
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<tr>
<td>III</td>
<td>19</td>
<td>30</td>
</tr>
<tr>
<td>DC component percentage (%)</td>
<td>35</td>
<td>18</td>
</tr>
<tr>
<td>CLVD component percentage (%)</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>Seismic moment (%)</td>
<td>9</td>
<td>0</td>
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<tr>
<td>Strike (°)</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Dip (°)</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

Figure 12. Horizontal plane view of microseismic event locations for the Bonner data set. Seven selected test events for moment tensor inversion are shown as red circles.
Figure 13. Constrained inversion for test event 1 with the type I constraint. The figure description is analogous to Figure 9.

Figure 14. Waveform fitting for test event 1. Modeled seismograms derived from constrained inversion are shown in black, while the observed seismograms are plotted in red. (a) North component. (b) East component.
Full-waveform moment tensor inversion

Table 4. Results of source parameter determinations for the seven selected test events using constrained inversion with the type I constraint.

<table>
<thead>
<tr>
<th>Event</th>
<th>$M_0$</th>
<th>$M_W$</th>
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<th>ISO%</th>
<th>CLVD%</th>
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<td>7</td>
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<table>
<thead>
<tr>
<th>Event</th>
<th>Strike</th>
<th>Dip</th>
<th>Rake</th>
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<td>137</td>
<td>73</td>
</tr>
<tr>
<td>6</td>
<td>83</td>
<td>−17</td>
<td>63</td>
</tr>
<tr>
<td>7</td>
<td>138</td>
<td>−116</td>
<td>73</td>
</tr>
</tbody>
</table>

Note: The strike, dip, and rake values are defined according to the conventions set forth by Aki & Richards [2002].

For each event, the corner frequency is estimated from the far-field S-wave displacement spectrum (Walter and Brune, 1993). The approximate source radius is then determined from the corner frequency estimate according to Madariaga’s model (Madariaga, 1976; Talebi and Boone, 1998). The corner frequencies of all seven test events range from 450 Hz to 750 Hz. The derived source radii indicate a small rupture area on the order of 1 m². The moment magnitude of the test events ranges from −4 to −2, which is consistent with previous studies of hydrofracture events from downhole observations (Warpinski, 2009).

CONCLUSIONS

In this paper, we developed a full-waveform-based complete moment tensor inversion approach for hydraulic fracture monitoring using microseismic data recorded at a vertical borehole. The study involved both synthetic data and field data. The condition number study showed that two monitoring wells at an azimuthal separation of 45° have a similar resolving power of the moment tensor as eight wells with full azimuthal coverage. By exploring full wavefields in a layered medium instead of using only far-field direct P- and S-wave amplitudes, we demonstrated that the complete moment tensor can be retrieved for events that are close to the monitoring well. The near-field information and nondirect waves (i.e., reflected/refracted waves) propagated through a layered medium contribute to the decrease in the condition number. On the other hand, when the events are in the far-field range, two monitoring wells are desirable for complete moment tensor inversion.

With synthetic tests, we demonstrated that the complete moment tensor from one-well data at far-field is possible if one imposes some constraints. Far-field tests with different constraints indicate that a priori information on fracture orientation helps to recover the complete moment tensor and reduces the uncertainty of not only the fracture plane solution but also the seismic moment and moment component percentages. The synthetic study also shows that a reasonable amount of error in source location and the velocity model, together with random noise, does not cause a serious distortion in the inverted moment tensors and source parameters.

Proper constraints on the source play a big role in complete moment tensor retrieval using one-well data at far-field. The strike and dip range constraints were applied in a field study to invert for the complete moment tensor from one-well data at far-field. The results indicate the existence of both double-couple and nondouble-couple components in the source. The fracture strike values, derived by the inversion, generally agree with the average fracture trend determined from multiple event locations.

Potential errors in source parameter estimates from one-well data at far-field primarily come from the inaccuracies in the a priori information that has been used in the inversion. Future work will include testing the method against the results from two-well inversion. An extended study on the influence of velocity model errors will also be carried out in the future. The full-waveform approach has the potential to improve the source properties study of microseismic events monitored using borehole sensors even in a single well.

ACKNOWLEDGMENTS

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APPENDIX A

RETRIEVAL OF $M_{22}$ FROM ONE-WELL DATA AT NEAR-FIELD

In this appendix, we study the ability to retrieve $M_{22}$ using two horizontal component data from one vertical well at near-field. Previous studies have shown that, with far-field P- and S-wave amplitudes, it is impossible to invert for $M_{22}$ using data from one vertical well (Nolen-Hoeksema and Ruff, 2001; Vavryčuk, 2007). In this study, we use a pure $m_{22}$ source to generate synthetic seismograms. The true moment tensor in this case, has only one nonzero element, $m_{22} = 1$. The source is comprised of 66.7% CLVD and 33.3% isotropic component. The source-receiver configuration is the same as the near-field study. We invert for the complete moment tensor with band-pass filtered horizontal component data after adding 10% Gaussian noise. During the inversion, we use the approximate velocity model and a spatial grid search around the
There is no definition for the strike, dip, and rake of the source parameters derived from the inverted complete moment tensor inversion can be inverted from near-field waveforms.

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Feasibility of estimating vertical transverse isotropy from microseismic data recorded by surface monitoring arrays

Davide Gei¹, Leo Eisner², and Peter Suhadolc³

ABSTRACT

Microseismic data recorded by surface monitoring arrays can be used to estimate the effective anisotropy of the overburden and reservoir. In this study we used the inversion of picked P-wave arrival times to estimate the Thomsen parameter δ and the anellipticity coefficient η. This inversion employs an analytic equation of P-wave traveltimes as a function of offset in homogeneous, transversely isotropic media with a vertical axis of symmetry. We considered a star-like distribution of receivers and, for this geometry, we analyzed the sensitivity of the inversion method to picking noise and to uncertainties in the P-wave vertical velocity and source depth. Long offsets, as well as a high number of receivers per line, improve the estimation of δ and η from noisy arrival times. However, if we do not use the correct value of the P-wave vertical velocity or source depth, the long-offset may increase the inaccuracy in the estimation of the anisotropic parameters. Such inaccuracy cannot be detected from time residuals. We also applied this inversion to field data acquired during the hydraulic fracturing of a gas shale reservoir and compared the results with the anisotropic parameters estimated from synthetic arrival times computed for an isotropic layered medium. The effective anisotropy from the inversion of the field data cannot be explained by layering only and is partially due to the intrinsic anisotropy of the reservoir and/or overburden. This study emphasizes the importance of using accurate values of the vertical velocity and source depth in the P-wave arrival time inversion for estimating anisotropic parameters from passive seismic data.

INTRODUCTION

Elastic media, for which seismic velocities depend on the direction of wave propagation, are called anisotropic (Thomsen, 1986). Most crustal rocks are found experimentally to be anisotropic. Anisotropy in sedimentary rock sequences may be caused by the preferred orientation of anisotropic mineral grains (such as in a massive shale formation), the preferred orientation of the shapes of isotropic minerals (such as flat-lying platelets), or the preferred orientation of cracks or thin bedding of isotropic or anisotropic layers (Thomsen, 1986). A transversely isotropic medium with vertical axis of symmetry (VTI) is believed to be the most common anisotropic model for sedimentary basins (Grechka et al., 2002). In this study we consider homogeneous VTI media, which are equivalent (in travel-times) to complex 1D media of isotropic or VTI layers (Backus, 1962; Grechka and Tsvankin, 1998). We do not use a more complex anisotropy because the observed P-wave arrival times described in the field data section do not seem to require a higher order of symmetry (Eisner et al., 2011).

Ignoring the contribution of the anisotropy to the normal-moveout (NMO) velocity in shale reservoirs leads to misties in time-to-depth conversion (Banik, 1984; Alkhalifah et al., 1996; Sarkar and Tsvankin, 2006). Not only velocity analysis, but practically all other conventional seismic processing and interpretation techniques become inaccurate if the medium is anisotropic (Lynn et al., 1991; Alkhalifah and Larner, 1994; Tsvankin, 1995; Tsvankin and Thomsen, 1995).

A simple methodology of estimating effective anisotropic parameters from long-offset seismic data is the inversion of P-wave arrival times. VTI media are characterized by nonhyperbolic...
reflection moveouts, which are more significant in large-offset arrivals for P-waves. The nonhyperbolicity of the moveout can also be related to vertical and lateral heterogeneity and reflector curvature (Fomel and Grechka, 1997). This technique, widely used for active seismic prospecting (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995; Grechka and Tsvankin, 1998), can be efficiently applied to microseismic data analysis as well. Although microseismic events also generate split S-waves that provide an evidence of anisotropy (e.g., Verdon et al., 2009), the first arrivals of P-waves can generally be unambiguously picked in noisy (surface) data. Thus, this study will focus on P-waves only.

Anisotropy is routinely inverted from active data through various techniques. The most commonly used configuration is the VSP-type of acquisition (e.g., Jech and Pšenčík, 1992), where receivers are lowered into a borehole and active sources (shots or vibroseis) are located at the surface. In such experiments both receiver position and shot timing are known with sufficient precision. Furthermore, receivers are distributed over some depth interval. In passive seismic geometry, discussed in this study, only one depth interval of calibration shots is usually available, and the timing of the shot is usually unknown. Therefore, this study focuses on the feasibility of this poorer inversion with additional unknowns. Alternatively, the presence of anisotropy can be inferred from the S-wave splitting observed in downhole data (e.g., Verdon et al., 2009). However, we are studying the feasibility of using P-waves only from surface data because they might provide an alternative measurement to S-wave splitting when the S-wave model is not sufficiently known.

Hydraulic fracture stimulation (fracking) is commonly used to enhance hydrocarbon recovery by increasing the reservoir permeability. These stimulations consist of injecting high-pressure fluids into rock formations. Such injections induce microseismic events that are monitored to optimize the hydraulic fracturing. A star-shaped array of surface (or near-surface) geophones can be used to monitor the induced microseismicity. The monitoring of induced seismicity differs from active seismic prospecting by having more unknowns. The origin time is obviously not known for microseismic events and often even for perforation shots. The locations of perforation shots are known with a limited precision, which depends on the accuracy of a well-deviation survey (Bulant et al., 2007). A velocity model is usually calibrated from seismic signals of perforation shots at known positions along the treatment well.

Chambers et al., (2010b) test the ability to detect microseismic events with a surface star-like array by locating several perforation shots in isotropic layered media. The application of isotropic velocity models to surface monitoring of induced microseismicity is also discussed by Chambers et al. (2010a) in an application of data from Valhall. In their study, the authors used an isotropic model built on active seismic monitoring, whereas this study investigates the possibility of building an effective anisotropic model.

In this study, we investigate the feasibility of inverting passive seismic data for effective anisotropic parameters, assuming a VTI model of the subsurface. We investigate the sensitivity of the P-wave arrival time inversion to picking errors and uncertainties in the P-wave vertical velocity and source location. We also aim to better understand the effects of the possible incorrectness of these input parameters on the results of the inversion and to verify the effectiveness and limits of using large offsets and short receiver spacing. Moreover, we show some results of the application of the P-wave arrival time inversion to four perforation shots at a gas shale reservoir as an example of practical velocity model calibration.

We use star-array geometry for our synthetic tests as this layout is similar to the field test of this study. We believe our conclusions are not significantly affected by the chosen geometry, as illustrated by Zhang et al. (2011) (Zhang, Y., L. Eisner, W. Barker, M. C. Mueller, K. Smith, 2011, Consistent imaging of hydraulic fracture treatments from permanent arrays through calibrated velocity model: submitted to Geophysical Prospecting).

**P-WAVE TRAVELTIME INVERSION FOR HOMOGENEOUS TRANSVERSELY ISOTROPIC (TI) MEDIA**

Traditionally, the inversion of P-wave traveltime has been used for active seismic applications (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995; Grechka and Tsvankin, 1998). Considering a straight-line raypath in a single horizontal homogeneous isotropic layer, the traveltime of the direct P-wave from a subsurface source is given by

\[ t(x) = t(0)^2 + x^2/V_{NMO}^2. \]  

where \( t(0) \) is the zero-offset one-way traveltime, \( x \) is the offset, i.e., the horizontal distance from the source, and \( V_{NMO} \) is the normal-moveout velocity, being the P-wave velocity of the isotropic medium (Figure 1). The moveout travel-time described by equation 1 is hyperbolic.

Let us consider a single horizontal homogeneous transversely isotropic layer with a vertical symmetry axis (VTI). In the small offset approximation equation 1 still holds, but

\[ V_{NMO} = V_{P0}(1 + 2\delta)^{1/2}, \]

where \( V_{P0} \) is the P-wave vertical velocity and \( \delta \) is one of the Thomsen parameters (Thomsen, 1986). In small offsets the moveout is still hyperbolic, but \( V_{NMO} \) is neither the vertical, nor the horizontal P-wave velocity of the medium.

Alkhalifah and Tsvankin (1995) showed that, in laterally homogeneous VTI media, the traveltimes of qP-waves depend mainly on the zero-dip normal-moveout velocity \( V_{NMO} \) and the anellipticity parameter \( \eta \), controlling the nonhyperbolic moveout:

![Figure 1. Schematic representation of a passive seismic monitoring experiment.](image-url)
VTI parameters from microseismic data

$$\eta = \frac{\epsilon - \delta}{1 + 2\delta},$$  \hspace{1cm} (3)

where $\epsilon$ is one of the Thomsen parameters, whose value in VTI media is close to the fractional difference between the horizontal and vertical P-wave velocities (Thomsen, 1986).

Alkhilafah and Tsvankin (1995) modified a three-term Taylor series expansion of the moveout given by Tsvankin and Thomsen (1994) as

$$t^2(x) = t^2(0) + \frac{x^2}{V_{NMO}^2} - \frac{2px^4}{V_{NMO}^2[r^2(0)V_{NMO}^2 + (1 + 2\eta)x^2]},$$  \hspace{1cm} (4)

where $V_{NMO}$ is given by equation 2. Here the coefficient of $x^4$ is modified to fit the horizontal velocity. This moveout equation is suitable for computation of P-wave traveltimes in the large-offset approximation. Furthermore, the equation holds even for computing the traveltime from a source at depth to the surface, if $x$ is the horizontal distance of the receiver from the source and $t(0)$ is the one-way traveltime. This equation can be used to invert arrival times from microseismic events or perforation or calibration shots. Arrival times can be inverted in a nonlinear iterative inversion minimizing the residuals between observed and synthetic traveltimes. This inversion uses as input arrival times $t_A(x)$ determined along various offsets. The traveltimes $t(x)$ in equation 4 are matched with the arrival times corrected for an origin time $t_0$ as

$$t(x) = t_A(x) - t_0.$$  \hspace{1cm} (5)

Thus, to invert VTI parameters $\delta$ and $\eta$, we need to either know or invert origin time $t_0$, vertical P-wave velocity $V_{P0}$, the depth of the source, and the one-way vertical traveltime $t(0))$. Without restriction to generality, we assume that the horizontal position of the source is either known with sufficient accuracy (perforation shots) or determined from symmetry of the moveout, independently of the velocity model calibration.

**SYNTHETIC DATA**

In this section we describe the synthetic data set, which is similar to the real data sets discussed in the field data section. Here we investigate synthetic arrival times for homogeneous anisotropic media equivalent to a layered model with a vertical axis of symmetry; such a model is appropriate for most fractured shale basins. Figure 1 shows a vertical cross section through the synthetic passive seismic monitoring experiment of this study. The microseismic source is located at depth $z_S$ and the receiver at offset $x$, $t(x)$ is the traveltime at offset $x$ and $t(0)$ is the one-way vertical traveltime. To compute synthetic travel times and perform the P-wave arrival time inversion, receivers are arranged in eight regularly spaced lines (45° spacing) radiating from a central point, in a star-like pattern. The source is located in the center of the star at a depth of 2100 m, as illustrated in Figure 2. The coordinates of the source are $x_S = 3350$ m, $y_S = 3350$ m, and $z_S = 2100$ m. The effective vertical velocity $V_{P0}$ is 2906 m/s and the anisotropic parameters are $\delta = 0.1$ and $\eta = 0.1$. We chose $\delta$ to be equal to $\eta$, as we want to compare the relative changes of these parameters in the inversion. From the computed traveltimes we subtract 0.5 s, corresponding to the origin time $t_0 = -0.5$ s.

The traveltimes are computed using equation 4, i.e., we use the same equation for forward and inverse modeling. We have also used alternative computations of traveltimes (e.g., traveltimes computed with equations listed in Bulant et al. (2007) and full-wave numerical modeling (Carcione, 2007) and picking) and have obtained similar results. Therefore, we assume that this choice does not affect our conclusions. To simulate picking noise, we perturb synthetic arrival times with Gaussian noise with zero mean and increasing values of standard deviation $\sigma_n$ from 0 to 4 ms. The standard deviation of the Gaussian noise is given by

$$\sigma_n = \sqrt{\frac{\sum_{i=1}^{N} t_i^2}{N}},$$  \hspace{1cm} (6)

where $\sigma_n$ is the noise added to the arrival times and $N$ is the number of arrival times in the data set. And analogously the rms of residuals, either from synthetic or real data sets, is

$$\text{rms} = \sqrt{\frac{\sum_{i=1}^{N} (t_i^{\text{data}} - t_i^{\text{synth}})^2}{N}},$$  \hspace{1cm} (7)

where $t_i^{\text{data}}$ are input arrival times and $t_i^{\text{synth}}$ are computed arrival times.

**SENSITIVITY ANALYSIS OF P-WAVE TRAVELTIME INVERSION**

In the following inversion tests we invert for anisotropic parameters $\delta$ and $\eta$ and origin time $t_0$. For input parameters we use arrival times with variable noise levels, the vertical P-wave velocity $V_{P0}$, and the source location ($x_S$, $y_S$, $z_S$). We investigate the sensitivity of the inverted parameters to input parameters, particularly the source depth and vertical P-wave velocity. We consider the horizontal coordinates of the source ($x_S$ and $y_S$) as known parameters, as they can be robustly inverted in VTI media. We do not invert more
than three parameters ($\delta$, $\eta$, and $t_0$) because equation 4 shows that only three coefficients of the Taylor expansion can be determined independently. This is analogous to the inversion described in Bulant et al. (2007), in which only three parameters were determined independently. In our case, we chose the origin time and anisotropy as unknowns. However, if the origin time is known we can invert for the vertical velocity instead.

Sensitivity to picking noise

In principle, once the VTI character of the subsoil has been ascertained, the inversion of P-wave arrival times can be performed for a single seismic line, e.g., a single arm of the star-array. However, in the presence of picking noise, a large number of arrival times given by an equally large number of receivers provides better statistical sampling resulting in a more precise estimate of anisotropic parameters $\delta$ and $\eta$. Figure 3 shows the results of 100 inversions, each of them characterized by a different realization of the Gaussian noise ($\sigma_n = 4$ ms). The maximum offset-to-source-depth ratio (MO/SD) is 1.5, the receivers spacing is 16 m, and the number of receivers per line is 200. Triangles and circles show inverted $\delta$ and $\eta$ using the eight-arm star-array and line 1 alone, respectively. The two anisotropic parameters are characterized by a linear trend as values of $\delta$ and $\eta$ trade off with one another. Inversions of the data set from the star-array result in tighter clustering than inversions of the data from single line 1, both centered on the true (input) values. All inversions in this study are performed using the star-array geometry.

Figure 4a and b shows $\delta$ and $\eta$, respectively, inverted from synthetic arrival times perturbed with different levels of Gaussian noise $\sigma_n$. Figure 4c and d shows inverted origin times $t_0$ and the root mean square of time residuals (rms). The maximum offset-to-source-depth ratio (MO/SD) is 1.5, the receiver spacing is 16 m, and the number of receivers per line is 200. For each noise level ($\sigma_n$) we compute 100 realizations. Inaccuracies in the estimated anisotropic parameters are proportional to the picking noise, and $\eta$ is more sensitive than $\delta$ to the noise level. $t_0$ is little affected by the picking noise. The rms of time residuals (Figure 4d) has the same value of $\sigma_n$; thus, we may use the measured rms values in a real data set as an estimate of the noise in picked arrival times.

Figure 5a and b shows the standard deviation of $\delta$ and $\eta$, respectively, versus the MO/SD with $\sigma_n$ of 4 ms. For each MO/SD value, the curves represent the standard deviation of 100 estimated anisotropic parameters, corresponding to 100 noise realizations in the synthetic arrival-times. Figure 5 shows three curves in each plot. To obtain the curves defined by circles, we increase the offset by adding receivers to each arm of the star-array; the receiver spacing is constant (25 m) and the number of receivers varies between 63 (MO/SD = 0.75) and 168 (MO/SD = 2). The curves defined by asterisks are computed by keeping constant the number of receivers per line (100) and increasing the receiver spacing from 16 m (MO/SD = 0.75) to 42 m (MO/SD = 2). The curves given by triangles use 200 receivers per line and receiver spacing from 8 m (MO/SD = 0.75) to 21 m (MO/SD = 2). The standard deviation of the inverted $\delta$ and $\eta$ decreases as MO/SD increases. The fastest decrease of the error is for MO/SD smaller than one, and the improvement to the inverted anisotropic parameters from offsets larger than 1.5 source depth is negligible. A greater number of receivers per line provides a better estimate of anisotropic parameters by means of

![Figure 3](image-url)

Figure 3. Anisotropic parameters $\delta$ versus $\eta$ as results of inversions of traveltimes perturbed with 100 different realizations of Gaussian noise ($\sigma_n = 4$ ms). Triangles correspond to inversions with the star geometry and circles indicate inversions with a single-line geometry. The cross indicates the actual values of the anisotropic parameters ($\delta = 0.1$ and $\eta = 0.1$).

![Figure 4](image-url)

Figure 4. Estimated anisotropic parameter $\delta$ (a), $\eta$ (b), origin time $t_0$ (c), and rms of time residuals (d) for nine values of the standard deviation of Gaussian noise ($\sigma_n$). The maximum offset-to-source depth ratio (MO/SD) is 1.5, the receiver spacing is 16 m, and the number of receivers per line is 200.
a better statistical sampling of noise. This conclusion is similar to
the one obtained with the test in Figure 3. Increasing the maxi-
mum offset by adding receivers to the seismic lines results in the
most effective reduction of the standard deviations of \( \delta \) and \( \eta \).

This is also shown in Figure 6a and b by plotting the standard
deviation of \( \delta \) and \( \eta \), respectively, as a function of the number
of receivers per line (\( nr \)) for a fixed MO/SD. For each value \( nr \)
we show the standard deviations of \( \delta \) and \( \eta \), resulting from 100 noise
realization of arrival times. MO/SD is 1.5 and the arrival times are
affected by the picking noise with the \( \sigma_p \) of the normal distribution
of 4 ms. The maximum offset is kept constant, and increasing the
number of receivers reduces the receiver spacing. The estimated va-
ues of \( \delta \) and \( \eta \) dramatically improve as more receivers are added to
each arm of the star-array: up to 200 receivers corresponding to a
receiver spacing of 15 m. Note that the uncertainty reduction falls
off approximately as \( \frac{1}{\sqrt{N}} \), where \( N \) is the number of receivers.

**Sensitivity to P-wave vertical velocity**

To study the sensitivity of this P-wave arrival
time inversion technique to the input value of the
P-wave velocity in the vertical direction (\( V_{P0} \)) we
perform arrival-time inversions considering se-
ven different velocity values, ranging from
\( -10\% \) to \( +10\% \) of the actual value \( V_{P0}^{true} = 2960 \text{ m/s} \).
Synthetic traveltimes are computed with the actual values of the P-wave vertical ve-
locity \( V_{P0}^{true} \), but they are inverted in view of the
incorrect values of \( V_{P0} \).

Figure 7 shows the inverted \( \delta \), \( \eta \), origin time \( t_0 \),
and the rms of the time residuals, respectively, as
a function of the assumed input P-wave vertical velocity \( V_{P0} \).
For each value of the P-wave
vertical velocity, we perform 100 inversions, cor-
responding to 100 noise realizations in the syn-
thetic arrival-times. The standard deviation of
the Gaussian noise is 4 ms. The maximum
offset-to-source-depth ratio is 1.5 and the num-
ber of receivers per arm is 200. Each circle in
Figures 7a-d represents the result of one of
the 100 performed inversions. For P-wave verti-
cal velocities differing from the actual value
\( V_{P0}^{true} \), the estimated \( \delta \) (Figure 7a) and \( \eta \)
(Figure 7b) reveal a systematic bias increasing
almost linearly with the actual difference be-
tween the correct and input vertical velocity.
However, the scatter (or standard deviation) of
the inverted anisotropic parameters remains ap-
proximately constant and depends on the level
of noise in the arrival-times (Figure 7d). The
bias, or accuracy, of the two inverted anisotro-
pic parameters is proportional to \( |V_{P0} - V_{P0}^{true}| \). The scatter, or precision, depends only on the level
of noise in the arrival times. Because the noise
level was kept constant in this test, the inverted
\( \delta \) and \( \eta \) show a constant scatter for all the \( V_{P0} \)
values. Origin time \( t_0 \), shown in Figure 7c, is
not affected by the Gaussian noise but depends
on the input P-wave vertical velocity. The rms
of the time residuals (Figure 7d) equals the noise
level (\( \sigma_t \)) but is unaffected by \( V_{P0} \). The presence of picking noise in
the input arrival times can be inferred from the root mean square of
the time residuals. Instead, inaccurate values of the input P-wave
vertical velocity cause proportional inaccuracies in the estimated
anisotropic parameters that cannot be detected from the rms or
any other result of the inversion.

Figure 8a and b shows the dependence of the means (solid lines)
and standard deviations (shaded areas) of 100 inverted \( \delta \) and \( \eta \) on
the maximum offset-to-source-depth ratio (MO/SD). We show three
dependences for three different input vertical velocities:
\( V_{P0} = 0.9V_{P0}^{true} \), \( V_{P0} = V_{P0}^{true} \), and \( V_{P0} = 1.1V_{P0}^{true} \). The input arrival
times are perturbed with Gaussian noise (\( \sigma_p = 4 \text{ ms} \)). The symbols
in Figure 8a and b are the mean values of the anisotropic parameters
inverted from 100 realizations of randomly distributed Gaussian
noise in the synthetic arrival times. They represent the accuracy
of the inversion method as a function of MO/SD and \( V_{P0} \). The stan-
dard deviations \( \sigma_\delta \) and \( \sigma_\eta \) (shaded area) for the increasing maximum

![Figure 5. Standard deviation of 100 estimations of anisotropic parameters \( \delta \) (a) and \( \eta \) (b) versus maximum offset-to-source-depth ratio (MO/SD). The standard deviation for the Gaussian noise perturbing the synthetic traveltimes is 4 ms. For curves defined by circles, the maximum offset is increased by increasing the number of receivers in each line of the star-array from 63 (MO/SD = 0.75) to 168 (MO/SD = 2). For curves, given by asterisks and triangles, the arms are stretched, increasing the receiver spacing with a constant number of receivers per line of 100 and 200, respectively.](image)

![Figure 6. The standard deviation of anisotropic parameter \( \delta \) (a) and anellipticity coefficient \( \eta \) (b) versus the number of receivers per line (\( nr \)). Each circle is the standard deviation of 100 estimations corresponding to the same number of different noise realizations. The maximum offset-to-source-depth ratio is 1.5. The standard deviation for the Gaussian noise is 4 ms.](image)
offset-to-source-depth ratio are related to the Gaussian noise in the arrival times. The surprising result is increasing bias (inaccuracy) for \( \eta \) with increasing maximum offset for \( V_{P0} \neq V_{P0}^{true} \). There are two sources of uncertainty in the estimated parameters: the lack of accuracy, highlighted by the means and related to the incorrect value of \( V_{P0} \), and the lack of precision, highlighted by the standard deviations and due to the picking noise. For the Thomsen parameter \( \delta \), both the accuracy and precision improve with increasing MO/SD.

The accuracy of the anellipticity coefficient \( \eta \) strongly decreases as the maximum offset increases, and the precision slightly increases.

**Sensitivity to source depth**

Similar to the test used for the sensitivity analysis to the P-wave vertical velocity, we invert the arrival times perturbed with white Gaussian noise (\( \sigma_\delta = 4 \text{ ms} \)) with various source depth values \( z_0' \) ranging from \( 0.95z_0 \) to \( 1.05z_0 \), where \( z_0 = 2100 \text{ m} \) is the actual source depth used to compute the arrival times.

Figure 9 shows \( \delta, \eta, \) origin time \( t_0 \), and the rms of the time residuals, respectively, as a function of \( z_0' \). For each value \( z_0' \) we perform 100 inversions, corresponding to 100 noise realizations. The standard deviation of the Gaussian noise is 4 ms. The maximum offset to the source depth ratio is 1.5, and the number of receivers in each line is 200. For source depths different from the actual value, the inverted \( \delta \) and \( \eta \) are characterized by a systematic error proportional to \( |z_0' - z_0| \). The depth \( z_0' \) affects the accuracy, but not the precision, of the inversion results. The precision is again controlled by noise in the arrival times only, because the scatter remains constant in Figure 9a, b, and d. The inverted origin times (Figure 9c) are not affected by the Gaussian noise, but they strongly depend on the source depth.

Figure 10a and b shows the means (solid lines) and standard deviations (shaded areas) of \( \delta \) and \( \eta \) as a function of the source-to-depth ratio (MO/SD); three values of source depths were tested: \( z_0' = 0.95z_0, 0.95z_0, \) and \( 0.95z_0 = 1.05z_0. \) The accuracy of the inversion of both anisotropic parameters decreases with increasing offset. This can be understood from the fact that the VTI anisotropy mainly affects horizontal traveltimes. Thus, as the offset increases, the raypaths become more horizontal, and the erroneous depth is compensated by larger values of the VTI parameters. The precision of the inversion method, emphasized by standard deviations \( \sigma_\delta \) and \( \sigma_\eta \), strongly increases with MO/SD. The increasing of the precision, inferred by the narrowing of the shaded areas with increasing MO/SD, could give an incorrect impression of obtaining more accurate results. Similar to the results of the P-wave vertical velocity test, the noise level in the arrival times can be gathered from the rms of the time residuals. An incorrect source depth causes inaccuracies in the estimated \( \delta \) and \( \eta \), and this inaccuracy increases with greater maximum offsets. Such inaccuracies do not occur with any output parameter of the inversion.

The results of testing the P-wave arrival time inversion with synthetic traveltimes are summarized in Table 1. Uncertainties in the input parameters are given in roman type and their effect on the results of the inversions are given in italic type.
FIELD DATA

Microseismic monitoring was performed by Microseismic Inc. during the hydraulic fracturing of a gas shale reservoir located in North America and operated by Newfield Exploration Mid-Continent Inc. They used a 10-line Fracstar® array (Figure 11) with 1C geophones located on the earth’s surface. The number of receivers per arm varied between 54 (line four) and 122 (lines two and 10) and the average receiver distance was 23 m. The formation was accessed by perforating the casing at reservoir depth. Such perforation shots are generally used to calibrate the velocity model in downhole and surface monitoring. As the tests on synthetic data sets in previous sections revealed, the inverted anisotropic parameters depend on the correct depth of the microseismic event. The depth of induced microseismic events is unknown; therefore, the inversion was performed from perforation shots, whose positions are known with high accuracy (less than 2% error, as discussed in Bulant et al., 2007). We apply the previously described inversion to P-wave arrival times measured from perforation shots. All of the shots belong to the same stage with a horizontal shot separation of 37 m; the vertical separation of shots is negligible. Perforation shot coordinates are given in Table 2.

Figure 11 shows the manually picked and interpolated arrival times for perforation shot 1. Microseismic data from the northwestern part of the array are too noisy to pick the first arrivals.

Figure 12 shows an example of seismic sections relative to shot 1. We apply a band-pass frequency filter with corner frequencies 6, 12, 60, and 70 Hz.

To apply the P-wave traveltime inversion for homogenous-anisotropic media to this data set, we compute the effective vertical velocity at the source depth: for a given depth of a seismic source, the effective velocity is the P-wave velocity of an equivalent homogeneous medium yielding the same zero-offset travel-time as the layered medium. Figure 13 shows the 1D P-wave vertical velocity profile derived from 3D active seismic over the reservoir (continuous lines) and the effective vertical velocity (dashed curve). The effective velocity at the depth of the perforation shots is \( V_p = 2906 \) m/s.

Figure 14a and b shows the time residuals of perforation shot 1 interpolated in a map-view plot and as a function of offset, respectively. The results of the inversions of picked arrival times from the four perforation shots are given in Table 3. \( \delta \) and \( \eta \) are high, indicating an anisotropic medium. The anisotropic parameters shown in Table 3 are similar from shot to shot, although they were obtained from independently picked arrival times of the four perforation shots. Shot 4 yields a slightly higher \( \delta \) and a lower \( \eta \) than the other shots. Because of the strong noise level in the northwestern part of the array, we could pick only arrivals from lines (1), (2), (3), (10), and part of (9) (see Figure 11 for line number reference). This makes shot 4 the least constrained. Finally, the resulting rms is comparable to the test of Figure 3; we can see that the scatter of the inverted anisotropic parameters is very consistent, and even the “error” of inverted parameters from shot 4 is consistent with the \( \delta \)-\( \eta \) trade-off observed in the test on the synthetic data set.

The nonhyperbolicity of the moveout can also be caused by vertical and lateral heterogeneity (Backus, 1962; Fomel and Grechka,
Inversions of arrival times in isotropic layered media can result in apparent anisotropy. To estimate the influence of layering on the above inverted effective anisotropy we compute and invert synthetic arrival times for a layered isotropic medium. We consider a horizontally layered model suitable for this data set with the P-wave velocity profile shown in Figure 13 (continuous line). We compute the synthetic traveltimes in the isotropic layered model and add Gaussian noise with zero mean and $\sigma_n = 4$ ms, similar to the residuals observed in the inversions of the field data. Figure 15 shows the time residuals from the inversion of synthetic arrival times computed with the same geometry of the field data (see Figure 14). Table 2 shows the inverted anisotropic parameters, which are only about 50% of the anisotropic parameters observed in the inversion of the real data set, assuming a homogeneous medium. Hence, we conclude that the isotropic layers seem to cause only about 50% of the effective anisotropy, indicating that the medium is also anisotropic.

**Table 1. Summary of the results of P-wave arrival time inversions of synthetic data.**

<table>
<thead>
<tr>
<th>Picking noise</th>
<th>$\delta$</th>
<th>$\eta$</th>
<th>$t_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scattered$^+$</td>
<td>Scattered$^+$</td>
<td>No effect</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Lower than $V_{P0}^{true}$</th>
<th>Higher$^*$</th>
<th>Higher</th>
<th>Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source depth</td>
<td>Lower than $z_S$</td>
<td>Higher</td>
<td>Lower</td>
<td>Higher</td>
</tr>
<tr>
<td>Source depth</td>
<td>Higher than $z_S$</td>
<td>Lower</td>
<td>Higher</td>
<td>Lower</td>
</tr>
</tbody>
</table>

$^*$Long offsets improve the estimation.
$^+$A high number of receivers per line improves the estimation.

**Table 2. Input parameters for P-wave arrival time inversions of field data.**

<table>
<thead>
<tr>
<th>Shot</th>
<th>Easting (m)</th>
<th>Northing (m)</th>
<th>Depth (m)</th>
<th>$V_{P0}^{true}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2343</td>
<td>2410</td>
<td>2100</td>
<td>2906</td>
</tr>
<tr>
<td>2</td>
<td>2341</td>
<td>2517</td>
<td>2100</td>
<td>2906</td>
</tr>
<tr>
<td>3</td>
<td>2341</td>
<td>2552</td>
<td>2099</td>
<td>2906</td>
</tr>
<tr>
<td>4</td>
<td>2342</td>
<td>2590</td>
<td>2100</td>
<td>2906</td>
</tr>
</tbody>
</table>

Figure 11. A contour plot showing the picked arrival times for perforation shot 1. The straight, gray, numbered lines represent the 10 seismic lines of the Fracstar. The white star is the source location.

Figure 12. Seismic sections of lines 1–3 for shot 1. The first arrivals are indicated by arrows.

Figure 13. A 1D vertical P-wave velocity profile of the study area. The interval velocity is given by the continuous line and the dashed line is the effective vertical velocity. The asterisk represents the effective velocity ($V_{P0} = 2906$ m/s) with the source located at 2100 m, the depth of the perforation shots.
A more general conclusion about the inverted anisotropic parameters requires the generalization of the methodology to at least 1D layered media. The observed anisotropy strength seems to be consistent with that observed in active seismics. The residuals observed in the test on the real data set are surprisingly low, suggesting small effects of the near-surface structure in this area.

CONCLUSIONS

In this study, we investigate reservoir characterization from an array of sensors with a star geometry, deployed on the earth’s surface, to monitor hydraulic stimulations. We explore the sensitivity of the P-wave arrival time inversion for homogeneous transversely isotropic media with a vertical axis of symmetry to picking errors and uncertainties in the P-wave vertical velocity and source depth.

The Gaussian noise perturbing the synthetic arrival times affects the precision of the resulting anisotropic parameters δ and η, whereas the origin time is estimated accurately. Long offsets of the lines forming the star-pattern array improve the estimation of anisotropic parameters, most effectively up to 1.5 of the maximum offset-to-depth ratio. Increasing the number of receivers per line of the star-array also increases the precision of the resulting anisotropic parameters, because they improve statistical sampling. Long offsets, and a high number of receivers per line improve the estimation of δ and η from noisy arrival times only if accurate values of the source depth and vertical P-wave velocity are used. If the latter input values are not correctly estimated, the increase of the maximum offset causes a bias of the estimated anisotropic parameters. The precision increases with increasing the length of the maximum offset and the number of receivers, giving an incorrect impression of more accurate results.

Operators and service companies need to use an accurate vertical velocity and good calibration shots to obtain reliable unbiased estimates of anisotropy for microseismic monitoring. A well-calibrated anisotropic velocity model is needed for accurate and unbiased locations of microseismic events.

We also apply the P-wave arrival time inversion to four perforation shots recorded from microseismic monitoring and obtain consistent results from the four independent inversions, resulting in approximately δ = 0.27 and η = 0.12. Furthermore, we invert the synthetic arrival times computed with an isotropic layered model suitable for this reservoir and obtain an effective anisotropy approximately 50% in strength. Thus, we conclude that the effective anisotropy observed in the field data is caused partially by the intrinsic anisotropic properties of the formations.

ACKNOWLEDGMENTS

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Narrow-angle representations of the phase and group velocities and their applications in anisotropic velocity-model building for microseismic monitoring

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\textbf{ABSTRACT}

It is usually believed that angular aperture of seismic data should be at least 20° to allow estimation of the subsurface anisotropy. Although this is certainly true for reflection data, for which anisotropy parameters are inverted from the stacking velocities or the nonhyperbolic moveout, traveltimes of direct P- and S-waves recorded in typical downhole microseismic geometries make it possible to infer seismic anisotropy in angular apertures as narrow as about 10°. To ensure the uniqueness of such an inversion, it has to be performed in a local coordinate frame tailored to a particular data set. Because any narrow fan of vectors is naturally characterized by its average direction, we choose the axes of the local frame to coincide with the polarization vectors of three plane waves corresponding to such a direction. This choice results in a significant simplification of the conventional equations for the phase and group velocities in anisotropic media and makes it possible to predict which elements of the elastic stiffness tensor are constrained by the available data.

We illustrate our approach on traveltime synthetics and then apply it to perforation-shot data recorded in a shale-gas field. Our case study indicates that isotropic velocity models are inadequate and accounting for seismic anisotropy is a prerequisite for building a physically sound model that explains the recorded traveltimes.

\textbf{INTRODUCTION}

Much of the progress in the estimation of seismic anisotropy and the building of anisotropic velocity models can be attributed to the choice of appropriate parametrization of the stiffness tensor. Such a parametrization is usually designed to capture the influence of elastic anisotropy on a seismic signature in question and to facilitate the inversion of anisotropy-related quantities from that signature. The most well-known example of reparameterizing the stiffnesses is Thomsen (1986) notation that identifies the combinations of stiffness coefficients of vertically transversely isotropic (VTI) media obtainable in a unique fashion from seismic velocities routinely measured in the exploration practice.

Thomsen parameters are no longer convenient when data contain both kinematic and polarization signatures. The data of this kind, for instance, components of the slowness vectors and directions of the particle motions, are recorded in vertical seismic profiling (VSP) surveys (Zheng and Pšenčík, 2002; Xiao and Leaney, 2010). To estimate seismic anisotropy in the vicinity of downhole geophones from such measurements, one has to define a different set of Thomsen-style parameters, specifically, the one that governs the dependence of the slowness component along a borehole on the direction of the polarization vector and can be unambiguously inverted from this dependence (Grechka and Mateeva, 2007).

Here we examine the same anisotropic parameter-estimation problem but for geometries encountered in downhole microseismic monitoring. The main objective of acquiring microseismic surveys in tight-gas and shale-gas fields is to delineate hydraulic fractures, which are artificially created to make production from low-permeability formations economically viable (e.g., Maxwell, 2010; Maxwell et al., 2010). The shapes and orientations of hydraulic fractures are inferred from locations of microseismic events triggered in the course of hydraulic well stimulations. Clearly, a velocity model is required to locate the microseismicity. This model is conventionally derived from sonic logs and traveltimes of the direct P- and S-waves excited by perforation shots and recorded by geophones placed in an adjacent borehole (e.g., Warpinski et al., 2005; Pei et al., 2009). Hence, the goal of our study is to invert the observed perforation-shot times for a velocity model.
Although one might think that Thomsen-type parameterization could be helpful in estimating seismic anisotropy from such traveltime data, this is seldom the case. Two typical reasons are narrow angular apertures of ray trajectories recorded by geophones placed in a single well and oblique orientations of those trajectories with respect to layering or principal stress directions, which might be taken as plausible symmetry elements for the elastic properties of the formation. As a consequence of the narrow ray coverage, an analog of the stacking velocity is difficult to measure and, therefore, the $\delta$-type parameters governing the P-wave normal-moveout velocities are poorly constrained. Also, the symmetry-direction velocities in a single well and oblique orientations of those trajectories with respect to uppercase ones.

The Christoffel equation (e.g., Auld, 1973; Červený, 2001)

$$g_Q = c_{ijkl} U_{ij} Q_{kl} U_{Q}, \quad (j, Q = 1, 2, 3),$$

where

$$p_Q = n / V_Q$$

are the slowness vectors. The (scalar) group velocities are defined as

$$g_Q = \| g_Q \| = \sqrt{g_Q \cdot g_Q}.$$  (4)

To resolve these issues and infer anisotropy from perforation-shot data, we propose a strategy that consists of (1) finding a limited subset of the stiffness coefficients that govern the velocities of waves recorded in a given narrow fan of directions and (2) estimating only those particular stiffnesses while leaving the other, not as tightly constrained stiffness components, undetermined. We begin the paper by discussing a coordinate transform that identifies the stiffnes coefficients that should be targeted in the inversion, confirm our analytical findings on ray-tracing synthetics, and apply the developed methodology to perforation-shot data from a shale-gas field in the continental United States.

**THEORY**

**Statement of the problem**

Consider three plane body waves that have the same unit wavefront normal $n$ (such waves are termed isonormal) and propagate in a homogeneous anisotropic medium specified by the density-normalized stiffness tensor $c$. The phase velocities $V_Q$ ($Q = 1, 2, 3$) and the polarization vectors $U_Q$ of these waves can be found from the Christoffel equation (e.g., Auld, 1973; Červený, 2001)

$$(c_{ijkl} p_i p_k - V_Q^2 \delta_{ij}) U_{ij} = 0, \quad (i, j, k = 1, 2, 3),$$

where $\delta_{ij}$ is the $3 \times 3$ identity matrix (the so-called Kronecker delta), the subscript $Q = 1, 2, 3$ or $Q = P, S_1, S_2$ is used to denote the wave type (P-, fast shear, or slow shear), and the waves are sorted in accordance with inequalities $V_1 \geq V_2 \geq V_3$ for the phase velocities. Hereinafter, we assume summation from 1 to 3 with respect to all repeating lowercase roman indexes and no summation with respect to uppercase ones.

The group-velocity vectors $g_Q$ of plane waves are given by (e.g., Auld, 1973; Červený, 2001)

$$g_Q = c_{ijkl} U_{ij} Q_{kl} U_{Q}, \quad (j, Q = 1, 2, 3),$$

where

$$p_Q = n / V_Q$$

are the slowness vectors. The (scalar) group velocities are defined as

$$g_Q = \| g_Q \| = \sqrt{g_Q \cdot g_Q}.$$  (4)

Suppose we compute $g_Q(n)$ from equations 1–4 for a set of the wavefront normals $n \in \Omega$ (Figure 1). We would like to know which components of tensor $c$ are constrained by velocities $V_Q$ or $g_Q$, especially when the directions of vectors $n$ compose a narrow fan $\Omega$.

**Examples**

A solution to the problem posed above appears to depend on the orientation of fan $\Omega$ with respect to the symmetry elements of a given anisotropic solid. Let us illustrate this statement with two simple examples, for which we take an orthorhombic medium that has Tsvankin’s (1997) coefficients $V_{P0} = 3 \text{ km/s}$, $V_{S0} = 1.3 \text{ km/s}$, $\epsilon^{(1)} = 0.4$, $\epsilon^{(2)} = 0.2$, $\delta^{(1)} = 0.1$, $\delta^{(2)} = 0.3$, $\delta^{(3)} = -0.2$, $\gamma^{(1)} = 0.15$, and $\gamma^{(2)} = -0.2$ or the density-normalized stiffness matrix (in $\text{km}^2 / \text{sec}^2$)

$$c = \begin{pmatrix}
12.600 & 5.272 & 7.949 & 0 & 0 & 0 \\
5.272 & 16.200 & 2.511 & 0 & 0 & 0 \\
7.949 & 2.511 & 9.000 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.662 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.690 & 0 \\
0 & 0 & 0 & 0 & 0 & 2.197
\end{pmatrix}.$$  (5)

Matrix 5 represents stiffness tensor $c$ in Voigt notation, which is defined by substitutions $11 \rightarrow 1$, $22 \rightarrow 2$, $33 \rightarrow 3$, $23 \rightarrow 4$, $13 \rightarrow 5$, and $12 \rightarrow 6$ for the pairs of indexes $(ij)$ and $(kl)$ of tensor $c_{ijkl}$.

In our first example, we choose a set of the wavefront normals

$$n = [\sin \theta_1 \cos \theta_2, \sin \theta_1 \sin \theta_2, \cos \theta_1]$$  (6)

whose polar angles $\theta_1$ and azimuths $\theta_2$ belong to the fan

$$\Omega = [89^\circ \leq \theta_1 \leq 91^\circ, -1^\circ \leq \theta_2 \leq +1^\circ].$$  (7)

Because the directions of $n$ within $\Omega$ concentrate in a close vicinity of the axis $x_1$, either velocities $V_Q$ or $g_Q$ constrain three elements $c_{11}$, $c_{55}$, and $c_{66}$ of the stiffness matrix 5. This follows from equations 1–4, which result in equalities $V_1 = g_1 = \sqrt{c_{11}}$, $V_2 = g_2 = \sqrt{c_{55}}$, and $V_3 = g_3 = \sqrt{c_{66}}$ for $n = [1, 0, 0]$. If velocities of one wave mode (for instance, slow S-wave) are unavailable, the corresponding stiffness coefficient ($c_{55}$, in this particular example) cannot be determined. Whether or not it is possible to estimate

**Figure 1. Fan $\Omega$ of wave-propagation directions in anisotropic medium.**
other stiffness components from the velocities measured for directions \( \mathbf{n} \) in fan 7 requires further investigation; this issue is addressed below.

In our second example, we change fan 7 to

\[
\Omega = [44^\circ \leq \theta_1 \leq 46^\circ, 44^\circ \leq \theta_2 \leq 46^\circ].
\]

(8)

If we now substitute the wavefront normals \( \mathbf{n} \) (equation 6) whose directions are given by angles 8 into the Christoffel equation 1, all nonzero stiffness elements in matrix 5 also appear in the Christoffel matrix \( \Gamma_{ij} \equiv c_{ij} \mathbf{n}_i \mathbf{n}_j \). Such a result suggests that the phase velocities might depend on all nine orthorhombic \( c_{ij} \)'s, rather than on their small subset, which was the case in the previous example. Switching from the stiffnesses to Tsvankin’s (1997) notation leads to a similar conclusion. Indeed, the weak anisotropy approximation of the P-wave phase velocity \( V_p \) given by equations 1.107–1.109 in Tsvankin (2001) contains five anisotropy coefficients: \( e^{(1)}, e^{(2)}, \delta^{(1)}, \delta^{(2)}, \) and \( \delta^{(3)} \) multiplied by comparable trigonometric factors, which range from 1/16 to 3/16 for \( \theta_1 = \theta_2 = 45^\circ \). Hence, all five anisotropy coefficients influence \( V_p \) in a significant manner. Their estimation from the P-wave velocities in the examined fan \( \Omega \) (equation 8), however, is likely to be ambiguous.

**Singular value decomposition**

The qualitative assessment of the uniqueness of the inversion made in the previous section can be quantified by computing the Frechét derivatives \( \mathbf{F}_V \equiv \partial V_0 / \partial c_{ij} \) and \( \mathbf{F}_g \equiv \partial g_Q / \partial c_{ij} \) of the velocities with respect to the stiffness components. Matrices, in which derivatives with respect to each \( c_{ij} \) form the column-vectors, are derived in Appendix A. Applying the SVD to either matrix \( \mathbf{F}_V \) or \( \mathbf{F}_g \), we represent it as the product of three matrices: a column-orthogonal matrix \( \mathbf{u} \), a diagonal matrix \( \mathbf{s} \), whose positive or zero elements known as the singular values are conventionally arranged in the descending order, and the transpose of an orthogonal eigenvector matrix \( \mathbf{w} \) (e.g., Press et al., 2003):

\[
\mathbf{F}_A = \mathbf{u}_A \mathbf{s}_A \mathbf{w}_A^T,
\]

where the subscript \( A = V \) or \( A = g \) denotes the phase or group velocity, respectively.

To illustrate the usefulness of this approach, we apply it to the second example from the previous section. Figure 2 shows the singular values and eigenvectors of matrices \( \mathbf{F}_V \) and \( \mathbf{F}_g \) computed for the P-, S1-, and S2-waves propagating along the wavefront normal directions that belong to fan 8. An intuitive understanding that our narrow-angle data constrain just the P-, S1-, and S2-waves velocities in the vicinity of \( \theta_1 = \theta_2 = 45^\circ \) is corroborated by the presence of a pronounced drop in the singular values after the three greatest ones, which presumably correspond to those velocities. A difference between the third and the fourth singular values (approximately by a factor of 30) suggests that perhaps only three stiffness combinations can be recovered from noise-contaminated velocities. The eigenvector matrices (gray squares), however, indicate that all nine orthorhombic stiffness coefficients contribute to these combinations, leading to an obvious ambiguity in the inversion for \( c_{ij} \); three equations cannot be uniquely solved for nine unknowns.

As expected, Figures 2a and 2b look very similar because the Frechét-derivative matrices \( \mathbf{F}_V \) and \( \mathbf{F}_g \) coincide in weakly anisotropic media. Their identity for weak anisotropy follows from the well-known equality \( V_0(\mathbf{n}) = g_0(\mathbf{r}) \), in which the group velocity is evaluated along the ray direction \( \mathbf{r} \equiv \mathbf{g}_0 / g_0 \). Although the equality \( \mathbf{F}_V = \mathbf{F}_g \) is satisfied only approximately in moderately and strongly anisotropic media, it allows us to restrict our further analysis to either matrix \( \mathbf{F}_V \) or \( \mathbf{F}_g \). In practice, the choice of inverting either phase or group velocities would depend on a particular data processing goal. For example, a local slowness (reciprocal to the phase velocity) component along a borehole can be always computed from arrival times measured on a common-shot gather (e.g., Zheng and Pšenčík, 2002; Grechka and Mateeva, 2007). On the other hand, the group velocities, needed to model the traveltimes, are useful for estimating the effective medium properties, such as those discussed in our field-data example below.

Performing the SVD for the phase velocities in the wavefront normal fan 7 results in the singular values and eigenvectors displayed in Figure 3. This time, eigenvector matrix \( \mathbf{w}_V \) is quite sparse and the elements of \( \mathbf{w}_V \) corresponding to \( c_{11}, c_{45}, \) and \( c_{66} \) are the only significant entries in the first three columns that contain the combinations of stiffness coefficients constrained by the three greatest singular values. Thus, these coefficients can be unambiguously estimated from the measured velocities. Although we foresaw such an outcome in the previous section, the SVD provides additional information. The high ratio of the third and the fourth singular values \( s_{V,33}/s_{V,44} \sim 10^3 \) in Figure 3 suggests that nothing except for \( c_{11}, c_{45}, \) and \( c_{66} \) can be estimated from the velocities contaminated with any realistic noise.

Comparison of the SVDs in Figures 2 and 3 might lead one to conclude that the ability to infer stiffnesses in anisotropic media from the velocity measurements primarily depends on the direction of wave propagation. In the next section, we show that this is not the case.
case and certain stiffness coefficients can be always obtained in a unique and robust fashion in a specially rotated coordinate system.

**Solution**

Here we discuss a coordinate transform designed to make many elements of matrix \( \mathbf{w} \) close to zero for an arbitrary orientation of narrow fan \( \Omega \) of the wave-propagation directions. We choose the matrix \( \mathbf{U}^T \) of our transform to be the transpose of the orthogonal polarization vectors \( \mathbf{U}_Q \) of three isominal waves propagating along an arbitrarily selected wavefront normal \( \mathbf{n} \) within \( \Omega \). Applying the Bond transformation (e.g., Auld, 1973) with matrix \( \mathbf{U}^T \) to stiffness tensor \( \mathbf{c} \) produces tensor \( \mathbf{c}' \) that generally has no zero elements (Helbig, 1994) and appears as the one describing a triclinic solid even when the original anisotropic medium has a higher symmetry. Importantly, the polarization vectors \( \mathbf{U}' \) after the transform,

\[
\mathbf{U}' = \mathbf{U}^T \mathbf{U} = \mathbf{U}_i \mathbf{U}_j = \delta_{ij}, \quad (i, j = 1, 2, 3) \tag{10}
\]

are oriented along the new coordinate axes \( \mathbf{x}'_i \), as is schematically shown in Figure 4.

**Phase velocity**

The simplification expressed by equation 10 is critical for gaining an analytic insight into which stiffness coefficients control the phase and group velocities of plane waves propagating in direction

\[
\mathbf{n}' = \mathbf{U}^T \mathbf{n}. \tag{11}
\]

Indeed, according to the Christoffel equation 1, the squared phase velocities are

\[
V_{Q}^2 \equiv V_{Q}'^2 = c_{Q\delta\epsilon\theta}^{n'} n'_i n'_j, \quad (Q = 1, 2, 3), \tag{12}
\]

or, in expanded form,

\[
V_{1}'^2 \equiv V_{1}'^2 = c_{11}^{n'} (n'_1)^2 + \left[ 2c_{16}^{n'} n'_1 n'_2 + 2c_{15}^{n'} n'_3 n'_1 \right], \tag{13}
\]

\[
\quad + \left[ c_{66}^{n'} (n'_2)^2 + 2c_{65}^{n'} n'_2 n'_3 + c_{55}^{n'} (n'_3)^2 \right].
\]

Equations 13–15 are exact for plane waves. To reveal their significance, we notice that in most anisotropic materials the P-wave polarization vector deviates from the wavefront normal by a few degrees; that is, \( \mathbf{n} \approx \mathbf{U}_p \) (e.g., Tsvankin, 2001). Hence, \( \mathbf{n}' \approx \mathbf{x}'_1 = [1, 0, 0]' \) and the terms in equations 13–15 can be placed into groups (i), (ii), and (iii) in accordance with their magnitudes expressed by inequalities

\[
|n'_1|^2 \gg |n'_1 n'_2| \gg |n'_2 n'_3|, \quad (j, k = 2, 3). \tag{16}
\]

Although the inequalities 16 might break down for extremely strong anisotropy (Helbig and Schoenberg, 1987), they are expected to be valid for typical anisotropic rocks encountered in the subsurface.

Combining equations 13–15 with inequalities 16, we conclude that stiffness coefficients \( c_{11}, c_{66}, \) and \( c_{55} \) are best constrained by the P-, S1-, and S2-wave velocities in narrow-angle geometries. These coefficients comprise group (i) in equations 13–15. This

\[
V_{S_1}^2 \equiv V_{2}'^2 = c_{66}^{n'} (n'_1)^2 + \left[ 2c_{16}^{n'} n'_1 n'_2 + 2c_{15}^{n'} n'_3 n'_1 \right], \tag{14}
\]

\[
\quad + \left[ c_{22}^{n'} (n'_2)^2 + 2c_{23}^{n'} n'_2 n'_3 + c_{33}^{n'} (n'_3)^2 \right].
\]

\[
V_{S_2}^2 \equiv V_{3}'^2 = c_{55}^{n'} (n'_1)^2 + \left[ 2c_{45}^{n'} n'_1 n'_2 + 2c_{35}^{n'} n'_3 n'_1 \right], \tag{15}
\]

\[
\quad + \left[ c_{44}^{n'} (n'_2)^2 + 2c_{46}^{n'} n'_2 n'_3 + c_{66}^{n'} (n'_3)^2 \right].
\]
finding is hardly surprising in light of our discussion in the previous sections. Stiffnesses $c_{11}$, $c'_{66}$, and $c_{55}$ are followed by six $c'_{ij}$s grouped in (ii) in 13–15. All stiffnesses in groups (iii) in equations 13–15, except for $c_{66}$ and $c_{10}$ that already appear in (i), are not as tightly constrained. The fourth group consists of six coefficients $c'_{12}$, $c'_{13}$, $c'_{14}$, $c'_{23}$, $c'_{35}$, and $c'_{46}$, which are not present in equations 13–15. For this reason, their estimation in the examined geometries should be regarded as questionable.

The last statement might be better understood if we notice that $c'_{12}$, $c'_{13}$, and $c'_{23}$ are related to Tsvankin’s $\delta$-coefficients that control the P-wave normal-moveout (NMO) velocities in orthorhombic media (Tsvankin, 1997; Grechka and Tsvankin, 1999). Because the NMO velocities cannot be accurately estimated in our narrow fans of wave-propagation directions, one might expect large errors in the $\delta$-coefficients and, consequently, in the stiffnesses $c'_{12}$, $c'_{13}$, and $c'_{23}$.

A qualitative discussion above is, in fact, directly supported by the SVD. Figure 5 shows the singular values and the eigenvector matrix for the orthorhombic model given by equation 5 but performed with respect to the Bond-transformed stiffness coefficients $c'$ with matrix

$$
U^T = \begin{pmatrix}
0.529 & 0.565 & 0.634 \\
0.350 & -0.825 & 0.444 \\
0.774 & -0.013 & -0.634
\end{pmatrix},
$$

which is obtained from the polarization vectors of waves that have the wavefront normal $\mathbf{n} = [1, 1, \sqrt{2}]/2$. The rows of matrix $\mathbf{w}_V$ in Figure 5 are sorted to make the eigenvector matrix as diagonally dominant as possible. Such a sorting is helpful because a diagonal eigenvector matrix would be ideal for the inversion. Although matrix $\mathbf{w}_V$ in Figure 5 is not diagonal, it has a large number of zero and nearly zero elements (compare with Figure 2), which imply that information about each stiffness coefficient $c'_{ij}$ is concentrated in

![Figure 5](image)

Figure 5. Logarithms of normalized singular values (black dots) and absolute values of the elements of eigenvector matrix $\mathbf{w}_V$ (gray squares) in a coordinate frame rotated with matrix 17. The rows of $\mathbf{w}_V$ are sorted to make it as diagonally dominant as possible. The fan $\Omega$ of the wavefront normal directions is given by inequalities 8.

The respective singular value and relatively little trade-off between different $c'_{ij}$s is expected. The lower the position of a given $c'_{ij}$ on the right-hand side of Figure 5, the less tightly it is constrained by the data.

Let us note a staircase behavior of the singular values (dots in Figure 5) predicted by equations 13–15. Indeed, the three greatest singular values correspond to $c'_{11}$, $c'_{66}$, and $c'_{10}$, in accordance with equations 13–15. The next six parameter combinations in Figure 5 are dominated by the stiffnesses $c'_{13}$, $c'_{14}$, $c'_{15}$, $c'_{26}$, $c'_{35}$, and $c'_{46}$ that appear in groups (ii) in equations 13–15. Their order depends on the relative magnitudes of the wavefront normal components $n'_1$ and $n'_2$ and, thus, unimportant for our discussion. Although the exact number of $c'_{ij}$s that can be estimated from the velocities depends on the noise level, the remaining 12 stiffnesses are unlikely to be accurately recovered because the normalized singular values corresponding to them are smaller than $10^{-9}$.

Because one’s ability of inferring certain stiffness components significantly depends on the data aperture, it is important to investigate whether our previous assessment holds when solid angles of the wavefront normal directions cover more than just a few square degrees. To address this issue, we open up the data aperture by a factor of approximately 100 (by increasing the ranges of both angles $\theta_1$ and $\theta_2$ 10 times) and repeat the previous numerical experiment. The result in Figure 6 indicates that equations 13–15 keep predicting the correct order of sensitivities of the phase velocities to $c'_{ij}$s, even though the staircase behavior of the singular values (black dots) disappears and their overall range gets smaller. Both those features are expected because of the widening of the data aperture.

**Group velocity**

The use of polarization vectors expressed by equation 10 is especially beneficial for the group velocities. Transforming equation 2 to the rotated coordinate frame yields

![Figure 6](image)

Figure 6. The same as Figure 5 but for a fan of polar angles $\theta_1$ and azimuths $\theta_2$ of the wavefront normals $\Omega = [35^\circ \leq \theta_1 \leq 55^\circ$, $35^\circ \leq \theta_2 \leq 55^\circ]$. 
where \( p'_Q = n'_i / V'_Q \) are the rotated slowness vectors. Introducing the \( 3 \times 3 \) symmetric positive-definite matrices

\[
V'_Q = V'_Q = c'_j Q,  \quad (j, Q = 1, 2, 3),
\]

which are related to the so-called Voigt stiffness tensor \( V' \) (Cowin, 1989; Helbig, 1994) as \( V' = \sum_{i=1}^{3} V'_Q \), we rewrite equation 18 in the form of a product of matrix \( V'_Q \) and vector \( p'_Q \)

\[
g'_Q = V'_Q p'_Q,  \quad (Q = 1, 2, 3).
\]

One can easily verify that the inverse relationship \( p'_Q = (V'_Q)^{-1} g'_Q \) is also valid.

To the best of our knowledge, equation 20 is the simplest exact equation for the group velocities of plane waves propagating in triclinic media. It turns out to be the most computationally efficient too. Table 1 compares the cost of applying equation 20 with those of other known equations. While the group velocities calculated with various equations are numerically identical, the computational expenses differ substantially. We see that equation 20 performs particularly well when \( g_0 \) of all three isonormal waves have to be computed. This happens because transformation \( c \rightarrow c' \) needs to be applied only once. Even for a single wave mode, however, equation 20 is superior to all its competitors. Although the issue of the efficient calculation of the group velocities might seem insignificant, we note that this computation comprises the innermost core of any anisotropic ray-tracing code and, therefore, directly influences the overall performance of all subsequent traveltime calculations.

Like equation 12, which can be written in terms of matrices \( V'_Q \) as

\[
V^2_Q = n'^T V'_Q n',  \quad (Q = 1, 2, 3),
\]
equation 20 leads to representation

\[
g^2_Q = p'^T Q (V'_Q)^2 p'_Q,  \quad (Q = 1, 2, 3)
\]

and results in analytic expressions of the group velocities similar to those given by equations 13–15. We do not present these expressions here because they are rather lengthy and entail the same conclusions pertaining to the sensitivities of \( g_Q \) to \( c_{ij} \) as those that we have already discussed for the sensitivities of \( V_Q \).

Figure 7 exemplifies this statement. We performed the SVD for the same fan of the wavefront normal directions as that in Figure 5 but used only two group velocities, \( g_P \) and \( g_S \), to construct the Frechét-derivative matrix \( F_g \). The absence of the slow shear-wave data is evident in Figure 7. The stiffness coefficients contributing to the singular values greater than \( 10^{-3} \) are exactly those that appear in groups (i) and (ii) in equations 13 and 14. They are separated from the rest of the stiffnesses in Figure 7 by a large drop in the singular values (black dots), which likely divides the \( c_{ij} \) into those that can and cannot be realistically recovered from the velocities.

### INVERSION OF PHASE VELOCITIES

Having shown that inversion of the phase, \( V_Q \), and group, \( g_Q \), velocities with respect to the stiffness coefficients needs to be performed in a local coordinate frame, which relates to a given narrow-angle data geometry, we proceed with estimation of these stiffnesses. Because \( V_Q \) and \( g_Q \) depend on the stiffness components in a similar manner (compare Figure 2a and 2b), we discuss inversion of the phase velocities in this section and turn our attention to the group velocities in the next section, where we examine our field data.

Here we compute the phase velocities in a narrow fan of the wavefront normal directions (described in the caption to Figure 8), add a 0.5% Gaussian noise to the velocity values and invert these noise-contaminated velocities for the stiffness coefficients. An important consequence of the fact that our velocities are inherently incapable of constraining all elements of stiffness tensor \( c' \) is that we have to devise a strategy for infilling the \( c_{ij} \) that cannot be inferred from the available data. While several choices for such an infill strategy exist (see the Discussion section), we opt for perhaps the simplest one, which is to make \( c' \) as close to isotropy as possible at each stage of the inversion. Appendix B describes the operational details of our approach.

![Figure 7](image-url)
Our inversion procedure is organized as follows. We begin with the SVD of the Frechét-derivative matrix constructed for a purely isotropic model. The velocities in this model are unimportant because the goal of this SVD is just to reveal a sequence of $c_{ij}$s that we then iteratively target in the least-squares, nonlinear, gradient-based inversion. In this section, we use the same orthorhombic model as before (equation 5) so that the column of stiffness elements on the right-hand side of Figure 8a represents the order in which the $c_{ij}$s are estimated in our first test. Next, we select the number of stiffnesses to be obtained at the initial nonlinear iteration of the inversion. Because the noise-contaminated velocities of all three wave modes are used and the S-waves exhibit splitting, it makes sense to honor it and invert the velocities for three best constrained stiffness coefficients $c_{66}^0$, $c_{55}^0$, and $c_{11}^0$. The inversion results in 1.4% root-mean-square (RMS) error in the velocities (the leftmost dot in Figure 8b).

We use the estimated $c_{66}^0$, $c_{55}^0$, $c_{11}^0$ as an initial guess for the second iteration of the nonlinear inversion, in which we add another unknown $c_{45}^0$ (see the right-hand side column in Figure 8a) and update all four stiffnesses. This inversion step yields a slightly smaller RMS error (Figure 8b). We keep adding one stiffness coefficient at a time, updating all relevant $c_{ij}$s, and observing the reduction in the RMS error. At some point, specifically, after adding the ninth stiffness coefficient, the RMS error reaches the level comparable to that of noise (0.5%) and does not significantly decrease any further (Figure 8b). Our interpretation of such a behavior is that the top nine $c_{ij}$s in Figure 8a are sufficient to explain the data; when we relax other stiffnesses, we start fitting the noise. Therefore, it is important to find out whether the nine stiffness coefficients result in a satisfactory solution in the model space.

To do so, we calculate the Bond transformation of the exact tensor $c$ (equation 5) with matrix $U^T$ (equation 17)

$$
c' = \begin{pmatrix}
10.869 & 5.216 & 6.596 & 1.136 & 0.492 & -0.658 \\
5.216 & 12.837 & 5.264 & 0.182 & 1.378 & -1.909 \\
6.596 & 5.264 & 11.409 & 0.451 & 0.698 & 1.634 \\
1.136 & 0.182 & 0.451 & 2.353 & -0.630 & -0.374 \\
0.492 & 1.378 & 0.698 & -0.630 & 1.869 & 0.028 \\
-0.658 & -1.909 & 1.634 & -0.374 & 0.028 & 4.664
\end{pmatrix}.
$$ (23)

This tensor is to be compared with the inverted stiffness tensor (in km$^2$/s$^2$)

$$
c'^{\text{inv}} = \begin{pmatrix}
10.885 & 4.335 & 4.335 & 0 & 0.427 & -0.695 \\
4.335 & 10.885 & 4.335 & 0 & 0 & -1.847 \\
4.335 & 4.335 & 10.885 & 0 & 0.648 & 0 \\
0 & 0 & 0 & 3.275 & -0.761 & -0.240 \\
0.427 & 0 & 0.648 & -0.761 & 1.864 & 0 \\
-0.695 & -1.847 & 0 & -0.240 & 0 & 4.686
\end{pmatrix}.
$$ (24)

The maximum error in the estimated stiffness components, which are typeset in bold in matrix 24 to facilitate the comparison, is 0.13 km$^2$/s$^2$ for $c_{45}$ and $c_{66}$. This result is certainly satisfactory. Note that rotating tensor $c^{\text{inv}}$ with matrix $(U^T)^T = U$ back to the original coordinate frame would lead to greater errors. They are caused by the optimal closeness of $c^{\text{inv}}$ to isotropy and the absence of this property in tensor $c'$ (compare the upper-left $3 \times 3$ blocks in matrices 23 and 24).

In our second example, we use the same fan of directions but remove the slow S-wave phase velocities from the data. Although Figure 9a indicates a different sequence of $c_{ij}$s than Figure 9b, our inversion strategy remains the same. This time, the RMS errors reach a plateau when the data are fitted with a model that contains eight stiffness coefficients. A comparison of the model stiffnesses (equation 23 with the estimated ones (in km$^2$/s$^2$),

$$
c^{\text{inv}} = \begin{pmatrix}
10.930 & 4.634 & 4.634 & 0 & 0.310 & -0.780 \\
4.634 & 10.930 & 4.634 & 0 & 0 & -1.866 \\
4.634 & 4.634 & 10.930 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.148 & 0 & -0.327 \\
0.310 & 0 & 0 & 0 & 1.669 & 0.415 \\
-0.780 & -1.866 & 0 & -0.327 & 0.415 & 4.626
\end{pmatrix}.
$$ (25)

reveals overall greater errors with the largest being about 0.4 km$^2$/s$^2$ for $c_{56}$.

Even though a higher uncertainty can be expected in the second test because we use less data to infer the same model parameters as those in the previous example, the obtained results are remarkable in their ability to establish the presence of seismic anisotropy in our

![Figure 8.](image-url)
narrow-angle geometry without relying on the shear-wave splitting. We believe that this happens because of the data sensitivity to the difference in slopes of the wavefront segments corresponding to the P- and S1-waves. Once a medium homogeneity is assumed, anisotropy provides the only plausible explanation for a misalignment of portions of the wavefronts (or the phase-velocity surfaces) corresponding to different wave modes.

In the next section, we show that the same logic would allow us to establish and estimate seismic anisotropy from our field data despite the velocity heterogeneity of the subsurface.

FIELD-DATA EXAMPLE

Here we apply the developed methodology to build an anisotropic velocity model from perforation-shot data recorded at one stage of hydraulic well treatment in a shale-gas play in the continental United States. The shot-receiver geometry is displayed in Figure 10 in a local coordinate frame whose origin is placed at the shallowest geophone. Figure 10d shows the ray directions plotted on a unit sphere under the coordinate frame whose origin is placed at the shallowest geophone.

The recorded seismic traces are presented in Figure 11. We rotate the original three-component (3C) data to project particle motions of the direct P-waves (their times are marked with the red ticks) on the first component (Figure 11a, 11c, and 11e). Next, we rotate the remaining two components of data around this component to enhance a wave that arrives next on the second component (Figure 11b, 11d, and 11f). Because, by definition, the second component is orthogonal to the first component, the wave whose travel-times are shown with the blue ticks in Figure 11 is the direct shear wave. After those two rotations, the third data component contains little coherent energy (not shown), making it impossible for us to determine whether the shear arrival is the fast S1- or slow S2-wave. We interpret it as the S1-wave and acknowledge the implications of our interpretation for the stiffness coefficients that we intend to estimate.

In the following, we will be constructing a velocity model using traveltimes \( t_Q \) of the P- and (presumed) S1-waves. These traveltimes were picked from the data (red and blue ticks in Figure 11) with a precision of one time sample \( \Delta t = 0.375 \text{ ms} \).

Isotropic velocity models

We begin building a seismic velocity model under the simplest assumption that the subsurface is isotropic and homogeneous. Therefore, our model has just two parameters: the P- and S-wave velocities, \( V_{P,iso} \) and \( V_{S,iso} \), respectively. To those we have to add the origin times \( t_i \) (\( i = 1,2,3 \)) of three perforations shots because the shots were not timed. Fitting the traveltimes picks in Figure 11 with these five parameters results in the RMS traveltimes misfit \( \Delta t_{RMS} = 0.810 \text{ ms} \). Even though it is significantly greater than the picking precision \( \Delta t = 0.375 \text{ ms} \), the issue of whether or not the obtained homogeneous isotropic model is reasonable can be resolved based on the values of the best-fit isotropic velocities \( V_{P,iso} = 2.870 \text{ km/s} \) and \( V_{S,iso} = 1.993 \text{ km/s} \). They yield the Poisson’s ratio

\[
\nu = \frac{V_{P,iso}^2 - 2V_{S,iso}^2}{2(V_{P,iso}^2 - V_{S,iso}^2)} = 0.034,
\]

which is too low for any realistic subsurface rocks. Clearly, this model should be deemed unacceptable on the grounds of rock physics.

To understand what might have caused such a low Poisson’s ratio, let us observe that the moveouts of direct arrivals in Figure 11 are close to straight lines (in fact, they are slightly curved, but the curvatures are too small to see them clearly). Therefore, the moveout slopes comprise the key data feature that has to be explained by our homogeneous isotropic model. Because the well containing the geophones is nearly vertical (Figure 10a, 10b, and 10c), the slopes in Figure 11 are approximately equal to the vertical slowness components

\[
p_{3,P} = \frac{\cos \theta_{1,P}}{V_{P,iso}} \tag{27}
\]

and

\[
p_{3,S} = \frac{\cos \theta_{1,S}}{V_{S,iso}} \tag{28}
\]

It is important to note that two polar angles in equations 27 and 28 are equal

\[
\theta_{1,P} = \theta_{1,S} \tag{29}
\]

Figure 9. The same as Figure 8 but when phase velocities of the P- and S1-waves only comprise the data for inversion.
for each shot-receiver pair because these angles are calculated in a homogeneous isotropic model. Hence, the measured slopes $p_{3,P}$ and $p_{3,S}$ relate to the velocity ratio as

$$\frac{p_{3,S}}{p_{3,P}} = \frac{V_{P,iso}}{V_{S,iso}}$$

and can be used to estimate the effective Poisson’s ratio.

Table 2 lists the ratios of the moveout slopes picked from perforation-shot gathers in Figure 11 and the Poisson’s ratios calculated using equations 26 and 30. Clearly, the quoted value $\nu = 0.034$ is close to an average Poisson’s ratio and, thus, is required to fit the data by a homogeneous isotropic model.

To make further progress and obtain a more satisfactory velocity model, we have the following three options. First, we might keep the assumption of isotropy and build a heterogeneous model. The heterogeneity is supposed to bend rays in such a way as to break down equations 29 and 30 while keeping the correct moveout slopes and yielding physically reasonable Poisson’s ratios. Second, we might maintain the homogeneity and evaluate the influence of anisotropy on the obtained traveltimes $t_0$. And, third, we might combine the two options above and allow our model to be both

Figure 10. (a, b) Depth and (c) plan views of the locations of three perforation shots (stars) and eleven geophones (circles), and (d) directions of straight rays plotted on a unit sphere.
Figure 11. Records of perforation shots number 1 (a, b), 2 (c, d), and 3 (e, f). The 3C traces are rotated to enhance the direct P-waves on one component (a, c, e) and the direct (presumably fast) S-waves – on another (b, d, f). Traveltime picks $t_Q$ of the P- and $S_1$-waves are shown with red and blue ticks, respectively.
heterogeneous and anisotropic. Let us investigate these options starting with the first.

Conventional velocity models for microseismic data processing consist of isotropic horizontal layers (e.g., Pei et al., 2009, and references therein). Rather than discuss our model of this kind, we analyze those provided to us by microseismic service companies. The first model, constructed by a service company that acquired the data, is shown in Figure 12 (blue) along with the sonic logs (black), which were used as soft constraints to seismic velocities. Let us note that, to honor traveltimes of the perforation shots (Figure 11), the P-wave velocity has been slowed down compared to the sonic by about 700 m/s in the depth range covered by the geophones, that is, exactly in the interval in which the velocities are constrained by the data.

If we take the model velocities (blue in Figure 12) and calculate the Poisson’s ratios using equation 26, we obtain the blue staircase line in Figure 13. It is to be compared with the Poisson’s ratios (black) derived from the sonic logs in Figure 12. Clearly, the blue staircase Poisson’s ratio exhibits the same problem that we experienced with our homogeneous isotropic velocity model: the implausible zero values in the depth range covered by the geophones.

Shell E&P Company has contracted another microseismic service provider to reprocess the data. Its velocity model yields the Poisson’s ratios shown in Figure 13 with a dashed green line. We observe not only a “confirmation” of \( \nu = 0 \) in a portion of the interval containing the geophones but also a layer at about 300 m depth characterized by the negative Poisson’s ratio \( \nu = -0.05 \). Evidently, the adopted assumption of isotropy fails to produce physically reasonable velocity models.

### Anisotropic models

Let us explore the second option and invert the perforation-shot times \( t_Q \) for a suite of homogeneous anisotropic models. In general, we proceed the same way as we did in the section on inversion of the phase velocities but with two obvious differences: we include the origin times \( t_i \) \( (i = 1, 2, 3) \) of the perforation shots as additional unknowns and use the Frechét derivatives relevant to the travelt ime inversion

\[
\mathbf{F} \equiv \frac{\partial t_Q}{\partial \mathbf{c}_I} = -t_Q \frac{\partial g_0}{\partial \mathbf{c}_I}, \quad (Q = 1, 2),
\]

where derivatives of the group velocities \( \partial g_Q / \partial \mathbf{c}_I \) are given in Appendix A. The stiffnesses \( c_{ij} \) relate to those in the global coordinates in Figure 10 via the matrix

<table>
<thead>
<tr>
<th>Perforation-shot number</th>
<th>Ratio of slopes ( p_{3,S}/p_{3,P} )</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.480</td>
<td>0.080</td>
</tr>
<tr>
<td>2</td>
<td>1.446</td>
<td>0.041</td>
</tr>
<tr>
<td>3</td>
<td>1.370</td>
<td>-0.070</td>
</tr>
</tbody>
</table>

Figure 12. Sonic logs (black) and a horizontally layered isotropic velocity model (blue) built by a microseismic service company. The circles and (overlapping) stars indicate the depths of the geophones and perforation shots, respectively.

Figure 13. Poisson’s ratios calculated using seismic velocities shown in blue in Figure 12 (solid blue) and provided by a different microseismic service company for the same data set (dashed green). Thin black line is the Poisson’s ratio derived from the sonic logs in Figure 12.
which was derived from the polarization (or hodogram) analysis of seismic data in Figure 11 and corresponds to the source-receiver pair exhibiting the greatest linearity of the P-wave particle motion (the linearity is defined by equation C-2 in Grechka and Mateeva, 2007).

Figure 14a displays the results of the SVD of matrix \( \mathbf{F} \), constructed for the homogeneous isotropic velocity model discussed in the previous section. (Note that full Frechét-derivative matrix for the examined inverse problem contains \( \mathbf{F} \), augmented by derivatives with respect to the origin times \( \partial_{\tau_i} / \partial \tau_i = \delta_{ij} \), where \( i, j = 1, 2, 3 \). The latter are excluded from the analysis presented in Figure 14a.) As we gradually increase the number of stiffnesses estimated by the nonlinear inversion, the traveltime misfit reduces to levels below the sampling interval \( \Delta t = 0.375 \text{ ms} \) (horizontal dashed line Figure 14b).

While there is some ambiguity about which anisotropic model might be accepted as a solution, we note that such a model cannot contain more than 15 uniquely determined parameters. The number 15 follows from a straightforward analysis of the perforation-shot data. Indeed, the traveltimes in each shot gather in Figure 11 are fully represented by six quantities: the origin time \( \tau_i \), the difference between arrival times of the shear- and P-waves at any geophone, two moveout slopes, and two moveout curvatures. Six constraints per shot times three shots yield 18 constraints; three of which have to be used to resolve the event-origin times \( \tau_i \). With these considerations in mind, we choose the model described by 14 stiffness coefficients. It has the RMS traveltime misfit \( \Delta t_{\text{RMS}} = 0.361 \text{ ms} \), which is just below the time sample, and the stiffness coefficients (in \( \text{km}^2/\text{s}^2 \))

\[
\mathbf{c}^T = \begin{pmatrix}
8.136 & 1.803 & 3.742 & 0 & -0.179 & 0.079 \\
1.803 & 9.669 & 2.389 & 0 & 1.762 & -0.025 \\
3.742 & 2.389 & 9.669 & 0 & -0.057 & 0.263 \\
0 & 0 & 0 & 4.146 & 0 & -0.129 \\
-0.179 & 1.762 & -0.057 & 0 & 3.623 & 0.067 \\
0.079 & -0.025 & 0.263 & -0.129 & 0.067 & 3.916
\end{pmatrix}
\]

There exist several ways of assessing whether the obtained stiffness tensor is plausible. First, we compare its prediction of the vertical velocities with the well logs. Although we realize that high precision of these velocities cannot be expected because they have to be calculated based on extrapolation of the inversion results by more than 30° away from the recorded aperture, we nevertheless apply matrix \( \mathbf{U} \) (its transpose is given by equation (32)) to rotate tensor 33 to the geographic coordinates. This yields (in \( \text{km}^2/\text{s}^2 \))

\[
\mathbf{c} = \begin{pmatrix}
9.350 & 1.923 & 1.960 & 1.003 & -0.017 & -0.401 \\
1.923 & 10.359 & 3.171 & 0.047 & 0.049 & 0.381 \\
1.960 & 3.171 & 9.526 & 0.173 & -0.917 & -0.499 \\
1.003 & 0.047 & 0.173 & 2.937 & -0.226 & 0.119 \\
-0.017 & 0.049 & -0.917 & -0.226 & 3.874 & -0.828 \\
-0.401 & 0.381 & -0.499 & 0.119 & -0.828 & 3.993
\end{pmatrix}
\]

and results in the P- and fast S-wave vertical velocities \( V_P = 3.169 \text{ km/s} \) and \( V_S = 2.074 \text{ km/s} \), respectively. These velocities are well within the intervals of variation of sonic logs in Figure 12. As another qualitative check, one might calculate the Poisson’s ratio corresponding to these \( V_P \) and \( V_S \). Even though using equation 26 for this purpose is not entirely appropriate because the medium is anisotropic, such a calculation gives \( \nu = 0.18 \), which is again a plausible value according to the log in Figure 13. In addition, the relative deviation \( \| \mathbf{e} - \mathbf{e}^{iso} \| / \| \mathbf{e} \| \) of tensor \( \mathbf{e} \) from its best-fit isotropic approximation \( \mathbf{e}^{iso} \) (computed in the euclidian norm B1) of about 20% suggests an overall moderate degree of seismic anisotropy, which is certainly expected in our shale-gas field.

The model given by equation 34 can be also verified by using it to find the locations of the perforation shots and compare them with those obtained from an available well-deviation survey (the crosses in Figure 15). To locate the shots, we define a rectangular grid in the vertical planes specified by the average shot azimuths (see Figure 10c) and treat each grid point \( \zeta \) as a potential shot position. We then calculate traveltimes \( t_{ij}(\zeta) + \tau_i \) (\( i = 1, 2, 3 \)) in our homogeneous triclinic model and post the RMS misfits \( \Delta t_{\text{RMS}}(\zeta) \) between these traveltimes and the times picked at Figure 11 at each \( \zeta \). Figure 15 presents the result of our computation performed.
on a square grid with the grid size $\Delta \zeta = 2 \text{ m}$. We accurately fit both the traveltime picks (the maximum of $\Delta t_{\text{RMS}}$ is 0.38 ms) and the spatial positions of the perforation shots (the maximum deviation of min $\Delta t_{\text{RMS}}$ from the corresponding shot location is 2 m), which, in retrospect, justifies our interpretation of the recorded shear arrivals as the $S_I$-waves. Clearly, comparison of the shot locations disregards possible errors in the well-deviation survey, which might not only influence the positions of crosses in Figure 15 but also bias the estimates of stiffness coefficients in matrices 33 and 34 (Bulant et al., 2007).

In summary, the obtained homogeneous anisotropic velocity model satisfactorily describes the available perforation-shot times, places the shots at their correct locations, and agrees with the well logs. Our model contains 14 stiffness coefficients, which nearly exhaust the maximum number of quantities that can be unambiguously estimated. Thus, we conclude that introducing heterogeneity is unnecessary to fit our field data.

**DISCUSSION**

The presented study had two main objectives. The first was to ensure the uniqueness of inversion of seismic anisotropy. We realized that the estimation of the elastic stiffness coefficients from seismic velocities measured in narrow-angle geometries (which are typical for downhole microseismic surveys) has to be performed in a specially rotated coordinate frame to avoid ambiguity. We suggested to select this frame by aligning its coordinate axes with the polarization vectors of three plane waves propagating along any direction within a given data aperture. This choice resulted in a significant simplification of equations describing the phase and group velocities in anisotropic media and made it possible to gain analytic insights into which stiffness coefficients $c_{ij}$ in the rotated frame are constrained by the data (see equations 13–15).

It was natural then that we targeted these $c_{ij}$s in the inversion. Our numerical tests corroborated the analytic results and demonstrated that, indeed, the accurate estimation of certain $c_{ij}$s from noise-contaminated seismic velocities is possible. Along the way, we encountered an interesting problem of infilling the $c_{ij}$s unconstrained by our data. It is quite obvious that some additional information had to be brought in to assign numerical values to these stiffness coefficients. We have chosen to draw this information from the requirement that the inverted stiffness tensor $c'$ is as close to isotropy as possible. Clearly, this is not the only option. Other options include approximating the symmetry of $c'$ with either transverse isotropy (TI) of orthorhombic symmetry that is also characterized by more independent stiffness coefficients than isotropy, they are expected to represent $c'$ better. We have not followed this path, however, primarily because it implies the availability of a priori knowledge of the medium symmetry. Although the symmetry was certainly known in our synthetic examples, this was not the case with the field data. For this reason, we decided to leave the problem of finding the best TI or orthorhombic approximation to an incompletely known stiffness tensor to a future study.

The second goal of our paper was to test the developed anisotropic parameter-estimation methodology on field data. Although, ideally, one would like to build a single, possible time-dependent, model that explains perforation-shot and microseismic data recorded in an entire survey (a more modest attempt is presented in a companion paper by Grechka et al., 2011), here we followed conventional practice, according to which velocity models for each

![Figure 15](image-url)  
*Figure 15. Sections of $\Delta t_{\text{RMS}}$ (in ms) for three perforation shots (arranged from north to south) in the vertical planes specified by the perforation-shot azimuths (Figure 10c). Calculations are performed in the triclinic model given by equation 34. The size of color bars in all panels is equal to 0.75 ms or to two time samples. The crosses mark locations of the perforation shots based on the well-deviation survey.*
stage of hydraulic well stimulation are constructed using perforation shots acquired at that stage. We managed to derive a homogeneous anisotropic velocity model that fits the times of the P- and fast S-waves picked from perforation-shot data in Figure 11 with the RMS error smaller than one time sample in the data. Importantly, our model predictions of the vertical velocities are consistent with the available well logs. In contrast, either homogeneous or layered isotropic velocity models examined in the paper exhibit unrealistically low or even negative Poisson’s ratios.

We conclude our paper by observing that anisotropic velocity-model building for microseismic data processing is in its infancy. While hundreds of authors discuss anisotropy in the context of seismic reflection data, we are aware of only a few publications (Teanby et al., 2004; Maxwell et al., 2006; Michaud et al., 2009) that describe estimation of anisotropy for microseismic monitoring. This makes us believe that a significant body of work needs to be done to develop practical approaches for measuring seismic anisotropy in microseismic geometries and using it to improve our knowledge of the properties of hydraulically treated formations.

CONCLUSIONS

Our paper contains both theoretical and practical contributions. On the theoretical side, we demonstrated how to estimate elements of the elastic stiffness tensor from seismic velocities or traveltimes measured in narrow-angle geometries. We showed that such an inversion has to be performed in rotated coordinates to ensure its uniqueness. In addition to identifying the stiffness components that can be unambiguously inverted, we derived the equation for the group velocities in generally anisotropic (triclinic) media that happened to be computationally superior to many other known equations.

Our practical contribution includes building a triclinic velocity model from field perforation-shot data. The constructed model fits the traveltime picks of the P- and fast S-waves within one time sample and resolves the issue with unrealistically low Poisson’s ratios inherent for isotropic velocity models derived from the same traveltimes.

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APPENDIX A

FRECHÉT DERIVATIVES OF SEISMIC VELOCITIES

The goal of this Appendix is to derive the Frechet derivatives $\mathcal{F}_V \equiv \partial V_Q / \partial c_{ij}$ and $\mathcal{F}_s \equiv \partial g_Q / \partial c_{ij}$ ($Q = 1, 2, 3; I, J = 1, \ldots, 6$) of the phase and group velocities $V_Q$ and $g_Q$, with respect to the stiffness components $c_{ij}$.

Derivatives of phase velocities

Because the phase velocities are calculated from the Christoffel equation $1$ in direction of the unit wavefront normal $\mathbf{n}$, it is natural to evaluate the derivatives $\partial V_Q / \partial c_{ij}$ at a fixed $\mathbf{n}$. Following the approach described in Zhou and Greenhalgh (2005), we rewrite equation 1 in the form

$$V_Q^2 U_{ijQ} = c_{ijkl} n_i n_k U_{ijQ}, \quad (i, Q = 1, 2, 3) \quad (A-1)$$

and use the fact that the polarization vectors $U_Q$ are mutually orthogonal, because they are the eigenvectors of equation $A-1$. Hence:

$$U_Q \cdot U_R = \delta_{QR}, \quad (Q, R = 1, 2, 3), \quad (A-2)$$

where $\delta_{QR}$ is the $3 \times 3$ identity matrix. Taking a dot product of equation $A-1$ and vector $U_R$ yields

$$V_Q^2 \delta_{QR} = c_{ijkl} U_{iQR} n_i n_k U_{ijQ}, \quad (Q, R = 1, 2, 3). \quad (A-3)$$

Next, we set $R = Q$ in equation $A-3$ and differentiate it with respect to components of the stiffness tensor. The result reads

$$2 V_Q^2 \frac{\partial V_Q}{\partial c_{ijkl}} = U_i^Q n_i n_k U_{ijQ} + 2 c_{ijkl} \left( \frac{\partial U_{ijQ}}{\partial c_{ijkl'}} \right) n_i n_k U_{ijQ}, \quad (i', j', k', l', Q = 1, 2, 3). \quad (A-4)$$

To prove that the second term in equation $A-4$ vanishes, we note that any derivative of $U_Q$ projects onto two orthogonal polarization vectors $U_R$, $U_K$, ($R_1 \neq R_2 \neq Q$) because $U_Q$ is a unit vector and apply equation $A-3$. Therefore,

$$\frac{\partial V_Q}{\partial c_{ijkl}} = \frac{1}{2 V_Q} U_{ijQ} n_i n_k U_{ijQ}, \quad (i, j, k, l, Q = 1, 2, 3). \quad (A-5)$$

The derivatives $\mathcal{F}_V = \partial V_Q / \partial c_{ij}$ are obtained from those in equation $A-5$ using the standard substitutions $11 \rightarrow 1, 22 \rightarrow 2, 33 \rightarrow 3, 23 \rightarrow 4, 13 \rightarrow 5$, and $12 \rightarrow 6$ for the pairs of indexes of $c_{ijkl}$ and the symmetry $c_{ij} = c_{ji}$ of a Voigt stiffness matrix. To account for this symmetry, the derivatives $\partial V_Q / \partial c_{ij}$ need to be multiplied by $(2 - \delta_{ij})$. We note that equation $A-5$ is inapplicable to the S-waves at their point singularities $n^3$, because the polarization vectors $U_Q (Q = 2, 3)$ are nonuniquely defined at $n = n^3$ (e.g., Fedorov, 1968).

Derivatives of group velocities

To derive the Frechet derivatives $\mathcal{F}_s \equiv \partial g_Q / \partial c_{ij}$, it is convenient to start with the equality (e.g., Auld, 1973)

$$\mathbf{g} \cdot \mathbf{p} = 1, \quad (A-6)$$

where $\mathbf{g}$ is the group-velocity vector, $\mathbf{p}$ is the corresponding slowness vector, and the wave-type identifier $Q = 1, 2, 3$ is omitted for brevity. Defining the unit ray vector $\mathbf{r}$ through equation

$$\mathbf{g} = |\mathbf{g}| \mathbf{r} \equiv \mathbf{g} \mathbf{r}, \quad (A-7)$$

we rewrite equation $A-6$ as

$$g(\mathbf{r} \cdot \mathbf{p}) = 1. \quad (A-8)$$
The reason for introducing vector \( \mathbf{r} \) is that the most useful derivatives \( \partial g / \partial c_{IJ} \) are obtained at a fixed ray direction, that is, \( \partial g / \partial c_{IJ} \rvert_{\mathbf{r} \text{ const}} \). Indeed, such derivatives can be directly applied to evaluate the Frechét derivatives of traveltimes (see equation A-31) because the latter correspond to stationary ray trajectories in accordance with Fermat’s principle. With this in mind, we differentiate equation A-8 with respect to \( c_{IJ} \)

\[
\frac{\partial g}{\partial c_{IJ}} (\mathbf{r} \cdot \mathbf{p}) + g \left( \frac{\partial \mathbf{r}}{\partial c_{IJ}} \cdot \mathbf{p} \right) + g \left( \mathbf{r} \cdot \frac{\partial \mathbf{p}}{\partial c_{IJ}} \right) = 0, \quad (I,J = 1, \ldots, 6).
\]

(A-9)

Taking into account that the second term in equation A-9 vanishes because \( \mathbf{r} = \text{const} \) and using equation A-8, we obtain

\[
\frac{\partial g}{\partial c_{IJ}} = -g^2 \left( \mathbf{r} \cdot \frac{\partial \mathbf{p}}{\partial c_{IJ}} \right), \quad (I,J = 1, \ldots, 6). \quad \text{(A-10)}
\]

To proceed further, we need to derive the Frechét derivatives of the slownesses \( \partial \mathbf{p} / \partial c_{IJ} \). Those are found by differentiating the condition

\[
F(\mathbf{p}, \mathbf{c}) \equiv \text{det}(c_{ijkl}p_{i}p_{k} - \delta_{ik}) = 0 \quad \text{(A-11)}
\]

for the existence of nonzero eigenvectors of the Christoffel equation. Applying the chain rule to equation A-11 yields

\[
\frac{\partial F}{\partial c_{IJ}} = -\frac{\partial F}{\partial p_{i}} \frac{\partial p_{i}}{\partial c_{IJ}} = -\left( \nabla_{\mathbf{p}} F \cdot \frac{\partial \mathbf{p}}{\partial c_{IJ}} \right), \quad (I,J = 1, \ldots, 6),
\]

(A-12)

where \( \nabla_{\mathbf{p}} F \) is the gradient of \( F \) in the slowness space. Being the gradient, \( \nabla_{\mathbf{p}} F \) is orthogonal to the slowness surface given by equation A-11 and, hence, parallel to the group-velocity vector and ray \( \mathbf{r} \) (e.g., Auld, 1973; Helbig, 1994). The latter means that

\[
\frac{\partial F}{\partial c_{IJ}} = -|\nabla_{\mathbf{p}} F| \left( \mathbf{r} \cdot \frac{\partial \mathbf{p}}{\partial c_{IJ}} \right), \quad (I,J = 1, \ldots, 6) \quad \text{(A-13)}
\]

and, as follows from equation A-10,

\[
\mathcal{F} = \frac{\partial g}{\partial c_{IJ}} - \frac{g^2}{|\nabla_{\mathbf{p}} F|} \frac{\partial F}{\partial c_{IJ}}, \quad (I,J = 1, \ldots, 6), \quad \text{(A-14)}
\]

where derivatives \( \partial F / \partial c_{IJ} \) and gradient \( \nabla_{\mathbf{p}} F \) are calculated by differentiating determinant A-11.

Like equation A-5, equation A-14 cannot be used at point singularities, where gradient \( \nabla_{\mathbf{p}} F \) is undefined. Also equation A-14 is expected to break down for rays \( \mathbf{r}^{c} \) corresponding to the tips of cusps that are always present at the shear wave group-velocity surfaces for symmetries lower than transverse isotropy. Indeed, a general stiffness perturbation is expected to move a cusp in such a way that \( \mathbf{r}^{c} \) would be placed either in a shadow zone or into a volume in which \( g(\mathbf{r}^{c}) \) is multivalued.

### APPENDIX B

## ISOTROPIC APPROXIMATION OF INCOMPLETE STIFFNESS TENSOR

Here we discuss how to find the best-fit isotropic stiffness tensor \( \mathbf{e}^{iso} \) to a given tensor \( \mathbf{c} \) when some elements of \( \mathbf{c} \) are unknown. Our analysis is based on the well-known Fedorov’s (1968) solution to the same problem obtained under the condition that all components of \( \mathbf{c} \) are available. We revisit the result of Fedorov first and then describe its appropriate modification.

Fedorov (1968) examines the problem of minimizing the function

\[
\mathcal{L} \equiv \min_{(\lambda, \mu)} \sum_{ijkl=1}^{3} (c_{ijkl} - c_{ijkl}^{iso})^2 \quad \text{(B-1)}
\]

in terms of the Lamé coefficients \( \lambda \) and \( \mu \) that define \( \mathbf{e}^{iso} \) according to equation

\[
c_{ijkl}^{iso} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \quad (i,j,k,l = 1,2,3), \quad \text{(B-2)}
\]

where \( \delta_{ij} \) is the Kronecker delta. Substituting equation B-2 into equation B-1 and switching to Voigt notation yields

\[
\mathcal{L} = \sum_{I=1}^{3} |c_{II} - (\lambda + 2\mu)|^2 + 2 \sum_{I=1}^{3} (c_{JK} - \lambda)^2 + 4 \sum_{M=4}^{6} (c_{MM} - \mu)^2. \quad \text{(B-3)}
\]

The Lamé coefficients are found using the standard optimization requirement

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{\partial \mathcal{L}}{\partial \mu} = 0, \quad \text{(B-4)}
\]

which leads to two linear equations for \( \lambda \) and \( \mu \). If values of all relevant stiffness coefficients are known, equations B-4 yield the system

\[
\begin{align*}
9\lambda + 6\mu &= \sum_{I=1}^{3} c_{II} + 2 \sum_{I=1}^{3} c_{JK}, \\
3\lambda + 12\mu &= \sum_{M=4}^{6} c_{MM}.
\end{align*}
\]

(B-5)

which was analyzed and solved by Fedorov (1968).

When some pertinent \( c_{ij} \)'s are unavailable, the corresponding terms have to be removed from the least squares formulation B-3 and derivatives in equations B-4 should be computed for the new misfit function. If, for example, only \( c_{11}, c_{55}, \) and \( c_{66} \) can be measured, as Figures 3 and 5 might suggest, then the equations to be solved are

\[
\lambda + 2\mu = c_{11}, \quad \lambda + 6\mu = c_{11} + 2(c_{55} + c_{66}). \quad \text{(B-6)}
\]
The solution of system B-6, \( \lambda = c_{11} - (c_{55} + c_{66}) \) and \( \mu = (c_{55} + c_{66})/2 \), is obviously different from that of system B-5.

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Estimation of effective anisotropy simultaneously with locations of microseismic events

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ABSTRACT

Passive seismic tomography, in which the event locations and the velocity model are inferred simultaneously, is seldom used to process microseismic surveys acquired in the oil and gas industry. We discuss advantages of applying tomographic ideas to typical microseismic data recorded in a single, nearly vertical well to monitor hydraulic stimulation of a shale-gas reservoir. Microseismic events are conventionally located in the energy-industry applications using a velocity model derived from sonic logs and perforation shots. Instead of fixing the model, as is normally done, we alter it while locating the events. This added flexibility not only makes it possible to accurately predict traveltimes of the recorded P- and S-waves, but also provides a convincing evidence for anisotropy of the examined shale formation. While we find that velocity heterogeneity does not need to be introduced to explain the data acquired at each stage of hydraulic fracturing, the obtained models are suggestive of possible time-lapse changes in the anisotropy parameters that characterize the stimulated reservoir volume.

INTRODUCTION

In global seismology, velocities are usually estimated simultaneously with hypocenters of the earthquakes using a suite of techniques collectively known as passive seismic tomography (Thurber, 1986; Iyer and Hirahara, 1993). Seismological applications of tomography typically assume isotropy and infer a 3D distribution of the P- and S-wave velocities in the subsurface (e.g., Thurber et al., 1995; Martakis et al., 2006). Although microseismic data acquired in tight-gas fields to delineate hydraulic fractures and assess the stimulated reservoir volume lend themselves to tomographic techniques, conventional velocity models for locating the microseismicity are constructed from well-logs and perforation-shots (e.g., Warpinski et al., 2005). Such an approach has at least two drawbacks (Grechka, 2010). First, because energy radiated from microseismic events often does not pass through subsurface regions illuminated by the available perforation shots, the velocities in those regions have to be obtained through extrapolation. Arguably, extrapolation is not the most reliable way to build a velocity model. Second, hydraulic stimulation alters the formation by increasing the pore pressure and creating fractures in the rock mass, which might entail observable time-lapse changes of the prestimulation velocities sampled by the perforation shots. Perhaps the only way to account for those velocity variations within the framework of traveltimes inversion is to derive the velocities from data provided by microseismic events.

We are aware of only a few case studies (Zhang et al., 2009; Concha et al., 2010; Zhou et al., 2010), in which tomographic ideas borrowed from global seismology are applied in the energy-industry settings. These studies use the double difference tomography (Zhang and Thurber, 2003) to locate microseismic events simultaneously with building isotropic velocity models. When data are recorded by geophones placed in several wells, such as in Zhang et al. (2009) and Concha et al. (2010), the P- and S-wave traveltimes suffice for the implementation of passive seismic tomography. Usually, however, microseismicity is monitored from a single vertical well, necessitating the use of polarization or hodogram analysis (e.g., Montalbetti and Kanasewich, 1970) to locate the events.

Here, we present an attempt to extend passive seismic tomography to anisotropic media and apply it to downhole microseismic geometries. In our study, we use data from the survey discussed by Grechka and Duchkov (2011; henceforth, called Paper I). Because they were able to accurately describe the perforation-shot times in a homogeneous anisotropic velocity model, we, too, target the velocity anisotropy rather than heterogeneity. We introduce...
the tomographic problem first and then use the singular value decomposition to find out which stiffness coefficients are constrained in our geometry. Next, we illustrate estimation of those coefficients on traveltime synthetic and, finally, apply the developed algorithm to field data.

**STATEMENT OF THE PROBLEM**

Consider a set of three-component (3C) seismic records of $N_p$ perforation shots, whose spatial locations $\xi_p\equiv\{\xi_{p,1}, \xi_{p,2}, \xi_{p,3}\}$ are known, and $N_e$ microseismic events, whose locations $\xi_e\equiv\{\xi_{e,1}, \xi_{e,2}, \xi_{e,3}\}$ are sought. Data for estimating the event locations $\xi_e$ are the traveltimes $t_Q(\xi_e; x_g)$ of the direct $P\text{-}, S_1\text{-}$, and $S_2\text{-}$waves ($Q=P, S_1$, and $S_2$); depending on the excitation, either $S_1$ or $S_2$ mode might be missing from the data) recorded by a string of $N_g$ geophones placed at $x_g\equiv\{x_{g,1}, x_{g,2}, x_{g,3}\}$. In addition to $\xi_e$, other unknowns of the problem include the origin times $\tau_p$ ($p=1, \ldots, N_p + N_e$) and the velocity-model parameters. Having in mind application to the field data discussed in Paper I and, specifically, the success of Paper I in explaining the perforation-shot times $t_Q(\xi_p; x_g)$ ($Q=P, S_1$) in a homogeneous triclinic model, we choose the velocity-related parameters to be the density-normalized stiffness coefficients $c_{ij}$ of a homogeneous anisotropic solid.

It is clear from the geometry of the problem that the azimuths $\alpha_e$ of microseismic events cannot be determined from traveltimes $t_Q$ of direct arrivals when geophones are placed in a single vertical well (which is the case of primary interest here) and the medium itself possesses rotational symmetry around the vertical. The presence of azimuthal anisotropy in the examined homogeneous anisotropic medium breaks down the rotational symmetry and, in principle, could help estimate all three components of the unknown vectors $\xi_e$, relatively weak azimuthal anisotropy of many realistic formations makes the azimuths of microseismic events poorly constrained by the traveltimes. To resolve this ambiguity, polarization azimuths of the P-waves are usually used to estimate $\alpha_e$ (e.g., Rutledge and Phillips, 2003). Adding the polarization information reduces the number of unknown components of $\xi_e$ from three to two

$$\xi_e = \begin{cases} 
\{e_{e,1}\cos \alpha_e, e_{e,1}\sin \alpha_e, e_{e,2}\} & (\text{for } e=1, \ldots, N_e),
\end{cases}$$

where $\alpha_e$ is the event azimuth, $\xi_e\equiv\{r_e, z_e\}$, and $r_e$ and $z_e$ are the radial distance between a microseismic event and the observation well and the event depth, respectively. The azimuth is considered known because it is independently measured.

Although, in general, the P-wave polarization vectors deviate from the corresponding ray directions in anisotropic media (e.g., Musgrave, 1970), these deviations typically do not exceed a few degrees (see Figure 1.15 in Tsvankin, 2001) and are comparable to the expected uncertainties in polarization analysis of strong microseismic events (Drew et al., 2008; Eisner et al., 2009b). Therefore, it is permissible to treat azimuths $\alpha_e$ as known quantities in theoretical analysis and numerically examine the influence of errors in $\alpha_e$ on the event locations.

Given all the considerations above, our inverse problem can be formulated as follows: find the parameter vector

$$m \equiv [c_{ij}, \xi_e, \tau_e]$$

that has the maximum size

$$\max(\dim m) = 21 + 3N_e + N_p$$

and contains 21 (or fewer) stiffness coefficients $c_{ij}$, $(2N_e$) event-location coordinates $\xi_e$, and $(N_p + N_e)$ event-origin times $\tau_e$ from the data vector

$$d \equiv t_Q(\xi_e; x_g) + \text{noise}.$$  

Vector $d$ is comprised of traveltimes of the direct P- and S1-waves and additive noise; the latter is assumed to be Gaussian. The maximum length of $d$,

$$\max(\dim d) = 2N_p(N_e + N_p),$$

is reached when the times of all events and perforation shots are recorded by all geophones. While working with the P- and S1-waves in synthetic study below is our choice, which reflects the ability to identify one S-wave rather than two split shear-waves in the field data, labeling the S-wave observed in the field as the fast shear (or $S_1$) as opposed to the slow shear (or $S_2$) will be our data-processing assumption.

**SENSITIVITY ANALYSIS**

To assess whether the inverse problem posed in the previous section has a unique solution, we perform the singular value decomposition (SVD) of the Frechét derivative matrix

$$\mathbf{F} \equiv \frac{\partial d}{\partial m} = \begin{pmatrix} \frac{\partial t_Q}{\partial \xi_1} & \frac{\partial t_Q}{\partial \xi_2} & \frac{\partial t_Q}{\partial \tau_p} \end{pmatrix}, \quad (Q = P, S_1).$$

Here $\frac{\partial t_Q}{\partial \xi} \equiv \frac{\partial t_Q}{\partial \xi}$ are the derivatives of traveltimes with respect to the stiffness coefficients derived in Paper I, $\frac{\partial t_Q}{\partial \xi} \equiv \frac{\partial t_Q}{\partial \xi}$, where $p_{ij}$ and $p_{iz}$ are the radial and vertical slowness components, and $\frac{\partial t_Q}{\partial \tau_p} = I_p$: $I$ is the $(N_p + N_e) \times (N_p + N_e)$ identity matrix, $N_p$ and $N_e$ are the numbers of perforation-shots and microseismic events. The singular value decomposition represents matrix $\mathbf{F}$ as the product of three matrices: a column-orthogonal matrix $\mathbf{u}$, a diagonal matrix $\sigma$, whose elements are the singular values, and the transpose of an orthogonal eigenvector matrix $\mathbf{w}$ (Press et al., 2003):

$$\mathbf{F} = \mathbf{u} \sigma \mathbf{w}^T.$$  

Having in mind a forthcoming field-data application, we chose for our modeling the locations of perforation shots, microseismic events, and 3C geophones (Figure 1) resembling those in the field (see Figure 12 below), and use the stiffness tensor (in $\text{km}^2/\text{s}^2$)

$$\epsilon = \begin{bmatrix}
9.350 & 1.923 & 1.960 & 1.003 & -0.017 & -0.401 \\
1.923 & 10.359 & 3.171 & 0.047 & 0.049 & 0.381 \\
1.960 & 3.171 & 9.526 & 0.173 & -0.917 & -0.499 \\
1.003 & 0.047 & 0.173 & 2.937 & -0.226 & 0.119 \\
-0.017 & 0.049 & -0.917 & -0.226 & 3.874 & -0.828 \\
-0.401 & 0.381 & -0.499 & 0.119 & -0.828 & 3.993
\end{bmatrix}$$

derived in Paper I.

We rely on the finding of Paper I that rotation of the stiffness tensor $\epsilon \rightarrow \epsilon^T$ with matrix $U^T$ composed of the polarization vectors of three plane waves that have the same wavefront normal and propagate within a fan of the examined rays makes it possible to
identify a limited subset of components $c'_{ij}$ constrained by the data. Following Paper I, we rotate tensor $\mathbf{c}$ (equation 8) using matrix

$$
\mathbf{U}^T = \begin{pmatrix}
-0.569 & -0.191 & -0.800 \\
0.590 & 0.583 & -0.558 \\
0.573 & -0.790 & -0.219
\end{pmatrix}
$$

obtained in the field-data application in Paper I and replace the derivatives $\partial q / \partial c$ with $\partial q / \partial c'$ in equation 6. Because the derivatives $\partial q / \partial c_x$, $\partial q / \partial c_z$, and $\partial q / \partial c'$ have different units (e.g., s/m, dimensionless, and $s^2/m^2$, respectively), we scale them by factors $f_z = \text{mean}(\|\mathbf{x} - \mathbf{x}\|)$, $f_t = \text{mean}(t_Q)$, and $f_{c'} = (f_c / f_c)^2$, which intend to normalize and balance the contributions of different unknowns.

The SVD of the resulting matrix

$$
\mathbf{F} = \begin{bmatrix}
\partial_{c'} q / \partial c_x, & \partial_{c'} q / \partial c_z, & \partial_{c'} q / \partial c_n
\end{bmatrix}
$$

is displayed in Figure 2. Even though the magnitudes of singular values $s$ and the exact values of elements of the eigenvector matrix depend on the selected scaling factors $f_z$ and $f_t$, Figure 2 unambiguously indicates the absence of zero singular values. This means that all unknowns listed on the right-hand side of Figure 2 can be inverted in a unique fashion from noise-free traveltimes of the direct P- and fast S-waves in the geometry shown in Figure 1 when an initial guess is sufficiently close to the true solution.

Further insights into the sensitivity of traveltimes $t_Q$ to different variables can be gained by resorting the rows of the eigenvector matrix with the goal of enhancing its diagonal dominance. Figure 3 demonstrates that the best constrained quantity of the problem is $c'_{ii}$; its square root is close to an average $S_z$-wave velocity in the fan of rays implied by the geometry in Figure 1. It is instructive to note that although our fan of rays is not as narrow as those discussed in Paper I, the key result of Paper I, namely, the ability to identify the stiffness coefficients constrained in narrow-angle geometries, remains valid.

The subsequent thirteen eigenvalues that follow the largest eigenvalue in Figure 3 correspond to a mixture of the coordinates and the origin times of events, which are likely to be accurately estimated from the data. A wider angular aperture of the examined ray trajectories than those analyzed in Paper I alters the order of influence of $c'_{ii}$'s on $t_Q$. Specifically, $c_{11}$, which relates to the P-wave velocity in the predominant ray direction, becomes coefficient number six rather than number two. This change in the order is inconsequential because a large singular value $s_{22} \equiv s_{c'_{ii}} = 0.05$ implies that $c_{11}$ is recoverable from the data. Let us also note a pronounced step between the eigenvalues number 36 and 37 in Figure 3. It suggests that nine stiffness coefficients $c_{36}$, $c_{56}$, $c_{44}$, $c_{54}$, $c_{24}$, $c_{13}$, $c_{22}$, $c_{23}$, and $c_{33}$, which correspond to the smallest singular values, are poorly constrained.

Figure 1. Locations of $N_p = 3$ perforation-shots (stars, numbered from 1 to 3), $N_e = 7$ microseismic events (circles, numbered from 4 to 10), and $N_g = 11$ geophones (triangles). Henceforth, the coordinate origin is placed at the shallowest geophone.

Figure 2. Logarithms of normalized singular values $s$ (dots) of matrix $\mathbf{F}$ (equation 10) overlaying the absolute values of elements of the eigenvector matrix $\mathbf{w}$ (gray squares). The rows of $\mathbf{w}$ are arranged in the order of variables shown on the right. Each column of $\mathbf{w}$ is an eigenvector that corresponds to the singular value displayed in that column. Black color of the squares denotes $|w_{ij}| = 1$, white --- $w_{ij} = 0$. The color scale is linear. Computations are performed with the stiffness matrix given by equation 8 for the ray trajectories generated by the perforation-shots and microseismic events in Figure 1.
Finally, we observe that almost every parameter combination (column) in Figures 2 and 3 couples $\xi$’s and $\tau$’s for different events. This coupling is caused by dependencies of the locations and origin times of all events on the stiffness coefficients.

Next, we verify conclusions of the presented sensitivity analysis by performing actual inversion of ray-traced synthetic data.

**INVERSION OF SYNTHETIC DATA**

Although the singular value decompositions examined in the previous section establish the uniqueness of inversion of noise-free traveltime data in the geometries similar to that in Figure 1, we emphasize the qualitative nature of those SVD’s. It stems from operating with the Frechét derivatives that contain quantities measured in different physical units. This point can be made explicit by changing, for example, the length units in equation 6 from meters to kilometers. By doing so, we would multiply the derivatives $\partial t_Q/\partial e$ by $10^6$, the derivatives $\partial t_Q/\partial \xi$, by $10^3$, and leave the derivatives $\partial t_Q/\partial \tau_n$ intact. Such a data reweighting dramatically changes Figures 2 and 3; they would indicate that the best constrained quantities are the stiffnesses, followed by the event coordinates and the origin times (not shown). Because of this profound difference, numerical tests on noise-contaminated data are important for understanding the properties of our inverse problem.

Here we describe inversion of synthetic traveltime data in a homogeneous triclinic medium specified by matrix 8. The traveltimes $t_Q$ of direct P- and $S_T$-waves in the geometry shown in Figure 1 were computed with anisotropic two-point ray tracing. We add Gaussian noise to those $t_Q$. The mean of noise is zero, its standard deviation is chosen to be equal to the sampling interval $\Delta t = 0.375$ ms in the field data discussed in Paper I and below. This choice of the standard deviation is motivated by the ability to find a homogeneous triclinic model that fits the perforation-shot times recorded in the field with root-mean-square (RMS) error of one time sample (see Paper I for detail).

The noise-contaminated $t_Q$ comprise the data vector $d$ in equation 4. Since the event azimuths $\alpha_e$ (equation 1) are derived from the polarization analysis of the P-waves, we simulate the polarization uncertainty by contaminating $\alpha_e$ with Gaussian noise that has the zero mean and the standard deviation of $5^\circ$; the latter corresponds to microseismic events with high signal-to-noise ratio (Drew et al., 2008; Eisner et al., 2009b; also Table 1 below).

Let us assume the locations of perforation shots to be known, which is usually the case in reality, and simultaneously estimate the locations and the origin times of all microseismic events in Figure 1. As to inferring the unknown stiffness components $c_{IJ}$ in equation 2, we replicate the strategy described in Paper I and obtain their values $c_{IJ}'$ in the local coordinate frame, which relates to the global (or geographic) coordinates via rotation matrix $U^T$ given by equation 9. Having recognized in the previous section that nine stiffness coefficients

$$c_{IJ}' \equiv [c'_{36}, c'_{35}, c'_{44}, c'_{24}, c'_{13}, c'_{22}, c'_{23}, c'_{13}, c'_{34}].$$

which mainly contribute to the smallest singular values in Figure 3, are likely to be unresolvable, we estimate the remaining twelve $c_{IJ}'$’s.

In what follows, we discuss the inversion results obtained for 100 realizations of random noise in traveltimes $t_Q$ and event azimuths $\alpha_e$.

**Stiffness coefficients**

We begin with the analysis of estimates of twelve best determined $c_{IJ}'$’s (Figure 3). In the inversion, we keep the remaining nine stiffnesses $c_{IJ}'$ (equation 11) at their exact values. As Figure 4 indicates, all sought $c_{IJ}'$’s are successfully recovered. The sizes of bars in Figure 4 represent the standard deviations of the obtained stiffness coefficients. These standard deviations originate from noise added to $t_Q$ and $\alpha_e$. It is interesting to note that estimates of some coefficients, e.g., $c'_{16}$ and $c'_{56}$, are biased (the centers of the bars do not coincide with the dots). This is, in fact, expected because of the nonlinearity of the inverse problem.

Rotating tensor $e'$ back to the global coordinates, $e' \rightarrow e$, results in the stiffnesses $c_{IJ}$ displayed in Figure 5. Clearly, all estimated components of $e$ match their model values within standard deviations. Such a result, however, can be only obtained when the exact values of loosely constrained $c_{IJ}'$ are used. If, for instance, we followed Paper I and kept tensor $e'$ close to isotropy in the course of inversion, elements of tensor $e$ would exhibit an imprint of this improper model assumption.

![Figure 3](image-url). Same as Figure 2, but the rows of eigenvector matrix $w$ are sorted to make it as diagonally dominant as possible.
Event locations

The event locations $\xi_e$ are certainly the main output of conventional microseismic data processing. Here we discuss two scenarios for finding $\xi_e$:

1) when $\xi_e$ are estimated simultaneously with the well constrained portion of the stiffness tensor; and
2) when the velocity-model is fixed, as is currently done in the industry.

Figure 6 displays the event locations (open circles) inverted simultaneously with the anisotropic velocity-model (shown in Figure 4 and Figure 5) for 100 realizations of noise in the travel-times and the events azimuths (scenario 1). The clouds of estimated $\xi_e$ are approximately centered at the correct locations (solid dots) and exhibit no significant biases. The sizes of microseismic clouds reflect the influence of noise on the event locations. (We remind the reader that noise is relatively mild: the standard deviations in $t_Q$ and $\alpha$, are 0.375 ms and 5°, respectively.) The cloud sizes, measured by the standard deviations in the estimated components of $\xi_e$, range from 9 m to 27 m, implying that the examined data-recording geometry is rather unfavorable for obtaining precise event locations.

To find out what portion of the errors in $\xi_e$ could be attributed to the fact that the events were located simultaneously with estimating the stiffnesses, we turn to scenario (2) and repeat the inversion, this time fixing tensor $\tau$ at its correct value. The obtained event locations are presented in Figure 7. Figures 6 and 7 reveal that the new event clouds became only slightly more compact. This indicates that random noise influences $\xi_e$ as much as trade-offs with $\tau$. We confirm this conclusion below by comparing locations of real microseismic events obtained in two different velocity models with the sizes of the location uncertainties (see Figure 13).

Origin times

Origin times $\tau_n$ of events and perforation-shots are the most easily determined quantities because they enter the inverse problem in a linear fashion. This can be made explicit by writing the times $t_Q^e(\xi_e)$ of a microseismic event or perforation-shot located at $\xi_e$ as

$$t_Q^e(\xi_e) = \tau + t_Q^m(\tau_e; \xi_e) + \text{noise}, \quad (12)$$

where $t_Q^m$ denotes the modeled traveltimes between $\xi$ and geophones at $x_g$. When noise has zero mean, the least-squares solution of equation 12 for the origin time $\tau$ is

$$\tau = \text{mean}[t_Q^e(\xi_e) - t_Q^m(\tau_e; \xi_e)], \quad (13)$$

where the mean is taken with respect to all wave types $Q$ recorded at geophones $x_g$.

Given the simplicity of solution 13, in principle, the origin times $\tau_n$ could have been removed from the unknowns. We, however, prefer to keep $\tau_n$ to make their trade-offs with other unknowns easier to quantify (Figures 2 and 3).

PROCESSING OF FIELD DATA

The study presented in the previous section allowed us to assess the level of uncertainties in the inverted stiffness coefficients (Figures 4 and 5) and the event locations (Figures 6 and 7). Here we apply the understanding we have gained to microseismic monitoring data acquired in a shale-gas field. Our goal is to locate a few strong events simultaneously with estimating a homogeneous anisotropic velocity-model described by the density-normalized stiffness tensor $\tau$. Even though vertical velocity heterogeneity is evident on well-logs (Figure 12 in Paper I), we do not include it in our models.
because it cannot be quantified from microseismic in our data-acquisition geometry. Indeed, since geophones are located some 400 m shallower than analyzed microseismic events (see Figures 12 and 14 below), the traveltimes of direct arrivals allow us to constrain only effective parameters of the medium between the geophones and the events. Our choice of the class of velocity models is in line with the results of Gajewski et al. (2009), who showed that homogeneous anisotropic media can be useful for processing of microseismic data even in a significantly more complex geologic environment than ours.

**Stage 1**

We begin our discussion with the stage of hydraulic stimulation, called stage 1 here, for which stiffness tensor $c$ (matrix 8) obtained in Paper I describes perforation-shot times $t_Q$ with the root-mean-square error of approximately one time sample $Δt = 0.375$ ms. For our detailed analysis, we select seven strong microseismic events, such as those shown in Figure 8.

The first step in our as well as in conventional processing of microseismic data recorded in a single-well is the P-wave time picking and hodogram analysis, which needs to be done to estimate an event azimuth $α_e$. Figure 9 presents an example of the P-wave hodogram. Direction of a straight line that approximates the recorded particle motion defines the P-wave polarization vector. Deviation of the particle motion from the best-fit straight line is a measure of the polarization uncertainty. We apply the SVD-based algorithm described by De Meersman et al. (2006) to calculate the P-wave polarizations and their standard deviations. Table 1 lists azimuths $α_e$ and std($α_e$) of strong microseismic events used in the inversion.

At the second step, we identify the next pickable arrival on two components orthogonal to the obtained P-wave polarization direction at each geophone, pick its times (blue ticks in Figure 8), and rotate those two components in their plane to enhance this wave. Because the identified waves exhibit particle motions in directions approximately orthogonal to the polarization vectors of the direct P-waves (Figure 8b and 8d), we interpret them as the direct shear-waves. After the two described rotations, the remaining third component contains little coherent energy around the picked S arrivals (not shown), making it impossible for us to conclusively state whether the picked S-waves are $S_1$ (fast) or $S_2$ (slow). To be consistent with Paper I, we assume them to be the fast $S_1$-waves, fully realizing that this is our interpretation of the data.

---

**Figure 6.** (a) East-depth, (b) north-depth, (c) plan, and (d) 3D views of locations of microseismic events (open circles) estimated simultaneously with the stiffness coefficients in Figures 4 and 5. The exact locations are shown with large solid dots whose colors correspond those of the inverted locations.

**Figure 7.** Same as Figure 6 but all components of the stiffness tensor are fixed to their correct values.
Third, we need to determine which stiffness coefficients should be estimated from the P- and (assumed) S1-wave times. To do so, we locate events number 4 through 10 in the best-fit isotropic velocity-model (its velocities are Vp,iso = 2.910 km/s and Vs,iso = 2.012 km/s) and use the estimated $\xi$, for the SVD performed exactly as was discussed in the section on sensitivity analysis. Specifically, we transform tensor $c$ to the local frame with matrix $U^T$ (equation 9), $U^T c \rightarrow c'$, and calculate the Frechet derivatives with respect to the new stiffness coefficients $c'_{ij}$. Figure 10a displays the resulting singular values and the eigenvector matrix.

Fourth, to find out how many $c'_{ij}$'s can be realistically constrained by our traveltime data $t_Q$ ($Q = P$, $S$), we run a series of inversions by gradually increasing the number of the estimated stiffness coefficients and observing a decrease in the RMS traveltime misfit $\Delta t_{RMS}$. The stiffnesses $c'_{ij}$ that are not updated have the values obtained from the perforation-shot times (matrix 33 in Paper I). As Figure 10b indicates, $\Delta t_{RMS}$ reduces until the number of unknown stiffnesses reaches 11 and then stays nearly constant. Hence, we conclude that the array of coefficients (see labels on the right-hand side of Figure 10a)

$$c'_{ij} = [c'_{00}, c'_{11}, c'_{10}, c'_{15}, c'_{14}, c'_{45}, c'_{55}, c'_{56}, c'_{66}]$$  

(14)

is sufficient to describe most of the data. Also, according to Figure 10b, fitting $t_Q$ ($Q = P$, $S$) with eleven stiffness coefficients in equation 14 entails the RMS error $\Delta t_{RMS} \approx 0.7$ ms, which is smaller than two time samples in the data.

It is instructive to note that $\Delta t_{RMS}$ for the stiffness tensor $c$ given by equation 8, which describes the perforation-shot times with the RMS misfit of one time sample (see Paper I), is $\Delta t_{RMS} = 1.24$ ms. Therefore, Figure 10b implies that even the simplest homogeneous isotropic velocity-model fits the times of microseismic events better than the fixed model obtained from the perforation-shots. This result by itself justifies our effort to estimate the velocity-model parameters simultaneously with locations of microseismic events.

In our fifth and final step, we essentially repeat the procedure described for traveltime synthetic and estimate parameter vector $m$ (equation 2) from noise-contaminated $t_Q$ and $\sigma_e$. We add Gaussian noise with the same standard deviation of 0.375 ms to $t_Q$ and use the standard deviations from Table 1 (instead of $S$) for noise in the event azimuths $\alpha_e$. The results — stiffness coefficients
c and event locations $\xi_e$ for approximately 100 random realizations of noise — are shown in Figures 11 and 12, respectively. Even though the uncertainties in $\xi_e$ appear to be substantial, all models in Figures 11 and 12 yield the RMS misfits ranging from $\Delta_{\text{RMS}} = 0.60$ ms to $\Delta_{\text{RMS}} = 0.95$ ms or from approximately two to three time samples.

The magnitudes of standard deviations in the estimated $c_{ij}$’s (Figure 11) are comparable to those obtained in our synthetic study (Figure 5). Perhaps more important is the fact that the inverted tensors $c$ are moderately anisotropic. The relative deviation of mean($c$) from isotropy is 24%. Such a strength of anisotropy in the examined shale formation is expected based on the existing laboratory measurements (e.g., Vernik and Liu, 1997; Wang, 2002).

We now turn our attention to uncertainties in the event locations $\xi_e$. An average uncertainty, calculated by taking the mean of $\sigma_{\xi_e}$ for approximately 100 random realizations (e.g., Vernik and Liu, 1997; Wang, 2002), is 0.60 ms.

Table 1. Azimuths (measured from east to north) and standard deviations of strong microseismic events recorded during stage 1 of the hydraulic treatment. The events are numbered sequentially in accordance with their occurrence time. The numbers 1, 2, and 3 are reserved for the perforations shots.

<table>
<thead>
<tr>
<th>Event number</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth $\alpha_e$ ($^\circ$)</td>
<td>36.5</td>
<td>35.1</td>
<td>39.3</td>
<td>20.9</td>
<td>32.6</td>
<td>24.9</td>
<td>45.6</td>
</tr>
<tr>
<td>Std $\alpha_e$ ($^\circ$)</td>
<td>5.8</td>
<td>7.4</td>
<td>5.1</td>
<td>8.7</td>
<td>8.1</td>
<td>7.0</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Figure 10. (a) SVD similar to that shown in Figure 3 but for strong microseismic events recorded at the stimulation stage one and (b) RMS errors of fitting the event times as a function of the number of estimated $c_{ij}$’s.
Again, these variations in $\xi$ have the same order of magnitude as those quoted earlier.

Figure 13 clearly demonstrates the superiority of the triclinic model over its isotropic counterpart. Ignoring anisotropy increases the RMS misfit from 0.8 ms to almost 1.2 ms or by 50% for event number 5 (Figure 13a and 13c) and from 0.6 ms to 1.4 ms or by 130% for event number 6 (Figure 13b and 13d). In addition, the assumption of isotropy approximately doubles the uncertainties in the event locations (compare the sizes of shaded areas in Figure 13a and 13b with those in Figure 13c and 13d). Hence, we conclude that accounting for anisotropy allows us to describe the traveltime data more accurately and locate the events more precisely. Red and blue open circles in Figure 8 display the quality of fitting the times of selected events.

Finally, let us discuss the shapes and orientations of the uncertainty plots in Figure 13. Their elongation in the direction approximately normal to the ray trajectories from the events to the geophones is caused solely by our relatively narrow-angle acquisition geometry. While an event placement along the rays is constrained by the differences ($t_S - t_P$) between the picked times of the $S_1$ and P-waves (Figure 8), its position in the orthogonal direction does not change the traveltime differences as much and, therefore, is constrained less tightly. This explanation and the appearance of uncertainties in Figure 13 are in agreement with the results of numerical study published by Eisner et al. (2009a).

**Stages 3 and 4**

We repeated our analysis for stages 3 and 4 (stimulation stage 2 had operational difficulties and did not produce a sufficient number of good-quality microseismic events) and built homogeneous triclinic velocity models simultaneously with locating twelve and ten events, respectively. Figure 14 shows the mean event locations computed after contaminating $t_Q$ and $\alpha_e$ with Gaussian noises that had the same standard deviations as before. We do not display $\xi$ for each realization of noise similarly to the open circles in Figure 12 to avoid making plots in Figure 14 too busy. The uncertainties in the event locations and the RMS misfits of noise-contaminated data happen to be very close to those at stage 1: mean[std($\xi$)] = 24 m and mean[ΔRMS] = 0.81 ms at stage 3, mean[std($\xi$)] = 22 m and mean[ΔRMS] = 0.87 ms at stage 4. Thus, we conclude that homogeneous triclinic models are capable of describing strong microseismic events recorded at each of the treatment stages.

**Comparison of estimated stiffnesses**

Figure 15 displays the stiffness coefficients estimated at stages 3 and 4. Their comparison with the stiffnesses obtained at stage 1 (Figure 11) reveals a feature, which we believe is worth discussing. Let us observe that stiffness coefficients $c_{33}$ corresponding to the squared P-wave velocities at near vertical propagation directions are...
greater at stages 3 and 4 than that at stage 1 by about 15% and 22%, respectively. Such a behavior of the P-wave velocity can be explained by lateral heterogeneity. Indeed, according to the gamma-ray log in Figure 16, the formation gets sandier as we move along the treatment well from north to south. Because shales usually are acoustically softer than sands, Figure 16 corroborates the inversion results for $c_{11}$.

The changes in stiffness coefficients $c_{11}$, however, are opposite to those in $c_{33}$. As evidenced by Figure 11 and Figure 15, $c_{11}$ decreases by approximately 13% at stage 3 and 19% at stage 4 compared to that at stage 1. The presence of this decrease is established with a high confidence because the magnitude of variation of $c_{11}$ exceeds the standard deviations in the estimated $c_{11}$. If the symmetry of the obtained stiffness tensors were orthorhombic, the relationship between $c_{11}$ and $c_{33}$ could be quantified in terms of Tsvankin’s (1997) anisotropy parameter

$$e(2) = \frac{c_{11} - c_{33}}{2c_{33}}$$

and would result in $e(2) = 0.07$ at stage 1, $e(2) = -0.08$ at stage 3, and $e(2) = -0.12$ at stage 4. A certain portion of this variation could be attributed to the change in lithology. Because the increase in sandiness (Figure 16) generally reduces the magnitude of seismic anisotropy of shales, the value of $e(2) = 0.07$ obtained at stage 1 is expected to decrease as the treatment proceeds to a sandier interval. The negative $e(2)$ estimated at stages 3 and 4, however, are not typical for either unfractured sands or shales (Wang, 2002). Therefore, we suggest that the hydraulic stimulation itself might cause these negative $e(2)$ values. Our arguments in favor of this hypothesis are as follows.

1) Since the well treatment proceeds from north to south, rays corresponding to microseismic events triggered at stage 1 propagate toward the geophones through mostly intact formation, resulting in a positive $e(2)$ as expected for shales. In contrast, portions of the ray trajectories generated by events at stages 3 and 4 have to cross the rock volume that has been already treated at the earlier stimulation stages. Fluids injected in this volume possibly open some natural fractures and fissures, which were closed prior to the treatment. Although Figure 14 displays only the events analyzed in conjunction with the discussed velocity-model building procedure, the entire event population comprises more than a thousand of events. The shallowest of them was...
located approximately 60 m deeper than the deepest geophone, suggesting that most of the formation between the perforation shots and the geophones has been stimulated.

2) Because the vertical component of the in-situ stress is known to exceed its horizontal components in the examined field, the newly opened fractures are likely to be approximately vertical. The presence of such fractures reduces stiffnesses $c_{11}$ and $c_{22}$ compared to $c_{33}$ and might result in negative $\epsilon$-type anisotropy parameters (e.g., Grechka, 2007). Rays in the geometry in Figures 12 and 14, however, sample the east-west direction better than the north-south direction, making it easier to estimate $c_{11}$ than $c_{22}$ from the data. While the density of fractures opened by the treatment cannot be unambiguously computed from the obtained stiffness tensors alone, the change from $\epsilon^{(2)} = 0.07$ to $\epsilon^{(2)} = -0.12$, if attributed to a single set of vertical gas-filled fractures, yields the effective crack density about 0.1 (see Tsvankin and Grechka, 2011, for an overview of crack-induced anisotropy and pertinent references).

Even though the above explanations and estimates seem plausible, they are in no way conclusive. A more convincing evidence for time-lapse anisotropy variation in the course of hydraulic well stimulation would be obtained by dividing population of microseismic events recorded at a given stage in accordance with their occurrence time into the early and late ones and comparing the stiffness tensors calculated for each group of events. We have attempted to do this using the available data, but could not arrive at unambiguous conclusion because of relatively large uncertainties in the estimated stiffnesses.

**DISCUSSION**

As has been mentioned in the introduction, velocity models used in conventional microseismic data processing are likely to be suboptimal because they usually ignore the velocity information carried by microseismic events themselves. Hence, the overarching idea of our study was to incorporate this information to improve both the velocity model and the event locations. In this sense, our study succeeded because we have shown that the model (given by equation 8, which perfectly fits the traveltimes of perforation shots at stage 1, nevertheless fails to adequately describe the times of microseismic events recorded at the same stage.

Although choosing an appropriate velocity parameterization was an important issue for our study, it has been resolved, at least partially, by the employed recording geometry, in which geophones were placed some 400 m shallower than perforation shots and the examined microseismic events. In such a geometry, only effective, that is, homogenous velocity model can be constrained. Once this has been recognized, we relied on the findings of Paper I that demonstrated the importance of accounting for anisotropy in our shale-gas field and allowed us to identify invertible elements of the elastic stiffness tensor. This understanding has led to building homogeneous triclinic models for the stimulation stages 1, 3, and 4. Those models describe traveltime picks of the direct P- and fast S-waves with the RMS errors of approximately two time samples. We emphasize that making the models anisotropic is essential for fitting the data. As illustrated in Figure 13, ignoring anisotropy significantly impairs one’s ability to predict the times of recorded microseismic events.

Despite fitting the data accurately, we acknowledge fairly large event-location errors. An overall location error in geometries that use a single nearly vertical monitor well is comprised of two types of uncertainties: uncertainty in the event azimuth, which comes from the hodogram analysis, and uncertainty in the vertical $f_{rz}$-plane, which originates from inaccuracies in the traveltime picks. While the encountered azimuthal errors, ranging from 5° to 8°, are usually observed for microseismic events with high signal-to-noise ratios, our significant errors in the $f_{rz}$-plane

![Figure 15](image-url)

Figure 15. Same as Figure 11, but $c_{ij}$'s are estimated from microseismic and perforation-shot data recorded at (a) stage 3 and (b) stage 4.
are caused by the unfavorable data-acquisition geometry, in which the P- and S-wave moveouts are close to straight lines (Figure 8). It is this lack of the moveout curvature that makes our average location errors as large as about 30 m.

The estimated effective stiffness coefficients represent an important output of our study. We have attempted to relate their values to the hydraulic stimulation activity but obtained only tentative results. The reason for that was our inability to use events recorded at the same treatment stage for deriving at least two distinctly different velocity models. However, because simultaneous inversion of microseismic data for the velocities and the event locations is demonstratively feasible, we expect time-lapse seismic observations in tight-gas and shale-gas environments to be just a matter of recording a suitable data set, especially because temporal variations in the shear-wave splitting coefficients in producing hydrocarbon fields have been already reported (Teanby et al., 2004; de Meersman et al. 2009).

Finally, building heterogeneous anisotropic models for microseismic data processing should be possible too, provided that an array of geophones covers the depth interval in which the recorded microseismicity takes place. Although our paper gives no proof of the uniqueness of such an inverse problem, its investigation is planned to be the subject of a sequel publication.

CONCLUSIONS

We demonstrated the feasibility of estimating the effective anisotropy simultaneously with locations of microseismic events in a single-well geometry, which is typical for contemporary monitoring of hydraulic stimulations. The results of the presented case study can be summarized as follows.

1) Accounting for seismic anisotropy improves description of the recorded traveltimes and reduces uncertainties in the locations of microseismic events compared to those obtained in purely isotropic models.
2) Velocity models constructed from perforation-shot data are found to be inferior to those that use microseismic events as additional sources of the velocity information.
3) Despite an accurate description of the recorded traveltimes (all our RMS errors are smaller than three time samples or 1 ms), uncertainties in the locations of microseismic events are large (between 20 m and 50 m). The reason for that is an unfavorable data-recording geometry in which moveouts of the observed direct P- and S-waves exhibit little curvature.

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Inferring rock fracture evolution during reservoir stimulation from seismic anisotropy

Andreas Wuestefeld, James P. Verdon, J-Michael Kendall, James Rutledge, Huw Clarke, and James Wookey

ABSTRACT

We have analyzed seismic anisotropy using shear-wave-splitting measurements made on microseismic events recorded during a hydraulic fracture experiment in a tight gas reservoir in Carthage, east Texas. Microseismic events were recorded on two downhole arrays of three-component sensors, the geometry of which provided good ray coverage for anisotropy analysis. A total of 16,633 seismograms from 888 located events yielded 1545 well-constrained shear-wave-splitting measurements. Manual analysis of splitting from a subset of this data set reveals temporal changes in splitting during fracturing. Inversion using the full data set allows the identification of fracture strike and density, which is observed to vary during fracturing. The recovered fracture strike in the rock mass is parallel to directions of regional borehole breakout, but oblique to the hydraulic fracture corridor as mapped by the microseismic event. We relate this to en-echelon fracturing of preexisting cracks. The magnitude of shear-wave splitting shows a clear temporal increase during each pumping stage, indicating the generation of cracks and fissures in a halo around the fracture corridor, which thus increase the overall permeability of the rock mass. Our results show that shear-wave-splitting analysis can provide a useful tool for monitoring spatial and temporal variations in fracture networks generated by hydraulic stimulation.

INTRODUCTION

Seismic studies of rock fractures provide valuable insights into the state of stress and are useful in a number of applications. In geotechnical and mining environments, knowledge of fracture orientation can be important in hazard assessment, but may also help guide excavation programs. In reservoirs, fractures provide important secondary porosity and permeability, which may or may not be desirable. To prevent unwanted leakage, subsurface CO2 storage projects require that injection does not stimulate fracturing in the sealing overburden (e.g., Verdon et al., 2009). In geothermal projects, fractures increase not only permeability, but also the effective contact surface, which facilitates the heat transport from rock to the transfer fluid. Reservoir formations may contain large quantities of oil or gas, but have a poor flow rate due to low permeability. This is particularly true for tight sands, shale gas, and coalbed methane. Cracks, or natural fractures, exist in most of these reservoirs, but they often lack connectivity. Hydraulic stimulation can be used to generate fracture networks and to improve the permeability. The resulting fractures and cracks will tend to align with direction of dominant horizontal stress, thus generating a permeability anisotropy.

A medium with aligned fractures can be approximated as an effectively homogeneous but anisotropic medium, if the dominant seismic wavelength is much longer than the fracture spacing and size (e.g., Kachanov, 1980; Crampin, 1984; Hall and Kendall, 2003). For a single set of vertical aligned fractures in an isotropic
medium, this results in hexagonal symmetry with a horizontal axis of symmetry (horizontal transverse isotropy (HTI)). However, other factors can also generate anisotropy. These include the alignment of grain-scale microcracks (Hall et al., 2008), which can be modulated by nonhydrostatic stress changes (Zatsepin and Crampin, 1997; Verdon et al., 2008), the periodic thin layering of contrasting lithologies (Backus, 1964), and the preferred alignment of intrinsically anisotropic crystals (Raymer et al., 2000; Valcke et al., 2006). In sedimentary rocks, these mechanisms can be also very effective in generating anisotropy with a hexagonal symmetry, but in general with a vertical axis of symmetry (vertical transverse isotropy, VTI).

In reality both mechanisms will be at play in sedimentary basins, which render the bulk rock anisotropic, but with an orthorhombic symmetry. Furthermore, structural deformation and stress anisotropy can lead to even more degenerate forms of anisotropy (e.g., monoclinic and triclinic symmetry).

A seismic shear wave entering an anisotropic medium will, in general, split into two independent, orthogonally polarized S-waves. The polarizations of the fast and slow shear waves and the separation in the arrival time of each of them can be used to characterize the magnitude and symmetry of the anisotropy. The delay time between the two waves is proportional to the strength of S-wave velocity anisotropy and the length of the raypath within the anisotropic medium. Wuestefeld et al. (2010) recently developed a workflow for the automated measurement of these shear-wave-splitting parameters with large data sets.

The splitting parameters will vary with ray direction, in a way that is characteristic of the anisotropy system, and hence, the underlying cause of the anisotropy. For example, shear-wave splitting can be used to characterize aligned fracture sets (Crampin, 1984; Kendall et al., 2005; Keir et al., 2005; Al-Harrasi et al., 2011a) and to study the temporal evolution of fractures (Gerst and Savage, 2004; Wuestefeld et al., 2011). Furthermore, the frequency-dependent nature of fracture-induced anisotropy can be used to estimate dominant fracture size (Al-Harrasi et al., 2011b). More generally, Verdon et al. (2009) introduced a method for inverting splitting parameters for fracture strike and density, which also accounts for the presence of shear-wave anisotropy induced by the intrinsic sedimentary fabric of the rock.

It is worth noting that more traditional passive seismic methods, such as those for estimating event locations and source mechanisms, give information about the source region, whereas shear-wave splitting is a raypath effect and thus provides valuable information about the surrounding rock mass. Used in conjunction with such other microseismic analysis techniques, shear-wave splitting can be used to help calibrate geomechanical and fluid-flow models of reservoirs (e.g., Angus et al., 2010).

Here, we study shear-wave splitting during hydraulic fracture stimulation in a gas field within the Cotton Valley formation in Carthage, East Texas (Walker, 1997; Rutledge et al., 2004). Hydraulic fracturing is a common technique used to enhance the production of oil and natural gas by stimulating fractures that extend from the wellbore into the surrounding rock. Typically, fluids injected underground at high pressures fracture the formation, and the oil or gas flows more freely out of the rock matrix. Usually, a “proppant” is suspended in the fluid to keep the fracture open (e.g., Jones and Britt, 2009). Monitoring microseismic activity helps better understand the response of the rock to changing stress conditions and map the fracture. In turn, mapping the microseismicity indicates the fracture network complexity and might thus help understand and improve proppant emplacement. We investigate what further insights into fracturing in the rock mass can be obtained from shear-wave-splitting studies.

The Cotton Valley data set

The Cotton Valley formation consists of siliciclastic sandstones with intermittent shale and carbonate horizons over a total thickness of approximately 325 m (Rutledge and Phillips, 2003). The reservoir is typical of low-permeability gas resources that require hydraulic fracture stimulation for economic production. The current understanding of the fracture process assumes that tensile opening dominates at the fracture tip, while shear failure occurs simultaneously along the entire fault length (e.g., Rutledge et al., 2004). In another deep hydraulic fracturing experiment, Ake et al. (2005) found that 89% of the events were strike-slip, while the remaining events where roughly equally divided between normal and thrust faulting events. More recent studies found evidence for a nondouble-couple component in event mechanisms associated with hydraulic fracturing (Foulger et al., 2004; Šilény et al., 2009).

We focus on microseismic events from a gel frac-job at the base of the Upper Cotton Valley

Figure 1. Events and receiver locations of the Cotton Valley data set in map-view (top-left) with inset showing details of the cluster region marked within a box, and in cross-section (top-right). Color indicates timing of the events during stimulation (see legend in upper left panel — time is in hours). A fracture corridor develops oriented at N80°E. A secondary fracture (see inset) opens oriented N65°E between 30 and 75 m east of the injection well (marked as a black cross). The lower panel shows a histogram of seismicity in 3 minute bins (green), the variation of pumping pressure (blue), slurry rate (black), and cumulative proppant density injected during stimulation (red).
formation, named “Treatment B” in Rutledge et al. (2004). The events generally occur in discrete, thin bands, which correlate with the perforation intervals targeted for fracturing (Figure 1). The events occur along a general trend of N83°E. Furthermore, Rutledge et al. (2004) note that an anomalously high event count is found within small clusters. Especially, one cluster contains 42% of the events and develops at a shallow angle from the general trend at N65°E. This structure develops westward, toward the injection well (Figure 1). Rutledge et al. (2004) associate this cluster with a pre-existing fault that has been reactivated by the back stream of fluid.

RESULTS

We performed manual shear-wave-splitting analysis (Table 1) on the waveforms of station 1–11 (Figure 1) using a version of the processing software SplitLab (Wuestefeld et al., 2008), adapted for reservoir studies. Station 1–11, located close to the top of Array 1, recorded a large number of good quality seismograms with clear S-wave arrivals. Relatively steep raypaths make events recorded at this station strongly susceptible to shear-wave anisotropy caused by vertical fractures (e.g., Verdon et al., 2009). The mean take-off angle from vertical toward the station is 30°. In general, any anisotropy generated by vertical fractures is best sensed with sensors from the top part of the array, whereas sensors near the bottom of the array will be more sensitive to anisotropy due to the sedimentary fabric (Verdon et al., 2009; Wuestefeld et al., 2010).

The focus here was on events from within the localized cluster of events during the third pressure stage (Figure 1). The anisotropy is smallest for events early in the development of fracturing and increases in time. The increase of anisotropy with time is clearly shown in Figure 2a. However, the event locations also propagate westward during this time (Figure 2b), and the increase may be explained by spatial variations in anisotropy. This entanglement of spatial and temporal variation is inherent to many production related studies: is the increase in observed change due to heterogeneity, or is it due to temporal changes caused by continuous fracturing? In the following, we address this issue by considering differences in the raypaths.

Our manual analysis of shear-wave splitting at station 1–11 (Table 1) shows an increase in delay time within a 30-minute period from 3 to 7.25 ms for events within 26 m of each other. This change cannot be explained by changes in raypath length (see Figure 2c): The maximum hypocentral distance between the events is 26 m, resulting in a maximum path length difference with station 1–11 of 15.05 m. As the raypaths are nearly identical, the increase in delay time from 3 to 7.25 ms would need to be accrued in this extra 15 m. Quantitatively, this would require 84% shear-wave anisotropy in this small isolated region (assuming a velocity of 3000 m/s, which is the mean S-wave velocity at the depth of the Cotton Valley event). Because this is unrealistically high, we conclude that the observed change in shear-wave splitting delay time is indeed a temporal effect.

This temporal increase can be explained by an increase in crack density in the subsurface due to fluid injection. This volume increase activates natural cracks and fissures of favorable orientation due to stress transfer (e.g., Rozhko, 2010; Schoenball et al., 2010). The alignment of the reactivated cracks will thus increase the anisotropy within a “halo” around the hydraulic fracture. If we allow for a shear-wave-velocity anisotropy of between 10% and 15% caused by aligned cracks, this suggests \( \Delta s = V_{s\text{mean}} \times 100 \times \Delta d(A) \) a halo radius between 127.5 and 85 m, respectively. The region affected by stress changes due to fluid injection can thus be narrowed to this radius around the observed seismicity. Note that the halo is not symmetric and this length is not necessarily the radius of a sphere, but rather that of an ellipsoid oriented between events and station.

Table 1. Parameters and shear-wave splitting results of the events used in the manual analysis of records at station 1–11. Note the increase in delay time over time at similar source-receiver distances. Event locations and magnitudes are from Rutledge et al. (2004). The errors in delay time, dterr, are calculated using an f-test as described in Silver and Chan (1991).

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<th>Depth (m)</th>
<th>Distance (m)</th>
<th>Magnitude (Mw)</th>
<th>Splitting delay time (ms)</th>
<th>Delay time error (ms)</th>
<th>Percent anisotropy</th>
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Full data set processing and inversion

Increasingly large data sets make manual shear-wave-splitting processing impractical. Automatic processing improves the objectivity and repeatability of measurements. Here, we use the automated shear-wave splitting technique presented by Wuestefeld et al. (2010), which is based on characteristic differences in two independent splitting inversion techniques (Wuestefeld and Bokelmann, 2007) and multiwindow analysis (Teanby et al., 2004). The automated analysis returns a quality index, which can vary from −1.0 for null to 0.0 for poor to +1.0 for good splitting measurements (see Wuestefeld et al., 2010).

For the Cotton Valley data set we extend the analysis to the entire period of monitoring. Rutledge et al. (2004) report 888 located events for this treatment. The data set thus consists of 16,633 source-receiver records of which 1545 resulted in good splitting measurements. Good splitting measurements are defined here as having a quality index of above 0.85, a signal-to-noise of $S/R > 4$, delay times between $0.5 \leq dt \leq 9$ ms, an error in delay time smaller than 2 ms and an error in fast-polarization directions smaller than 15º.

Figure 3 shows the resulting delay times from automation for all stations during the third stimulation phase. Note the equivalent increase in delay time for station 1–11 presented in Figure 2a. Overall, the shallower stations (cf. Figure 1), with steep incidence angles, show significant variations in delay time during this stimulation. Shallower stations exhibit only minimal variations, albeit with a higher starting level. The fast polarization directions for the entire data set are shown in Figure 4. Steep incident rays show fast polarizations subparallel to the ray direction, whereas shallower incident waves result in ray-perpendicular fast-polarization directions.

To understand these variations in delay times and fast polarization directions between stations in terms of changes in fracture strike ($\alpha$) and density ($\xi$), we invert our observations for fracture...
parameters using a rock-physics-based approach based on Verdon et al. (2009). For a similar approach to invert for the entire set of orthorhombic parameters, using P- and S-waves, see Tsvankin (1997), Grechka et al. (1999) and Bakulin et al. (2000). Our model assumes that the observed shear-wave splitting is caused by aligned vertical fractures set in a rock with VTI symmetry described by Thomsen’s (1986) $\epsilon$, $\gamma$, and $\delta$ parameters. Although such an approach can be generalized to allow for dipping cracks or multiple crack sets (e.g., Verdon et al., 2011), attempts to invert for two fracture sets with the Cotton Valley data set suggest that a single fracture set is most appropriate.

Initially, we invert the full data set of shear-wave-splitting measurements, using fixed values for $V_p$ and $V_S$ based on the velocity model used during event location, with $\epsilon$, $\gamma$, $\delta$, $\alpha$, and $\xi$ as free parameters. The results are plotted in Figure 5. Figure 5a shows the shear-wave-splitting observations overlain on the best fitting rock-physics model. Figure 5b and 5k shows slices of the misfit hypercube between rock-physics model and observation. The inversion recovers the following values: fracture strike ($\alpha$) = N66°E±3°, fracture density ($\xi$) = 0.024±0.002, and $\gamma$ = 0.12±0.01. Examining the misfit hypercube reveals a trade-off between $\epsilon$ and $\delta$ (Figure 5j), implying that neither is well-constrained. Further discussion on the ability of the inversion process to accurately recover the free parameters is included in Appendix A. The trade-off between $\epsilon$ and $\delta$ allows us to remove $\epsilon$ from the free parameters that we invert for, instead arbitrarily fixing it to $\epsilon = 2\gamma$, so long as we accept that $\delta$ is poorly constrained. This significantly improves the computational expense of our grid-search based inversion procedure, and so we follow this for our analysis of temporal variations below.

The inversion result of $\gamma$ of 0.12±0.01 is comparable, but not equal to, the value of $\gamma = 0.18$ measured by Thomsen (1986) for a laboratory sample of Cotton Valley shale. We note here the difficulties in comparing isolated rock samples measured in the laboratory, whereas our values are based on in situ measurements through the whole rock mass, and are thus arguably more representative for uses in other seismic or modeling methods.

The inversion images fractures striking at N66°E. The borehole breakout orientations given in the World Stress Map (Heidbach et al., 2008) rotate from north-northeast to south-southwest in southern Texas to east-northeast to west-southwest in the Cotton Valley area (Figure 6). Interestingly, two borehole breakout measurements in that area from a hydraulic fracture experiments deviate slightly from this general trend. In northwest Louisiana, Strubhar et al. (1975) report a borehole breakout orientation of N90°E, comparable to the N83°E hydraulic fracture trend observed in Cotton Valley. In a study of cores mainly from the Travis Peak formation, which overlies the Cotton Valley formation, Laubach (1988) found two distinct fracture trends: one set of natural and coring induced cracks with a mean strike of N83°E, and a second set of natural cracks that strikes between N65°E and N74°E. The crack strike of our inversion is oblique to the strike of the hydraulic fracture as mapped by the microseismic events (N83°E, Figure 1), but it agrees well with the orientation of that second natural cracks set and also with the general regional trend in east Texas as seen in borehole break outs (Figure 6). This indicates that stress change may reactiviate mostly the cracks of orientation N65°E, as seen by shear-wave splitting, and the hydraulic fracture develops oblique to these cracks in an en-echelon style.

We next analyzed temporal changes observed in the whole data set, similar to Figure 2, where we described the increase in delay time and thus magnitude of anisotropy during the third pumping stage. We attributed this to the development and reactivation of cracks and fissures in the surrounding rock. This hypothesis can be tested by applying the above inversion technique to results from sliding time windows. Such an approach has been successfully applied to an underground mining data set to monitor fracture evolution during a production cycle (Wuestefeld et al., 2011). We chose a 30-minute time window of events from the cluster region and calculated the inversion parameters in time steps of 6 minutes. We only used windows with more than 10 measurements. In these inversions, $\epsilon$ is fixed, $\gamma$, $\delta$, $\alpha$ and $\xi$ are free parameters. Our error analysis in Appendix A suggests that such an approach will allow $\alpha$, $\xi$ and $\gamma$ to be constrained, leaving $\delta$ unresolved. Figure 7 shows the resulting evolution of fracture density and strike found by the inversion. Interestingly, crack density increases during each injection phase, but returns to a lower level. There are also minor decreases in crack density at around 12:30 and 13:00. The increase in crack density is most prominent after pumping stopped. Our preferred interpretation is that the additional crack density caused by injection is offset by compression of the surrounding rock and thus crack closure. Immediately after “shut-in,” the rock begins to relax as the fluid leaks off, causing the preexisting cracks in the surrounding medium to open or be more compliant, which in turn increases the anisotropy. This also occurs after pumping the main stage (near
Figure 5. Inversion results for the entire data set, with $\epsilon$, $\gamma$, $\delta$, fracture density and fracture strike as free parameters. Panel (a) shows the splitting observations (white outlined ticks) overlain on the splitting predicted by the best fitting model (black ticks and contours). Panels (b–k) show slices through the misfit hypercube at the best fit values of the parameters not plotted, i.e., (c) shows misfit as a function of fracture density and $\epsilon$ at best fitting values of $\gamma$, $\delta$, and fracture strike. In (b), fracture density is the radial coordinate, strike is polar angle. Other panels are as labeled.
hour 15). Further, the anisotropy (i.e., fracture density) is minimum during the main and most sustained pumping stage. Note that detected seismic activity also trails the pumping pressure. We suggest that the fracture density does not remain at the higher level because the proppant does not reach these cracks to keep them open and fluid leaks off, which reduces the volume and thus stress on the rock.

DISCUSSION

We observe temporal variation in shear-wave-splitting delay time and our inversion of anisotropy parameters indicates a variable fracture density in the rock mass. We interpret that as crack opening and closure, whose seismic energy emission is too small to be detectable by the monitoring array. This cracking occurs within the rock mass in a region beyond the hydraulic fracture corridor, the latter being delineated by observed microseismic events. Three scenarios may explain the different orientation of cracks and the hydraulic fracture corridor (Figure 8). First, the fractures and cracks oriented at N66°E represent a joint set formed by a paleo (or regional) stress field with the maximum compressive horizontal stress orientation $\sigma_H$ parallel to that direction. The current (or local) stress field is oriented parallel to the strike of the hydraulic fracture, which opens in a purely tensile mode. Nolen-Hoeksema and Ruff (2001) argue that overpressuring the rock by injection can result in such a deviation of the local stress field and leads to preferred fracture orientations deviating from the regional stress field. Second, the current and past stress field are oriented parallel to the (again tensionally opening) hydraulic fracture at N83°E and the fracture set at N66°E represents Riedel shears of a regional scale shearing process. The third alternative is a stress field $\sigma_H$ parallel to the observed fracture orientation of N66°E. The hydraulic fracture corridor forms then as en echelon failure (e.g., Pollard and Aydin, 1988) in a combination of tensile and shears fracturing oblique to the orientation of $\sigma_H$. If natural cracks and fissures exist already in the rock mass, they are likely to be reactivated due to a change in state of stress, even if they are not optimally oriented. These cracks join in an en echelon fashion and form a larger scale hydraulic fracture, whose orientation deviates from the dominant crack strike (Figure 8).

The combination of shear and tensile failure in the Cotton Valley hydraulic fracture experiment has been inferred from moment tensor inversions. Generally, composite focal mechanisms determined by Rutledge et al. (2004) indicate both left- and right-lateral strike-slip faulting with slip planes subparallel to the strike of the hydraulic fracture. More recent full moment tensor inversions by Šišlený et al. (2009) show evidence for a larger nondouble-couple component for events of the main fracture and more shear component for events of the off-strike cluster. In an earlier study, Urbancic et al. (1999) identified large b-values in the area of the cluster. Such observations have been related to multiple fractures along different orientations (Kagan, 1992), thus supporting the interpretation of complex and varied rupture mechanisms. In contrast to current models of a single fracture opening, it thus appears that complex rupture processes generate the fracture corridor. Such complex processes require full moment tensor inversions instead of double-couple focal mechanism analysis and may explain why our inferred dominant strike of cracks is oriented between the orientation of the composite focal mechanism of the reactivated fracture and that of the main fracture (Figures 4 and 5 and Rutledge et al., 2004, respectively).

Our shear-wave-splitting analyses shows that the observed anisotropy can in part be attributed to preexisting cracks in the rock.
Figure 8. Model of Cotton Valley fracturing process. Top: joints and cracks form parallel to the regional stress field. The hydraulic fracture corridor opens at an angle to that orientation by a mixture of tensile and shear failure. Potential mechanisms for fracturing are outlined in the lower three panels.

CONCLUSION

Shear-wave-splitting analysis of microseismic events from the Cotton Valley hydraulic fracture experiment shows that microseismic events can be used for reservoir characterization well beyond the source regions. Inversion of the shear-wave-splitting data reveals a nonzero Thomsen’s $\gamma$ parameter, implying anisotropy due to the sedimentary fabric, which is in agreement with observations from previous in laboratory studies. The inversion also indicates a dominant fracture orientation of $N66^\circ E$ within the reservoir, in agreement with regional borehole breakout measurements, but oblique to the induced hydraulic fracture trend. We attribute this to a series of en-echelon ruptures, which connect preexisting joints and cracks.

Our shear-wave-splitting results show an increase in anisotropy within a localized cluster of events, and we conclude that only a temporal increase in the activation of fissures and cracks (at subseismic detection levels) can cause the observed change in anisotropy. The activation must, however, occur within an elliptical halo around the main fracture. The halo radius is estimated to be between 85 and 127.5 m and may well account for a large amount of fluid loss. The fracturing of rock beyond the fracture corridor has wide implications for the effective permeability and thus efficiency of a hydraulic fracture treatment.

We have shown that shear-wave-splitting analysis broadens the utility of microseismic data that are routinely acquired during hydraulic stimulations. It provides a means of monitoring the extent, orientation, and magnitude of hydraulic fracture stimulation beyond the actual event locations. A temporal variation of fracturing in the rock matrix correlates well with pumping stages and the accompanying seismicity. The observation that fracture density returns to background level after each cycle implies that in this experiment, the connectivity of the cracks with the main fracture is small and proppant does not penetrate very far into the surrounding rock. The overall decrease in inverted fracture density can be attributed to crack closure due to compressive stresses following the fluid injection. This suggests that shear-wave-splitting analysis could provide a very useful tool to monitor the extent of the subsseismically stimulated reservoir volume and the placement of proppant into a hydraulic fracture.

ACKNOWLEDGMENTS

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APPENDIX A

STABILITY OF INVERSION

To assess the ability of the inversion procedure to accurately identify fracture strike and density, we present an analysis of error using synthetic data. The parameters controlling the elastic stiffness tensor, from which we forward model shear-wave splitting to be compared with observation, are the background P- and S-wave velocities ($V_p$, $V_s$), fracture strike ($\alpha$), fracture density ($\xi$), and Thomsen’s parameters $\epsilon$, $\gamma$, and $\delta$. The free parameters we use in our inversion are $\alpha$, $\xi$, $\gamma$, and $\delta$. Therefore, erroneous choices of $\epsilon$, $V_p$, and $V_s$, which must be fixed in the inversion, have the potential to influence the inversion results. We constrain $V_p$ and $V_s$ based on the velocity model used to compute event locations. Thomsen’s $\epsilon$ parameter may be estimated using controlled source seismic data or laboratory core measurements, but for our purposes may be considered to be unconstrained. The purpose of this error analysis is to estimate whether fixing $\epsilon$, $V_p$, and $V_s$ using erroneous values impedes us in constraining fracture parameters.

We generated an initial elastic model using as input parameters: fracture strike = $N70^\circ E$, $\xi = 0.04$, $\epsilon = 0.24$, $\gamma = 0.12$, $\delta = 0.06$, $V_p = 5000$ m/s, $V_s = 3000$ m/s. This model was illuminated using a randomly selected subset of 150 of the 1500 Cotton Valley observations, and the resulting shear-wave splitting was modeled. We add noise to the synthetically generated data set, using a uniform distribution within $\pm 10^\circ$ for arrival azimuth and inclination, $\pm 10^\circ$ to the fast shear-wave polarization direction, and $\pm 0.5\%$ to the percentage velocity difference between fast and slow shear waves. These noisy data were then inverted using erroneous
values for \(\epsilon, V_p, \) and \(V_S\), randomly selected from a uniform distribution within \(V_{\text{PS}} = V \pm (V \times 10\%)\), \(\epsilon = \epsilon \pm (\epsilon \times 100\%)\). The inversions using incorrect \(\epsilon, V_p, \) and \(V_S\) was repeated 100 times. The ability (or failure) of the inversion to recover the initial model parameters, despite erroneous choices for \(\epsilon, V_p, \) and \(V_S\), will show how well-constrained we might consider our results to be.

The results of this error analysis are shown in Figure A-1. The lower three panels show the erroneously fixed values of \(\epsilon, V_p, \) and \(V_S\). The upper four panels show histograms of the free parameters recovered by the inversion. The standard deviations of the recovered distributions are also given. We note that strike, \(\xi\) and \(\eta\) appear to be well-constrained, while \(\delta\) is unconstrained. The inaccuracies in \(\delta\) can be understood by considering Figure 5j, where it is apparent that \(\delta\) and \(\epsilon\) trade-off against each other. Therefore, an inaccurate choice of fixed \(\epsilon\) must cause an inaccurate value of \(\delta\) to be recovered. Laboratory measurements or controlled source seismic data may help to constrain \(\epsilon\), permitting accurate recovery of \(\delta\). However, as no such data is available here, \(\delta\) is unconstrained. Importantly, however, Figure A-1 shows that the lack of constraint in \(\delta\) does not contaminate our estimates of \(\alpha\) and \(\xi\). The results presented in Figure A-1 should inform our analysis presented above, showing the extent to which the parameters can be constrained.

![Figure A-1. Error analysis results. The upper four plots show histograms of the values recovered by the inversion, using the erroneous values of \(\epsilon, V_p, \) and \(V_S\) shown in the lower three panels. The dashed lines show the parameters used to generate the initial model. Fracture density and strike, and \(\gamma\), are well recovered, while \(\delta\) is unconstrained.](image-url)

### References


Numerical modeling of seismicity induced by fluid injection in naturally fractured reservoirs

Xueping Zhao¹ and R. Paul Young²

ABSTRACT

The interaction between hydraulic and natural fractures is of great interest for the energy resource industry because natural fractures can significantly influence the overall geometry and effectiveness of hydraulic fractures. Microseismic monitoring provides a unique tool to monitor the evolution of fracturing around the treated rock reservoir, and seismic source mechanisms can yield information about the nature of deformation. We performed a numerical modeling study using a 2D distinct-element particle flow code (PFC²D) to simulate realistic conditions and increase understanding of fracturing mechanisms in naturally fractured reservoirs, through comparisons with results of the geometry of hydraulic fractures and seismic source information (locations, magnitudes, and mechanisms) from both laboratory experiments and field observations. A suite of numerical models with fully dynamic and hydromechanical coupling was used to examine the interaction between natural and induced fractures, the effect of orientation of a preexisting fracture, the influence of differential stress, and the relationship between the fluid front, fracture tip, and induced seismicity. The numerical results qualitatively agree with the laboratory and field observations, and suggest possible mechanics for new fracture development and their interaction with a natural fracture (e.g., a tectonic fault). Therefore, the tested model could help in investigating the potential extent of induced fracturing in naturally fractured reservoirs, and in interpreting microseismic monitoring results to assess the effectiveness of a hydraulic fracturing project.

INTRODUCTION

Microseismic (MS) monitoring is an established technique used to monitor the effectiveness and response to a hydrofracture well simulation intended to improve the production of hydrocarbons from unconventional oil and gas reservoirs (Maxwell and Urbancic, 2001; Young and Baker, 2001; Ramakrishnan et al., 2009), enhance design of geothermal energy extraction in hot dry rock (Norio et al., 2008), and to assure long-term storage of greenhouse gas (Lumley et al., 2008; White, 2009). Due to the ubiquity of natural fractures, the interaction between hydraulic (induced) and natural fractures is of great interest for the energy resource industry because natural fractures can significantly influence the overall geometry and effectiveness of hydraulic fractures.

A considerable amount of research has been carried out in the past few decades trying to understand the complexity and mechanics of hydraulic fractures in fractured reservoirs. Blanton (1986) conducted scaled laboratory hydrofracturing experiments on naturally fractured Devonian shale and hydrostone. These experiments showed that hydraulic fractures crossed preexisting fractures only under high differential stress and high approaching angles, while at low differential stress and low angles of approach the existing fracture opened, diverting the fracturing fluid and at least temporarily preventing the induced fracture from crossing. Warpinski and Teufel (1987) conducted mineback experiments to study the effect of geological discontinuities on hydraulic fracture propagation. They derived a fracture interaction criterion to predict whether the induced fracture causes a shear slippage on the natural fracture plane, leading to the arrest of the propagating fracture, or whether it dilates the natural fracture, causing excessive leak-off. Potluri et al. (2005) analytically and numerically presented criteria to predict the manner in which a hydraulic

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fractures, depending on the differential horizontal stresses, the angle of approach, and the shear strength of the preexisting fracture. Observations similar to Blanton (1986) were made with respect to the influence of the differential stress conditions and angle of approach on whether the hydraulic fracture crossed or opened the preexisting fracture. Field work undertaken in naturally fractured formations reveal that the effects of natural fractures on fracture propagation are enhanced fluid leak-off, premature screen-out, arrest of the fracture propagation, formation of multiple fractures, fracture offsets, and high net pressures (Britt and Hager, 1994; Vinod et al., 1997; Rodgerson, 2000; Azeemuddin et al., 2002; Sharma et al., 2004).

Passive MS monitoring provides a unique tool to monitor the evolution of fluid injection around the treated rock reservoir, and seismic source mechanisms can yield information about the nature of deformation. However, field observations record only a portion of the activity, depending on the sensitivity of the monitoring system, and the high S-/P-wave amplitude ratio (a common situation in hydraulic fracturing) limits the availability of recorded MS data when using traditional arrival-time-based location methods (Zhao et al., 2010). Furthermore, with use of predominantly single monitoring wells, the failure source mechanisms of microseismicity induced by hydraulic fracturing operations are unresolved, and we are unable to confirm induced tensile fractures suggested by theory (Economides and Nolte, 2000; Akulich and Zyvagin, 2008) and shear failure along preexisting but isolated natural fractures (Rutledge and Phillips, 2003; Maxwell et al., 2010), although Šílený et al. (2009) concluded that double-couple shear failure need not be the dominant process in the microseismicity induced by hydrofracture treatments. In addition, there is a suggestion that hydrofracture growth may not be symmetric from the injection borehole (Fischer et al., 2008), which could be the result of the position of the monitoring system and its magnitude-distance detection threshold (Rutledge and Phillips, 2003) or caused by preexisting natural fractures (Sharma et al., 2004). For these reasons, numerical simulations could be a good choice to examine these effects, because in the model we could directly observe and analyze all mechanical simulations could be a good choice to examine these effects, because in the model we could directly observe and analyze all mechanical changes in domain volumes causing changes in domain pressures. Moreover, there are still issues that cannot be understood from the MS data alone, including the relationship between the fluid front and induced fracture, and the location and nature of any aseismic deformation.

A fully dynamic 2D distinct-element particle flow code model was tested with respect to simulating fluid injection into a reservoir containing a natural fracture, by comparing modeling geometries of hydraulic fractures and induced seismicity with actual results in laboratory experiments presented by Zhou et al. (2008) and fluid stimulation in a tight gas sand at Dowdy Ranch field, USA (Sharma et al., 2004). As would be expected, the 2D nature of the model limits direct comparisons with 3D field observations, such as fluid injection rate, seismic locations, or lateral and vertical hydrofracture growth. However, the model enables us to examine in detail the interaction between fluid pressure, rock deformation, and slip on existing fractures. The tested numerical models can help elucidate our understanding of the mechanics underlying seismicity, and can also provide insight on the relationship between seismicity, stress/damage, and the fluid front.

**METHODOLOGY**

**Particle flow code (PFC)**

The particle flow code in two dimensions (PFC2D) is a distinct element geomechanical modeling program in which the rock material is modeled as an assembly of rigid round particles (unit-thickness disks). Contacts between the particles are assigned normal and shear stiffnesses, together with tensile and shear strengths, to model deformable and breakable bonds according to the relations described by Potyondy and Cundall (2004). Normally, the specified contact microstiffnesses influence the global macro stiffness (Young’s modulus) of the rock being modeled. The ratio of normal-to-shear stiffness influences the Poisson’s ratio of the rock. The measured macroscopic strength can be calibrated either to unconfined compressive strength or to Brazilian indirect tensile strength. The microstiffness and microstrength of particles can then be adjusted to reproduce realistic macrorock behavior. Under the applied load, if the maximum tensile or shear stress acting on a bond exceeds the tensile or shear strength of the bond, then the bond breaks, resulting in a tensile or shear crack, respectively. By further generation of these microcracks, a fracture can develop from the linking of individual microcracks. PFC has been applied to study mechanical behavior of sandstones, granites, and other rocks under different stress conditions with much success, including thermal fracturing experiments (Wanne and Young, 2008), hydraulic fracturing (Al-Busaidi et al., 2005), seismic velocities (Hazzard and Young, 2004b), in-situ failure tests (Potyondy and Autio, 2001), and large-scale underground excavations (Cai et al., 2007). A thorough description of the PFC model for rocks is given by Potyondy and Cundall (2004).

According to the studies by Potyondy and Cundall (2004), the particle size is an intrinsic part of the material characterization that affects the Brazilian indirect tensile strength (σ). Thus, particle size cannot be regarded as a free parameter that only controls model resolution; instead, it affects the fracture toughness. Potyondy...
and Cundall (2004) noted that the fracture toughness ($K_{f}$) scaling goes by the product of the tensile strength ($\sigma_t$) and the square root of the grain radius ($r$) for real rocks. Therefore, for a sample in the DEM, we have

$$K_{f} \propto \sigma_t \sqrt{r}. \quad (1)$$

**Synthetic seismicity in PFC**

PFC uses an explicit approach to solve the equation of motion. This allows a dynamic simulation in which seismic waves propagate out from new faults and fractures. Each bond breakage is assumed to be a microcrack. The crack location is assumed to be the contact between the two particles, and the orientation of the crack is assumed to be perpendicular to the line joining the two centers. Microcracks occurring closely in both space and time are considered a single seismic event, if the models are run dynamically, by specifying low levels of numerical damping that simulates realistic levels of attenuation in the rock (Hazzard and Young, 2004a). Seismic source information can therefore be calculated for seismicity.

As described by Hazzard and Young (2002), when a bond breaks, the two particles on either side of the crack (the source particles) will move and contacts surrounding the source particles will experience some deformation. There will therefore be a force change at the surrounding contacts due to the formation of the crack. A summation is then performed around the contacts surrounding the crack to calculate components of the moment tensor from the contact locations and force changes. The moment tensor can be decomposed into isotropic and deviatoric components to help analyze the nature of the event as well as the moment magnitude (Feignier and Young, 1992). Furthermore, the moment tensor is calculated for each step time over the duration of the event. The duration of an event is determined by assuming that a shear fracture propagates at half the shear wave velocity of the rock (Madariaga, 1976) to the edge of the source area (one particle diameter). If another crack forms adjacent to the active crack such that the source areas overlap within the duration of the event, then more realistic magnitude distributions are achieved. The event centroid is assumed to be the location of the previously bonded contact. If more than one bond breakage makes up the event, then the geometrical center of the event is used. The occurrence time of the event is presumed to be the time of maximum scalar moment (Hazzard and Young, 2002). An example of a synthetic seismic event is illustrated in Appendix A.

**Fluid flow modeling in PFC**

A technique for simulating fluid flow in PFC is adapted from the algorithm by Cundall (2000). Cundall’s fluid flow is simulated by assuming that each particle contact is a flow channel (pipe) and that these channels connect small reservoirs that store some fluid pressure. As shown in Figure 1, the fluid network topology is generated by drawing lines between the centers of all particles in contact. This creates a series of enclosed domains. The center of each of these domains is stored as a reservoir. The reservoirs are then connected by pipes. Each particle contact corresponds to one pipe with a length related to two particle radii at the contact. Therefore, each reservoir is surrounded completely by contacts and has some volume (void space in an enclosed domain) associated with it. A contact in which the particles are just touching is assumed to have residual aperture that allows fluid to flow. This is thought to be a realistic assumption since laboratory studies have shown that rock faces in contact always exhibit some residual aperture (Cook, 1992). When the contact is in compression, the aperture decreases as a function of the normal force at the contact. A thorough description of the mathematical equations for the fluid network in PFC was given by Al-Busaidi et al. (2005) and Zhao and Young (2009).

For a 2D fluid network model, fluid flow through a pipe is approximated by laminar flow through parallel plates with some aperture associated with it. The rate of volumetric flow ($Q$) is controlled by the Darcy’s law (Bear, 1972),

$$\frac{\Delta P}{\Delta t} \approx \frac{K_f}{\nu} \left( \sum Q \Delta t - \Delta V \right), \quad (2)$$

The second term in the equation represents the mechanical change in volume of the domain. Hazzard et al. (2002) and Al-Busaidi et al. (2005) considered the mechanical term negligible and did not include it in the calculation of domain pressure changes. However, this may not be the case in reality because the pressure variation caused by domain volumetric change could be significant especially for the areas with induced and natural fractures. Therefore, in this paper we use equation 2 in its entirety to enable the full hydro-mechanical coupling. Furthermore, to mimic a natural fracture in the model: Both normal and shear bonds are weakened compared with their intact strength along the line of the preexisting fracture, and residual apertures of the pipes along the line of the

![Image](https://example.com/image.png)

**Figure 1.** The fluid network modelled in PFC$^{2D}$, including solid particles (light orange circles), contacts (black lines), flow pipes (green lines), and fluid domains (solid red circles).
preexisting fracture are increased, resulting in an increased fracture permeability.

**HYDRAULIC FRACTURE MODELING AT THE LABORATORY SCALE**

Zhou et al. (2008) conducted a series of servo-controlled triaxial fracturing experiments to study the hydraulic fracture propagation behavior and fracture geometry in naturally fractured cubic cement-sand blocks of 300 mm. Figure 2 shows the schematic plan view of a hydraulic fracture intersecting a preexisting fracture for the laboratory experiments under a normal-faulting stress regime. The fluid pressure is applied in the center wellbore. Along the direction of hydraulic fracture propagation in the far field, parallel to the direction of maximum horizontal stress \( \sigma_3 \) (minimum horizontal stress), the fracture intersects with a single closed natural fracture with an angle of approach \( \theta \). According to the Mohr-Coulomb criteria (Jaeger et al., 2007), slip occurs if the shear stress \( \tau \), resulting from the normal stress \( \sigma_n \) and pore pressure acting on the plane of the natural fractures, is higher than the shear stress encountering the Mohr-Coulomb failure envelope.

Zhou et al. (2008) used three types of paper (rice paper, printer paper, and wrapping paper) with different thicknesses incorporated into cubic blocks to simulate natural fractures, and then tested under true triaxial loading. The friction coefficient and cohesion of the preexisting fracture were obtained from direct shear tests, which can be used to estimate the shear strength of the preexisting fracture, as well as to compare it to the unconfined compressive strength of the sample to find the strength ratio between the fracture and the intact rock. After simulating a hydraulic fracture, they observed three interaction types (cross, dilate, and arrest) between the induced fracture and preexisting fracture depending on the horizontal differential stress \( \Delta\sigma = \sigma_3 - \sigma_2 \), angle of approach \( \theta \), and shear strength of the preexisting fracture.

In this paper, PFC is utilized to model the fluid stimulation based on the results from Zhou et al. (2008), specifically where printer paper was used to form the fracture. These are then examined in detail to provide insights into the mechanism underlying the induced fractures through the synthetic seismicity generated.

**Model calibration**

First, a set of biaxial and Brazilian tests was run to match the macroproperties of the laboratory rock, such as strengths and elastic properties, following the routine described by Potyondy and Cundall (2004). Table 1 lists the best fit parameters used for hydraulic fracturing models. For simplicity, the symbols in Table 1: \( r \), \( L_d \), \( L_h \), \( E \), \( \nu \), \( \sigma_c \), \( \sigma_t \), and \( q_i \) are the mean grain radius of sample, distance between the wellbore and the center of preexisting fracture, half-length of preexisting fracture, thickness of preexisting fracture, Young’s modulus, Poisson’s ratio, unconfined compressive strength, tensile strength, and fluid injection rate, respectively. To reduce the computation time, the sizes of particles in the model do not represent the actual grain sizes and in total there are 27,379 bonded particles composing the 300 × 300 mm sample. Accordingly, Figure 3 shows the naturally fractured laboratory reservoir model. Table 1 also provides the best fit model results, referring to the actual laboratory values, which indicates that the macroproperties \( (E, \nu, \text{and } \sigma_t) \) are well reproduced by the PFC2D model.

Note that the injection rate used in the model is much larger than the actual value, which is due to the fact that the current coarse model has a larger particle size than the actual grain size listed in Table 1. According to equation 1, the toughness of the current model is within an order of magnitude larger than that of the actual rock. As a result, the injection rate in the PFC model cannot be easily related to the actual injection rate because of higher fracture toughness and the 2D nature of the model, so a rate was chosen that was fast enough to induce fracturing and

![Figure 2. Schematic view of a hydraulic fracture intersecting a preexisting fracture (after Zhou et al., 2008).](image)

**Table 1. Best-fit parameters for lab hydraulic fracturing tests and PFC model calibration results.**

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<th>Model</th>
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</tr>
<tr>
<td></td>
<td>( \nu )</td>
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<td></td>
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<td>( \sigma_t ) (MPa)</td>
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<td></td>
<td>( L_h ) (mm)</td>
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<td></td>
<td>( L_s ) (mm)</td>
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<tr>
<td>Fluid</td>
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</tr>
</tbody>
</table>

\( a \)From Zhou et al. (2008).

\( b \)Values are unknown.
result in reasonable model run times and comparable synthetic seismicity, but slow enough to maintain stability.

Model results and discussion

For a series of hydraulic fracturing experiments on blocks with the printer paper as the preexisting fracture, Zhou et al. (2008) tested three interaction angles (θ = 30°, 60°, and 90°) and four combinations of confining stresses (σ2 − σ3 = 13 MPa − 3 MPa, 10 MPa − 3 MPa, 10 MPa − 5 MPa, and 8 MPa − 5 MPa), i.e., the horizontal differential stress Δσ = 10 MPa, 7 MPa, 5 MPa, and 3 MPa). They found that the hydraulic fracture could cross the preexisting fracture, or be arrested by either the opening and diverting or shear slippage of the preexisting fracture, in response to the fluid flow redirected along the preexisting fracture. As shown in Figure 4 from the laboratory results, hydraulic fractures crossed the preexisting fracture only at high horizontal differential stress and angles of approach of 60° or higher; hydraulic fractures did not cross the preexisting fracture at low horizontal differential stress or low angles of approach, primarily because the fluid pressure in the hydraulic fracture was sufficient to open and divert fluid along the preexisting fracture. Moreover, hydraulic fractures were arrested by the shear slip along the preexisting fracture only at high differential stresses and at angles of approach of 30° (Zhou et al., 2008). These results are consistent with Blanton’s (1986) and Warpinski and Teufel’s (1987) experimental results.

Based on the parameters shown in Table 1, PFC was used to numerically model the laboratory hydraulic fracturing tests. Figure 4 also summarizes the modeling results including the resultant cracks, induced seismicity, and calculated moment tensors shown in Figures 5, 6, 7, 8, 9, 10, 11, 12, and 13 after the fluid injection of around 3 s. In addition, four more models at different Δσ and θ as well as the laboratory experiments by Zhou et al. (2008), were run to test the crossed and dilated trends shown in Figure 4. As shown in Figure 2, for each model the confining stresses were applied along the same directions. The height of each subfigure was less than 300 mm after being clipped, since the cut sections were not of interest here. The process and mechanics of hydraulic fracturing and the interaction between the preexisting and hydraulic fractures are discussed in detail below.

By virtue of the modeling, we can examine in detail the propagation of cracks outside and within the preexisting fracture and the evolution of induced seismicity and its mechanism. In Figure 5a, at Δσ = 10 MPa the induced fractures were clearly arrested by the preexisting fracture and higher fluid pressure was accumulated in front of the preexisting fracture. A large number of shear cracks were produced along the preexisting fracture. Compared to induced seismicity in Figure 8a and the moment tensors in Figure 11a, Figure 14 shows only shear events and corresponding moment tensors after the fluid injection of about 4 s. At the interaction region between preexisting and hydraulic fractures (Fig. 14), shear events were triggered and propagated along the preexisting fracture, but seldom ran through the fracture, which confirms that the fracture arrest was caused by the shear slip of the preexisting fracture. This was also the mechanical reason for the much lower fluid pressure within the preexisting fracture. Interestingly, Figure 14 also illustrates that shear events accompanied the propagation of hydraulic fractures. The fracture tip tends to be ahead of the fluid front in the current case. In general, as shown in Figures 5, 6, and 7, the higher the Δσ, the higher the likelihood of the fracture tip moving ahead of the fluid front.

Sometimes the fracture arrest and dilation are not easy to distinguish, since both could arrest the hydraulic fractures, at least temporally. Nevertheless, the dilation usually results in fluid flow

Figure 4. Laboratory (smaller solid symbols) (Zhou et al., 2008) and model (larger symbols) results showing the hydraulic fracture behavior for the printer paper as the preexisting fracture. Δσ and θ are the horizontal differential stress and the angle of approach. The crossed and dilated tendency lines were from Zhou et al. (2008) who estimated these based on their laboratory testing results. Four additional models, i.e., Δσ = 10 MPa at θ = 90°, Δσ = 7 MPa at θ = 30°, Δσ = 5 MPa at θ = 60°, and Δσ = 3 MPa at θ = 90°, were run to test the crossed and dilated tendency lines.
along the preexisting fracture or a high-pressure accumulation around preexisting fracture. The evolution of seismicity, as shown in Figures 8, 9, and 10, was used to ascertain whether the hydraulic fractures were arrested temporally. As shown in Figures 5c, 5d, 8c, and 8d, the interaction types for \(\Delta \sigma = 5 \text{ MPa} \) and 3 MPa at \(\theta = 30^\circ\) are thus regarded as the fracture dilation, since the induced seismicity propagated along the preexisting fracture for a certain period of time after the hydraulic fractures interacted with the preexisting fracture. More importantly, there was a significant amount of accumulated fluid pressure around the preexisting fracture, compared to the situation of \(\Delta \sigma = 10 \text{ MPa} \) at \(\theta = 30^\circ\).

In the case of dilation, for \(\Delta \sigma = 5 \text{ MPa} \) at \(\theta = 30^\circ\), the preexisting fracture was opened by the large tensile force at the interaction between the preexisting fracture and induced fracture shown in Figures 8c and 11c. The high fluid pressure in Figure 5c and 5d dominated a much larger region than the case of the fracture arrest in Figure 5a, which may result from the increasing leak-off into the preexisting fracture as mentioned by Warpinski (1991). Moreover, the hydraulic fracture was arrested temporarily by opening and dilating the preexisting fracture, which agrees with the interaction criterion described by Potluri et al. (2005). Similar to the case of \(\Delta \sigma = 3 \text{ MPa} \) at \(\theta = 60^\circ\) in Figures 6d.

Figure 5. Induced cracks (marked by black/red lines corresponding to tensile/shear cracks) and fluid flow (marked by light blue circles whose sizes are scaled to 40 MPa) under different \(\Delta \sigma \) at \(\theta = 30^\circ\) after an injection of about 3 s. The width of each subfigure is 300 mm. The injection point is located at the center of each subfigure that is the same as Figure 2. The light gray solid circles are particles within the preexisting fracture.

Figure 6. Induced cracks and fluid flow under different \(\Delta \sigma \) at \(\theta = 60^\circ\) after an injection of about 3 s. The marks have the same meanings as Figure 5.
and 9d, the hydraulic fracture was arrested temporarily by opening and dilating the preexisting fracture, which agrees with the interaction criterion described by Potluri et al. (2005). On the other hand, for $\Delta \sigma = 3$ MPa at $\theta = 30^\circ$ in Figures 5d and 8d, the hydraulic fractures were almost arrested along the direction of the maximum horizontal stress by opening and dilating the preexisting fracture with a significant resultant fluid flow around the preexisting fracture. In this case, a branch of hydraulic fractures finally detoured nearly $90^\circ$ away from the lower end of the preexisting fracture. As shown in Figure 8d, the occurrence times of the corresponding seismicity were directly affected by the distance between the injection center and the lower edge of the preexisting fracture. As a result, the fluid pressure, as well as fractures and seismicity, was accumulated earlier close to the lower boundary of the preexisting fracture.

In the case of crossing, the preexisting fracture can be easily bisected by a single hydraulic fracture when $\Delta \sigma = 10$ MPa and 7MPa at $\theta = 90^\circ$, as shown in Figure 7a and 7b. Hydraulic

**Figure 7.** Induced cracks and fluid flow under different $\Delta \sigma$ at $\theta = 90^\circ$ after an injection of about 3 s. The marks have the same meanings as Figure 5.

**Figure 8.** Induced seismicity for different $\Delta \sigma$ at $\theta = 30^\circ$ after an injection of about 3 s. The width of each subfigure is 300 mm. The sizes of seismic events are scaled according to their magnitudes and the color corresponds to the occurring time of seismic events (green/red = early/late). The central injection point is indicated by the black filled circle.
fractures can break out of the preexisting fracture with some offsets when $\Delta \sigma = 10$ MPa, 7 MPa, and 5 MPa at $\theta = 60^\circ$, as well as $\Delta \sigma = 7$ MPa at $\theta = 30^\circ$, as shown in Figure 6a and 6c and Figure 5b. Moreover, multiple fractures can be induced and fractures seem to propagate through the tip of the preexisting fracture when $\Delta \sigma = 5$ MPa and 3 MPa at $\theta = 90^\circ$, as shown in Figure 7c and 7d. These three modeled crossing types are also consistent with the three possible modes of fracture growth described by Potluri et al. (2005). In addition, the higher the $\Delta \sigma$, the fewer the multiple branches for induced fractures.

Figure 9. Induced seismicity for different $\Delta \sigma$ at $\theta = 60^\circ$ after an injection of about 3 s. The marks have the same meanings as Figure 8.

As shown in Figures 8, 9, and 10, a significant number of seismic events form away from the high-pressure areas in each case, especially with lower $\Delta \sigma$. As indicated by the times at which seismic events occurred, it appears that hydraulic fractures simultaneously propagate in opposite directions, away from the central injection area. After induced fractures reach the preexisting fracture, it seems that the interaction between the natural and induced fractures dominates the process. There is more seismicity triggered close to the preexisting fracture, due to the accumulation of higher fluid pressures. Note that the asymmetrical distribution of seismicity may result from the preexisting

Figure 10. Induced seismicity for different $\Delta \sigma$ at $\theta = 90^\circ$ after an injection of about 3 s. The marks have the same meanings as Figure 8.
fracture. Furthermore, the moment tensors shown in all cases reveal that the hydraulic fractures are opened subparallel to the minimum horizontal principal stress outside the preexisting fracture and subperpendicular to the preexisting fracture within the preexisting fracture. At the same time, the forces capable of crossing the preexisting fracture are generally much smaller than those cracking particles outside the preexisting fracture, except for the fracture arrest case. As a whole, the interactions between the preexisting and hydraulic fractures are well captured by the PFC models, and the model results provide detailed insights into the mechanism. Additionally, the modeling results suggest that the dilated and crossed tendency lines described by Zhou et al. (2008) need to be adjusted and the region of arrested behavior possibly decreased.

HYDRAULIC FRACTURE MODELING AT THE FIELD SCALE

As shown in the laboratory and corresponding modeling results, natural fractures can have a significant impact on fracture growth. This influence is more complex in the field because natural fractures vary across scales from a joint network to fault networks, although

Figure 11. Moment tensors of induced seismicity corresponding to Figure 8 under different $\Delta \sigma$ at $\theta = 30^\circ$. $\sigma_2$ is the maximum horizontal stress. The light gray solid circles are particles within the preexisting fracture, and the width of each subfigure is 300 mm. Note that moment tensors are plotted as equivalent forces, so that two sets of arrows of equal length but opposite polarity represent a perfect double-couple (shear) source.

Figure 12. Moment tensors of induced seismicity corresponding to Figure 9 under different $\Delta \sigma$ at $\theta = 60^\circ$. The marks have the same meanings as Figure 11.
they are isolated or discrete. The analysis at the field scale presented here is based on the data sets of the Bonner layer stimulation in the Dowdy Ranch field, East Texas (Sharma et al., 2004). As shown in Figure 15, owing to the preexisting fault, the fracture growth was asymmetrical and arrested in the Bonner layer, but the Bonner treatment was observed to have communicated upward into the Moore and Bossier Marker sands through a fault (Sharma et al., 2004), resulting in a significant amount of out-of-zone fracture height growth as indicated by the migration of seismicity in space and time. Furthermore, propped or effective fracture half-length, derived from pressure build-up analysis and history matching, were significantly shorter than the created fracture half-length deduced from MS locations (Sharma et al., 2004). Since seismic events were recorded only by a single array of receivers in the field, it is difficult to solve for the seismic source mechanism (Fischer et al., 2008). Fortunately, we can examine the mechanics and mechanism by the direct observation of the numerical model.

Model calibration

Following a similar procedure to the previous laboratory hydraulic fracturing model, the calibrated bonded particle model was created to simulate a 2D slice of the Bonner reservoir approximately 3974 m below the surface (the starting position for the fluid treatment). The 500 x 500 m model was made up of 13,880 particles with an average radius of 2 m to optimize calculation time. Clearly each particle does not represent a single mineral grain in the rock, and the particles are simply a way to discretize the medium. A physical interpretation of the particles might be blocks of sandstone separated by cohesive joints or planes of weakness.

The best-fit parameters and calibration results are shown in Table 2. Note that effective stress is assumed so that the applied stresses (σ2 and σ3) equal the actual in situ stresses, minus the pore fluid pressure. The tensile strength was calculated by use of the method described by Eaton (1975) when we estimated in-situ stresses and pore pressures assuming the overburden pressure gradient 22.6 KPa/m (1 psi/ft). In addition, estimated from the microseismic mapping result shown in Figure 15, the preexisting fault was created assuming θ = 120°, Ld = ∼50 m, Lh = ∼43 m, and Lt = ∼5.7 m. According to equation 1 the toughness of the sample is about one order of magnitude larger than that of the real rocks for the current coarse model. The resultant higher injection rate in the model is attributed to the same factors as in the laboratory models mentioned in the laboratory model.

Figure 13. Moment tensors of induced seismicity corresponding to Figure 10 under different Δσ at θ = 90°. The marks have the same meanings as Figure 11.

![Figure 13](image1)

Figure 14. Induced shear events and corresponding moment tensors for Δσ = 10 MPa at θ = 30° after an injection of about 4 s. The width of the subfigure is 300 mm. (a) Seismicity. The sizes of seismic events are scaled according to their magnitudes and the color corresponds to the occurring time of seismic events (green/red = early/late). (b) Moment tensors corresponding to (a).

![Figure 14](image2)
Model results and discussion

Due to limitations in the microseismic monitoring configuration used in the field, which was restricted to the deployment of a string of geophones in a single borehole, the resulting data could not fully resolve source location and source mechanism information. However, through direct examination of the calibrated PFC model, the mechanics underlying the recorded MS events can be projected. Figure 16 shows the total induced MS events and associated moment tensors, cracks, and fluid flow after about a 3-hour fluid injection in the model. Compared to Figure 15b, Figure 16 illustrates that the PFC model produces a scope and orientation of induced MS events similar to results obtained from the actual recorded MS events, and that the range of moment magnitudes in the model (−1.51–0.61) is within an order of magnitude of the actual events recorded at Bonner (−0.93–1.15, Pettitt and Young, 2007). This gives some confidence that the model is behaving in a realistic manner in consideration of the 2D nature of the current model.

Table 3 summarizes the results of the hydraulic fracturing of the naturally fractured field reservoir after about a 3-hour fluid injection. The tensile events dominated the propagation of hydraulic fracturing and the shear events only account for about 25% of the total induced MS. However, there are generally not enough events in the test to consider the statistics of the magnitude distributions. The reasons for the low number of MS events are twofold: First, the current model has a very low resolution; second, for the current synthetic seismic algorithm, once a bond along the preexisting fracture is broken, no more cracking or seismicity will be recorded, although unstable slip can occur and energy can be released (Marone, 1998), which could also result in a lower magnitude in the model as shown in the result above, and this must be accounted for in future developments.

A certain number of MS events seem to propagate out of the trajectory through the upper side of the fault, due to the opening of the fault. As a result, the hydraulic fracture was arrested initially, but extended further from the tip of the preexisting fracture. The synthetic moment tensor result is used to illustrate the mechanics underlying the recorded MS event. As shown in Figure 17, the significant tensile event in Figure 16c actually includes a large amount of tensile and shear cracks within and outside the fault, which results in the opening and dilation of the preexisting fault. From this dilation and the fluid leakage as shown in Figure 16, we could expect the upper and lower layers were communicated through the preexisting fault, resulting in out-of-zone seismicity in the field. Furthermore, the induced cracks appeared to be ahead of the fluid front, indicating that there is a fluid lag between the fracture tip and the fluid front. Since a tip screenout did not occur during the hydraulic fracturing of Bonner sand (Sharma et al., 2004), the fluid lag together with the fluid leak-off imply that the effective or propped half-length is shorter than the generated fracture half-length deduced from MS locations. This agrees with the results derived from pressure buildup and history matching production data during the multistage fracturing treatment in the Bossier formation (Sharma et al., 2004). In addition, the interaction type between

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</table>

*Values are unknown.*
hydraulic and natural fractures is also located below the possible dilated tendency for the current $\Delta \sigma = 3.3$ MPa, as shown in Figure 4. Therefore, these matches strongly suggest that the model is behaving in a realistic way, and that levels of deformation and associated energy release are similar in the model and in the field. Because of the 2D nature of the model, it can only qualitatively capture the propagation of the out-of-zone seismicity recorded in the field. If a 3D model were run, a direct measurement of fracture volume and fluid leak-off calculation could be realized.

Table 3. Hydraulic fracturing modeling results for the Bonner treatment.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cracks (normal/shear)</td>
<td>263 (228/35)</td>
</tr>
<tr>
<td>Number of induced MS events</td>
<td>52</td>
</tr>
<tr>
<td>Number of shear MS events</td>
<td>14</td>
</tr>
<tr>
<td>Number of explosive/implosive MS events</td>
<td>31/7</td>
</tr>
<tr>
<td>Magnitude of MS (Min/Max)</td>
<td>$-1.52/0.61$</td>
</tr>
</tbody>
</table>

Figure 16. Induced cracks, fluid flow, MS, and moment tensors after an injection of about 3 hours in the field model. The fluid injection point is indicated by the black filled circle at the center of each subfigure, the width of each subfigure is 500 m, and the axis values are in meters. (a) Induced cracks (marked by black/red lines corresponding to tensile/shear cracks) and fluid flow (marked by light blue circles whose sizes are scaled to 40 MPa). (b) Synthetic MS events. The sizes of seismic events are scaled to the maximum magnitude of 0.61 and the color corresponds to the occurring time of seismic events (green/red = early/late). (c) Moment tensors corresponding to (b). The seismic moment is scaled to the maximum of $8.2 \times 10^9$ N·m.

Figure 17. The tensile seismic event composed of 147 cracks corresponding to the largest event shown in Figure 16c. (a) Particle velocities (green arrows) after bond breakages. The crack is represented by a subvertical line (black for tensile and red for shear) between the two source particles. (b) The calculated moment tensor.
CONCLUSIONS

We have successfully tested a 2D discrete-element particle flow code model with fully dynamic and hydro-mechanical coupling, to simulate fluid stimulation on a reservoir containing a natural fracture by directly comparing the modeling geometries of hydraulic fractures and induced seismicity with actual laboratory and field data results. The numerical results qualitatively agree with the laboratory and field observations and confirm the possible mechanics of new fracture development and their interaction with natural fractures.

On the laboratory scale, the model captured three interaction types between preexisting and induced fractures. Depending on the differential stresses and orientations of the preexisting fracture, the model also verified three possible crossing modes of fracture growth. Through seismic mechanism analysis, the models confirmed that fracture arrest can be caused by the shear slippage of the preexisting fracture and that shear slippage can explain lower fluid pressures within the preexisting fracture. At the same time, the hydraulic fracture can also be arrested temporarily by opening and dilating the preexisting fracture, which may result in increased leak-off into the preexisting fracture. Furthermore, the less the differential stress, the more the fracture segments and the less the likelihood that the fracture tip will go ahead of the fluid front.

On the field scale, the model produced an extent and orientation of induced fractures similar to the results obtained in the actual recorded microseismicity. Moreover, the model showed the resultant fluid lag between the fracture tip and the fluid front and provided strong support for the hypothesis that the hydraulic fractures were arrested by the dilation of the fault. At the same time, the leakage of a large volume of fluid through the fault area was qualitatively predicted by the 2D model. This model observation was congruent with the engineering assessment that the propped half-length is shorter than the created fracture half-length as deduced from MS locations. The model is consistent with filed observations and can assist in the interpretation both of the micromechanism underlying the induced fracture and of the relationship between the induced seismicity and the fluid front through direct observation of the model.

Admittedly, the model is still a gross simplification of the actual rock mass conditions. The low resolution, the 2D nature of the models, and the shortage of actual parameters, all particularly limit the possible quantitative comparisons with actual data. However, through the use of synthetic seismicity, moment tensors, and fractures, the current model clearly demonstrates not only the reasonable physical mechanics underlying the interaction between hydraulic and natural fractures, but also the relationship between the fluid front and the fracture tip, all of which are difficult to ascertain from field data. If a higher resolution was used with further development in the seismic monitoring algorithm to account for unstable slip along the preexisting fracture, then the model would produce a realistic injection rate and more synthetic seismicity for better comparisons with field data. Moreover, modeling results in 2D may not be fully representative of fluid properties (fluid volume and injection fluid rate) and, thus, of the total number of induced microcracks. Three-dimensional modeling is more realistic, allowing the use of a realistic fluid injection rate and examination of both lateral and vertical hydrofracture growth, especially for reservoirs with natural fractures, such as the Bonner treatment.

ACKNOWLEDGMENTS

The authors would like to acknowledge Applied Seismology Consultants and the Halliburton company for providing the hydraulic fracturing data sets. The authors would also like to thank Dr. Jim Hazzard for providing the basic PFC2D fluid functions. A further acknowledgement goes to the four reviewers, and to the volume editor, whose comments have greatly improved this paper.

APPENDIX A

AN EXAMPLE OF A SYNTHETIC SEISMIC EVENT

According to the definitions by Feignier and Young (1992), an event will be considered predominantly implosive if the isotropic component is negative and makes up greater than 30% of the moment tensor. Similarly, an explosive event will be greater than 30% isotropic (positive). All other events are considered predominantly deviatoric (shear), i.e., its isotropic component is less than 30% of the moment tensor.

![Figure A-1. An example synthetic seismic shear event composed of seventeen tensile cracks and one shear crack. (a) Particle velocities (green arrows) and cracks (black line for tensile crack and red line for shear crack); (b) the calculated moment tensor.](image)
Figure A-1 shows an example of a synthetic seismic shear event resulting from a shear and several tension bond-breakages. The calculated moment tensor is shown in Figure A-1b. The moment tensor is plotted as equivalent forces (i.e., two arrows pointing away from each other indicate a tensile event, while two sets of arrows of equal length pointing in opposite directions indicate a double-couple, the signature of a shear event). The representation of the moment tensor demonstrates the principal values (eigenvalues) of the moment tensor matrix as sets of arrows whose direction and length indicate the orientation and magnitude, respectively, of the principal values. In addition, the PFC particle velocities show rapid shear motion.

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Investigation of injection-induced seismicity using a coupled fluid flow and rate/state friction model

Mark W. McClure¹ and Roland N. Horne¹

ABSTRACT

We describe a numerical investigation of seismicity induced by injection into a single isolated fracture. Injection into a single isolated fracture is a simple analog for shear stimulation in enhanced geothermal systems (EGS) during which water is injected into fractured, low permeability rock, triggering slip on preexisting large scale fracture zones. A model was developed and used that couples (1) fluid flow, (2) rate and state friction, and (3) mechanical stress interaction between fracture elements. Based on the results of this model, we propose a mechanism to describe the process by which the stimulated region grows during shear stimulation, which we refer to as the sequential stimulation (SS) mechanism. If the SS mechanism is realistic, it would undermine assumptions that are made for the estimation of the minimum principal stress and unstimulated hydraulic diffusivity. We investigated the effect of injection pressure on induced seismicity. For injection at constant pressure, there was not a significant dependence of maximum event magnitude on injection pressure, but there were more relatively large events for higher injection pressure. Decreasing injection pressure over time significantly reduced the maximum event magnitude. Significant seismicity occurred after shut-in, which was consistent with observations from EGS stimulations. Production of fluid from the well immediately after injection inhibited shut-in seismic events. The results of the model in this study were found to be broadly consistent with results from prior work using a simpler treatment of friction that we refer to as static/dynamic. We investigated the effect of shear-induced pore volume dilation and the rate and state characteristic length scale, $d_c$. Shear-induced pore dilation resulted in a larger number of lower magnitude events. A larger value of $d_c$ caused slip to occur aseismically.

INTRODUCTION

Overview

Enhanced geothermal systems (EGS) are characterized by the use of hydraulic stimulation to increase flow rate in high temperature, low productivity wells, most often located in crystalline rock. Water is typically injected at high pressure without proppant. In most cases, the elevated fluid pressure triggers slip on preexisting fractures. The process of using elevated fluid pressure to trigger fracture slip is often referred to as “shear stimulation.” When the fractures slip, their permeability is permanently enhanced, and well productivity can be improved by an order of magnitude or more (Tester, 2007).

An important challenge for the deployment of EGS is that shear stimulation triggers microseismicity, low magnitude seismic events that sometimes can be felt at the surface (Majer et al., 2007). Induced seismicity threatens public acceptance of EGS, and the possibility of triggering a truly damaging seismic event, while seemingly remote, deserves careful consideration. In 2005, one of the largest seismic events ever associated with an EGS project, a magnitude 3.4 earthquake, occurred following the hydraulic stimulation of an EGS well in Basel, Switzerland. As a result, the project was suspended and eventually canceled (Häring et al., 2007; Majer et al., 2007). Events strong enough to be felt at the surface have occurred during stimulations at several other EGS projects, including at Soultz-sous-Forêts, France, magnitude 2.9, and Cooper Basin, Australia, magnitude 3.7 (Majer et al., 2007).

There is a practical need for credible shear stimulation modeling because shear stimulation directly impacts induced seismicity,
well productivity, and the long term temperature decline of the system. Shear stimulation and induced seismic modeling have applications in areas outside of EGS, including gas shale hydraulic fracturing and carbon dioxide sequestration.

Shear stimulation models typically include (1) stochastic or deterministic generation of a preexisting fracture network, (2) simulation of fluid flow in the network, and (3) modeling of induced slip (Willis-Richards et al., 1996; Rahman et al., 2002; Ghassemi and Tarasovs, 2006; Kohl and Mégel, 2007; Bruehl, 2007; Baisch et al., 2010; McClure and Horne, 2010; Rachez and Gentier, 2010; Deng et al., 2011).

Significant issues remain in the development of shear stimulation modeling. The complexity of the physical processes taking place and the uncertainty and incompleteness of the data forces modelers to make significant simplifying assumptions. Two physical phenomena that are frequently subject to simplifying assumptions are fracture friction and stresses induced by fracture slip. A more detailed discussion of these topics can be found in Appendix A, “Seismicity modeling in EGS.”

A goal of the shear stimulation modeling described in this paper was to focus on realistic treatment of friction and induced stresses. Rate and state theory was used to describe fracture friction. Rate and state friction is based on laboratory observations of rock friction and has been successful in describing a variety of earthquake phenomena (Dieterich, 2007; Segall, 2010). Stress interaction was calculated using the Crouch and Starfield (1983) boundary element method which assumes that the rock material is homogenous, isotropic, and linearly elastic. These assumptions are reasonable for EGS reservoirs that are located in fractured granite.

Injection into a single, isolated, 1D fracture was modeled. The problem geometry was simple, but it was a reasonable analog for injection into faulted granite such as is found at the EGS project at Soultz-sous-Forêts, France. At Soultz, observations suggested that flow and seismicity was confined to a small number of large scale faults. A more detailed discussion can be found in Appendix B, “Relationship of our model to actual EGS reservoirs.”

**Summary of results**

Major results of this paper are summarized in this section. Many of the results were consistent with what has been described by other studies in the literature. There were several results that are novel and could have important practical implications.

The mechanism of growth of the stimulated region was a two-part cycle. The cycle began with a seismic event that spread slip and permeability enhancement into a region of the fracture where slip had not previously occurred and where there was low permeability and fluid pressure. The second part of the cycle was flow of water into the newly slipped patch of fracture, which eventually triggered the next seismic event. We refer to this process as the “sequential stimulation” (SS) mechanism.

The SS mechanism is distinctly different from a process that has been described in the literature in which pressure diffuses into unstimulated fracture regions, subsequently causing slip (Bruehl, 2007). We refer to this process as the “diffusion controlled” (DC) mechanism.

The difference is that in the DC mechanism, slip follows pressure diffusion. In the SS mechanism, pressure diffusion follows slip. SS behavior will only occur in a model if stress interaction between elements is included and friction is allowed to weaken (as it would during a seismic event). SS behavior is not unique to rate and state friction models, as similar behavior was described in McClure and Horne (2010) using an instantaneously weakening friction model that we refer to as static/dynamic.

If the SS, not the DC, mechanism controls the growth of the stimulated region, it would undermine several common assumptions. In the literature, unstimulated hydraulic diffusivity has been estimated using the assumption that it controls the rate of growth of the stimulated region during injection (Shapiro et al., 1999; Bruehl, 2007). In the SS mechanism, the rate of growth of the stimulated region during injection does not depend on the unstimulated diffusivity.

The SS mechanism could explain why injection pressure tends to increase only slightly when injection rate is increased during shear stimulation. Previously, such behavior has been interpreted as being caused by the opening of tensile fractures at a pore pressure equal to the least principal stress (Cornet and Béard, 2003; Valley and Evans, 2007; Cornet et al., 2007). The SS mechanism provides an alternative explanation that does not involve the propagation of opening mode tensile fractures and does not require the pore pressure to be equal to the least principal stress. The least principal stress would be underestimated if it was assumed incorrectly to be equal to the pore pressure during injection.

In the model, spreading and redistribution of pressure after the end of injection caused shut-in seismicity, a commonly observed phenomenon in which seismic events of significant magnitude continue to occur at the edge of the stimulated region after shut-in (Haring et al., 2007; Asanuma et al., 2006; Baisch et al., 2010). Redistribution of pressure has also been proposed as a mechanism for shut-in events by Baisch et al. (2006), Healy et al. (1968), and Hsieh and Bredehoeft (1981).

We investigated strategies to minimize induced seismicity. Reducing injection pressure over time reduced the maximum magnitude. With constant injection pressure over time, using a lower injection pressure led to fewer significant sized events, but did not affect the maximum magnitude. Producing fluid back after injection reduced shut-in seismicity. Baisch et al. (2006) also suggested producing fluid could reduce shut-in events.

We investigated the effect of two geological uncertainties on the results, namely pore volume dilation during slip and the value of the rate and state characteristic length scale, $d_c$. A larger $d_c$ caused slip to occur aseismically as opposed to seismically. Pore volume dilation caused slip to occur with a larger number of lower magnitude events. Both results were consistent with other studies in the literature (Ruina, 1983; Yamashita, 1999; Segall and Rice, 1995; Segall et al., 2010).

**METHODS**

**Problem definition**

Our numerical model required solution of five equations for five primary variables. The variables were velocity ($v$), state ($\theta$), mass of fluid in a cell ($m$), shear traction ($r$), and cumulative shear displacement ($D$). A full list of symbols is given in Table 1. The equations solved were unsteady-state fluid mass balance (with Darcy’s law), frictional force equilibrium (with a radiation damping approximation term), a stress strain relationship that related shear displacement to shear traction, the aging law for state evolution, and the time integral relationship between slip velocity and cumulative shear displacement.
The problem was solved on a 1D fracture embedded in a 2D homogenous, isotropic medium. It was assumed that the permeability of the surrounding medium was zero (reasonable for fractures embedded in granite), and so the injected water was located only in the fracture.

The 2D stress/strain problems were solved using plane stress, which assumes the thickness of the medium in the third dimension is infinite. For some calculations, an infinite height fracture would lead to unrealistic results. For example, an infinite height fracture would have an infinite flow rate. Therefore, for calculations not involving stress and strain, fracture height was defined to be \( b_{w0} \), which we set to 100 m.

The simulations were isothermal. The fluid was single-phase liquid water.

The unsteady-state fluid mass balance equation in a fracture is (Aziz and Settari, 1979; with fracture aperture \( E \) replacing porosity)
\[
\frac{\partial (\rho E)}{\partial t} = \nabla q + s,
\]
where \( q \) is the mass flux rate, \( s \) is a source term, \( t \) is time, \( E \) is the void aperture (the pore volume per cross-sectional area of fracture), and \( \rho \) is the fluid density. Darcy flow was assumed, in which mass flow across an area \( A \) in a direction \( x_i \) is (Aziz and Settari, 1979)
\[
q = \frac{k \rho A}{\mu} \frac{\partial P}{\partial x_i},
\]
where \( P \) is fluid pressure, \( \mu \) is fluid viscosity, and \( k \) is permeability.

The permeability \( k \) is given by the “cubic law” and defined as (Jaeger et al., 2007)
\[
k = \frac{e^2}{12},
\]
where \( e \) is hydraulic aperture, which is the effective aperture for flow in the fracture. Hydraulic aperture is equal to void aperture between two smooth plates, but can be lower than void aperture between rough surfaces such as a rock fracture.

For flow in a 1D fracture, the cross-sectional area \( A \) is \( b_w \ast e \), and so the mass flow rate is
\[
q = \frac{\rho b_w e^3}{12 \mu} \frac{\partial P}{\partial x_i}.
\]

For a closed fracture, force equilibrium requires that shear traction be equal to the frictional resistance to slip. An additional term, \( \nu \ast \eta \), called the radiation damping term, can be included to approximate the damping effect of inertia on sliding at high velocities (Rice, 1993). The variable \( \eta \) is on the order of one to tens of

<table>
<thead>
<tr>
<th>( v )</th>
<th>Sliding velocity</th>
<th>( t )</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>State</td>
<td>( T )</td>
<td>Stress tensor</td>
</tr>
<tr>
<td>( m )</td>
<td>Mass of fluid in cell</td>
<td>( G )</td>
<td>Shear modulus</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Shear traction</td>
<td>( \nu_p )</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>( \sigma_n )</td>
<td>Normal traction</td>
<td>( \varepsilon )</td>
<td>Strain tensor</td>
</tr>
<tr>
<td>( \sigma'_n )</td>
<td>Effective normal traction</td>
<td>( I )</td>
<td>Unit matrix</td>
</tr>
<tr>
<td>( D )</td>
<td>Cumulative shear displacement</td>
<td>( E_0 )</td>
<td>Void aperture constant</td>
</tr>
<tr>
<td>( b_w )</td>
<td>Out-of-plane width</td>
<td>( e_0 )</td>
<td>Hydraulic aperture constant</td>
</tr>
<tr>
<td>( E )</td>
<td>Void aperture</td>
<td>( E_{res} )</td>
<td>Residual void aperture</td>
</tr>
<tr>
<td>( q )</td>
<td>Mass flux rate</td>
<td>( e_{res} )</td>
<td>Residual hydraulic aperture</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Fluid density</td>
<td>( D_{max} )</td>
<td>Maximum hydraulic aperture displacement</td>
</tr>
<tr>
<td>( A )</td>
<td>Area</td>
<td>( D_{max} )</td>
<td>Maximum void aperture displacement</td>
</tr>
<tr>
<td>( P )</td>
<td>Fluid pressure</td>
<td>( \sigma_{Eref} )</td>
<td>Void aperture reference normal traction</td>
</tr>
<tr>
<td>( k )</td>
<td>Permeability</td>
<td>( \sigma_{Eref} )</td>
<td>Hydraulic aperture reference normal traction</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Fluid viscosity</td>
<td>( \Phi_{Edd(1,2)} )</td>
<td>Void aperture dilation angle</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Radiation damping coefficient</td>
<td>( \Phi_{Edd(1,2)} )</td>
<td>Hydraulic aperture dilation angle</td>
</tr>
<tr>
<td>( \mu_f )</td>
<td>Coefficient of friction</td>
<td>( M_w )</td>
<td>Moment magnitude</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>Nominal coefficient of friction</td>
<td>( B )</td>
<td>Matrix of sliding to shear traction interaction coefficients</td>
</tr>
<tr>
<td>( \nu_0 )</td>
<td>Velocity normalizing constant</td>
<td>( P_{inj} )</td>
<td>Injection pressure</td>
</tr>
<tr>
<td>( a )</td>
<td>Velocity effect coefficient</td>
<td>( P_{inj}' )</td>
<td>Injection pressure derivative</td>
</tr>
<tr>
<td>( b )</td>
<td>State effect coefficient</td>
<td>( \mu_s )</td>
<td>Static coefficient of friction from S/D friction</td>
</tr>
<tr>
<td>( d_c )</td>
<td>Characteristic displacement scale</td>
<td>( \mu_d )</td>
<td>Dynamic coefficient of friction from S/D friction</td>
</tr>
<tr>
<td>( \sigma_{yy} )</td>
<td>Remote compressive stress in the y-direction</td>
<td>( T_{inj} )</td>
<td>Injection temperature</td>
</tr>
<tr>
<td>( \sigma'_{yy} )</td>
<td>Remote shear stress</td>
<td>( T_{init} )</td>
<td>Initial temperature</td>
</tr>
</tbody>
</table>

Table 1. Table of symbols.
and referred to as the characteristic displacement scale. The parameters in the vector equation (Jaeger et al., 2007) are assumed, which means that the relationship between stress and strain is given by Hooke’s law (Jaeger et al., 2007)

\[ T = \frac{2G\nu_p - \text{trace}(\varepsilon)}{1 - 2\nu_p} I + 2G\varepsilon, \]  

where \( I \) is the unit matrix, \( \varepsilon \) is the strain tensor, \( \nu_p \) is Poisson’s ratio, and \( G \) is the shear modulus.

The cumulative displacement at any point is equal to the time integral of velocity

\[ D = \int v \, dt. \]  

Void and hydraulic aperture are related to effective normal traction and cumulative displacement. There is not a universally accepted equation in the literature for the relationship between these variables. A modified version of the equation was used by Willis-Richards et al. (1996), Rahman et al. (2002), Kohl and Mége (2007), and others was

\[ E = \frac{E_0}{1 + 9\sigma_n/\sigma_{\text{Emref}}} + D_1 \tan \frac{\phi_{\text{Edil1}}}{1 + 9\sigma_n/\sigma_{\text{Emref}}} + D_2 \tan \frac{\phi_{\text{Edil2}}}{1 + 9\sigma_n/\sigma_{\text{Emref}}} + E_{\text{res}}, \]  

where \( E_0, \sigma_{\text{Emref}}, \phi_{\text{Edil1}}, \) and \( \phi_{\text{Edil2}} \) are material constants. We allowed these constants to be different for hydraulic aperture, \( \epsilon \), and void aperture \( E \). In most simulations, \( \phi_{\text{Edil1}} \) and \( \phi_{\text{Edil2}} \) were set to zero so that there was no shear-induced pore volume dilation, only hydraulic aperture dilation.

Prior authors have used only one term for aperture enhancement from shear displacement. We used two terms to take into account the laboratory observation that hydraulic aperture of a fracture tends to increase more slowly after the initial shear displacement. Such a property was not recognized in early laboratory testing of shear displacement and aperture coupling in granite (Barton et al., 1985). More recent laboratory work has observed this phenomenon. Esaki et al. (1999) and Lee and Cho (2002) both found that for a shearing fracture in granite, permeability increased rapidly at first, but permeability increased slowly or not at all after 5–10 mm of slip. Esaki et al. (1999) and Lee and Cho (2002) both observed an increase in mechanical aperture with slip beyond 10 mm. It is not clear whether or not void aperture continued to increase after 10 mm of slip.

The state variable can be interpreted as the average contact time of asperities on the fault. The “aging law” of state evolution is (Segall, 2010)

\[ \frac{\partial \theta}{\partial t} = 1 - \frac{\theta v}{d_c}. \]  

Dieterich (1979) associated the state variable with asperity contact time. Dieterich and Kilgore (1994) demonstrated experimentally that surface contact area increased with contact time due to creep of asperities.

The stresses induced by fracture slip were calculated according to the equations of quasistatic equilibrium in a continuum assuming that body forces are equal to zero. These stresses are given by the vector equation (Jaeger et al., 2007)

\[ \nabla^T T = 0, \]  

where \( T \) is the stress tensor.

Linear elasticity in an isotropic, homogeneous body was assumed, which means that the relationship between stress and strain is given by Hooke’s law (Jaeger et al., 2007).

Fluid density and viscosity are related to fluid pressure (and temperature, but the simulations were isothermal). Values were interpolated from a large table of properties generated using the freeware Matlab code XSteam 2.6 by Magnus Holmgren (2007).

A microseismic event was considered to have begun when the maximum velocity on the fracture exceeded 5 mm/s. A slip event was considered finished when the highest velocity on the fracture dropped below 2.5 mm/s. Event durations were variable, but were at most a few seconds. Event hypocenters were defined as the location where slip velocity first exceeded 5 mm/s.

The total amount of displacement on the fracture during the event was correlated to seismic magnitude. The seismic moment \( M_0 \) is a measure of the size and energy release of an earthquake (Stein and Wysession, 2003). \( M_0 \) is defined as the integral of displacement over the fracture area times the shear modulus.

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A microseismic event was considered to have begun when the maximum velocity on the fracture exceeded 5 mm/s. A slip event was considered finished when the highest velocity on the fracture dropped below 2.5 mm/s. Event durations were variable, but were at most a few seconds. Event hypocenters were defined as the location where slip velocity first exceeded 5 mm/s.

The total amount of displacement on the fracture during the event was correlated to seismic magnitude. The seismic moment \( M_0 \) is a measure of the size and energy release of an earthquake (Stein and Wysession, 2003). \( M_0 \) is defined as the integral of displacement over the fracture area times the shear modulus.
\[ M_0 = G \int D \dd A. \tag{14} \]

From Hanks and Kanamori (1979), the seismic moment magnitude \( M_w \) is defined as
\[ M_w = \frac{\log_{10} M_0}{1.5} - 6.06, \tag{15} \]
where \( M_0 \) is defined in N-m. For calculation of slip surface area, the dimension of the fracture out of the plane (in the third dimension) was taken to be \( b_w \). For a one-dimensional fracture, a patch of slip \( dA \) is equal to \( b_w dl \), where \( dl \) is an increment of distance along the fracture.

Modeling a 1D fracture instead of a two-dimensional fracture had some consequences, but should not have had a major impact on the overall results. Total displacements were overestimated slightly because in the 2D plane strain elastic solution, displacement goes on infinitely in the third, out-of-plane coordinate direction. Surface area close to the wellbore was overestimated because flow from the wellbore was 1D, not radial. As a result, the magnitudes of events near the wellbore were overestimated and the magnitudes of distant events were underestimated. Because the fracture in this study was 1D, the magnitudes calculated should only be compared relative to each another and not be considered actual magnitude predictions.

Finally, the dimensionality of the fracture affected connectivity with respect to heterogeneity. For fluid or slip to travel from one location to another on a 1D fracture, it has to travel through all points in between. For a 2D fracture, fluid or slip can propagate around barriers.

We neglected elastodynamic transfer of stress. Stress changes were propagated instantaneously and calculated using the quasi-static boundary element solution. Dynamic stresses may have some effect on the results, but are computationally intensive to calculate. Lapusta (2001) found for a single fracture case, calculations neglecting dynamic stresses could be made consistent with dynamic stress calculations by using a lower value of \( \eta \). Lapusta suggested that for geometries more complex than a single fracture, dynamic stress transfer would play a more complicated role.

The injection well was modeled by including a source term \( s \) in the mass balance equation for the two elements at the center of the fracture. The source term for each was set to half of the total flow rate. In most simulations, injection was carried out at a specified pressure. The injection pressure was not specified directly in the model. Instead, the source term was adjusted at each time step to bring the injection pressure to the target level. The injection pressure was calculated by assuming Darcy flow between the elements adjacent to the injector and a constant pressure boundary.

In one of the simulations, fluid was produced from the wellbore at a specified rate following injection. In that case, the source term was set constant until the wellbore pressure reached the initial fluid pressure of the reservoir. At that point, production was ended.

To include some heterogeneity, the permeability of each element was multiplied by a coefficient. The coefficients did not change during the simulations and were set to be a random number between 0.1 and 2.0.

**Methods of solution**

The fracture was discretized into elements of constant length. The same discretization was used for the mechanical and the fluid flow parts of the problem.

The mass balance equation was solved using the finite volume method. The flow between two adjacent elements in a linear fracture was calculated from Darcy’s law. The transmissibility between two elements was calculated using the harmonic average. Flow between fracture elements was calculated according to the method of Karimi-Fard et al. (2004).

The force equilibrium and stress/strain relations were solved with the 2D displacement discontinuity method, a linear elastic boundary element method (BEM) from Crouch and Starfield (1983). The problem reduces to finding the induced stresses \( \Delta \tau \) at each element \( i \) caused by the cumulative shear displacements from each element \( j \). Stresses and displacements are linearly related so that
\[ \Delta \tau_i = \sum_{j=1}^{n} B_{ij} \Delta D_j \tag{16} \]
where \( B \) is a matrix of interaction coefficients calculated according to Crouch and Starfield (1983). Because the problem setup was a single, linear fracture, shear displacements only affected shear tractions, not normal tractions.

We neglected stresses induced by fracture normal displacement. The fracture in our simulations was never “open” because it never experienced tensile stress. Closed fractures can have some slight normal displacement due to loading or unloading, but these displacements are around 0.1 mm (Barton et al., 1985), and their effect should be slight compared to the effects of pore pressure change caused by injection. The fracture normal displacement in our model was around 0.1 mm.

As discussed in Appendix A, during actual EGS stimulation, injection sometimes occurs into fault zones that have much greater storativity than the crack that was used in our model. For injection of larger volumes of fluid into a fault zone, it is possible that normal traction interaction between adjacent areas of a fault could play a larger role.

**Time discretization**

The issue of solving mechanical and flow equations together has been discussed at length in the literature of poroelasticity. One way to solve the problem is to use implicit Euler time-stepping on every equation simultaneously and solve the entire problem as a large coupled system of equations. This is a “fully coupled” scheme (Kim et al., 2011). The fully coupled strategy is stable and accurate but is expensive computationally. We used an “explicitly coupled” scheme in which a rate and state time-step was taken, and then the time-step was repeated for the flow problem using updated values from the rate and state calculation.

The rate and state time-step was taken with an explicit, third-order Runge-Kutta scheme (Abramowitz and Stegun, 1972). In this time-step, state, shear traction, and cumulative displacement were updated. The frictional equilibrium equation, equation 5, is an algebraic constraint, not a differential equation. At the end of each substep in the Runge-Kutta scheme, the frictional equilibrium equation was solved to find velocity for each individual element. Next, a flow time-step was taken using implicit Euler to find \( m^{r+1} \). The flow
equations and the frictional equilibrium equation were solved using Newton-Raphson iteration. Figure 1 summarizes the coupling strategy.

The advantage to splitting the problem is that different parts of the problem are most appropriately solved in different ways. The implicit Euler scheme is always numerically stable and is necessary to solve flow equations such as equation 1. However, the implicit Euler scheme requires solving a large system of equations. It would be impractical to attempt implicit Euler with the equations from the Crouch and Starfield (1983) method because the boundary element method uses a dense matrix of interaction coefficients. The matrix inversion requirements would be very large. Explicit time-steps require only multiplications of the BEM matrix.

Adaptive time-stepping was used. The time-steps were chosen based on four criteria. The first was a built-in error estimation on the calculation of state and shear traction from the third-order Runge-Kutta method. The second was the change in fluid pressure during the previous time-step. The third was the number of iterations used by the flow simulator in solving the nonlinear system of equations. The fourth was the relative amount of velocity change for each of the elements at the previous time-step. There was a target value for each criterion, and a time-step adjustment factor (either up or down) was calculated to move each value toward its target based on the result from the previous time-step. The adjustment was chosen by taking the square root of the ratio of the target to the criterion. The subsequent time-step was equal to the previous time-step multiplied by the adjustment factor. Of the four criteria, the adjustment that resulted in the most conservative time-step was used. If any of the criteria exceeded four times the target, the entire time-step was discarded and repeated with a smaller time interval.

During seismic events when slip was very rapid, very small time-steps on the order of microseconds were necessary. In between seismic events, time-steps on the order of seconds, minutes, or hours were taken.

Problem setup

Simulations were performed of injection into the center of a single, isolated, 1D fracture embedded in a 2D whole space. The fracture was 500 m long and oriented 20° clockwise from the vertical y-axis. The 2D problem could be interpreted as viewing a strike-slip fault in plan view, a normal fault in side view, or a reverse fault in side view, rotated 90°. The fracture was discretized into 2000 elements of length 25 cm.

![Figure 1. Explicit coupling scheme. The time-step is split into two parts. First, an explicit time-step is taken to update D, θ, and τ. Then an implicit time-step is taken to update m, and v is calculated as an algebraic constraint to enforce equilibrium.](image)

The base case parameters are given in Table 2. $T_{init}$ was the initial temperature and was the same as the injection temperature $T_{inj}$, 200 °C because the simulation was isothermal. The variables $\sigma_{rxx}$, $\sigma_{ryy}$, and $\sigma_{rx}$ are the remote compressive stress in the x-direction, the y-direction, and the remote shear traction.

Once 75% of the fracture had slipped by a minimal amount, 0.1 mm, injection was ceased. The simulation was continued after injection stopped for a period equal to 20 times the duration of injection.

The frictional parameters $a$, $b$, and $d_\ell$ deserve some discussion. For unstable slip to occur, $a$ must be smaller than $b$ (Ruina, 1983). This is because to achieve runaway velocity acceleration, the friction weakening effect of state decrease must be greater than the friction strengthening effect of velocity increase. The parameter $d_\ell$ controls the minimum size of a patch of slip that can slip unstably and cause seismicity (Ruina, 1983). $d_\ell$ also limits the size of the spatial discretization. The element size must be significantly smaller than a characteristic length scale related to $a$, $b$, $d_\ell$, and $\sigma_{rxx}$, otherwise the result is numerically unstable (Lapusta, 2001).

Several different simulations were carried out. The simulations were performed by specifying injection pressure. In practical EGS stimulations, positive displacement pumps are used so that the flow rate is controlled directly, not the injection pressure. However, in our simulations, trends in the injection pressure were most relevant to the behavior of the hydraulic stimulation, so injection pressure boundary conditions were used to control directly for these effects. In practice, injection pressure could be controlled indirectly by adjusting injection rate over time.

To investigate the effect of injection pressure, eight simulations were carried out using constant injection pressure (Cases A1–A8). All the simulations used pressures at 1 MPa increments from 51 MPa to 58 MPa.

To investigate the effect of decreasing injection pressure with time, 27 simulations were carried out that began with an injection pressure of 58 MPa and decreased the injection pressure over time. The injection rate was kept constant until the first microseismic event occurred, and then the injection pressure was decreased at

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{init}$</td>
<td>40 MPa</td>
</tr>
<tr>
<td>$T_{init}$</td>
<td>200 °C</td>
</tr>
<tr>
<td>$\theta_{init}$</td>
<td>10e8 s</td>
</tr>
<tr>
<td>$\sigma_{rxx}$</td>
<td>65 MPa</td>
</tr>
<tr>
<td>$\sigma_{ryy}$</td>
<td>100 MPa</td>
</tr>
<tr>
<td>$\sigma_{rx}$</td>
<td>0 MPa</td>
</tr>
<tr>
<td>$a$</td>
<td>0.011</td>
</tr>
<tr>
<td>$b$</td>
<td>0.014</td>
</tr>
<tr>
<td>$G$</td>
<td>10 GPa</td>
</tr>
<tr>
<td>$v_p$</td>
<td>0.1</td>
</tr>
<tr>
<td>$E_0$</td>
<td>1 mm</td>
</tr>
<tr>
<td>$\epsilon_0$</td>
<td>0.01 mm</td>
</tr>
<tr>
<td>$E_{res}$</td>
<td>0.002 mm</td>
</tr>
<tr>
<td>$\eta$</td>
<td>20 MPa/(m/s)</td>
</tr>
</tbody>
</table>

Table 2. List of simulation base case parameters.
a constant rate, defined as $P'_{\text{inj}}$. A variety of rates were used. In 14 of the simulations, Cases B1–B14, the injection pressure was kept constant if it dropped to 51 MPa. In 13 simulations, Cases C1–C13, the injection pressure was kept constant if it dropped to 53 MPa. In some of the cases, the minimum injection pressure was not reached before injection was stopped.

Three additional simulations were performed (Cases D1–D3). All used constant pressure injection at 58 MPa, like Case A8. D1 tested the effect of void aperture dilation with slip by using a value of $\phi_{\text{Edil}}$ equal to 1°, instead of the baseline value of 0° (no void dilation with slip). D2 used $d_c$ equal to 5 mm, 100 times larger than the baseline value. D3 produced fluid at 3.0 l/s after the end of injection.

Table 3 specifies the settings for all the simulation runs. Table 4 provides the values for $P'_{\text{inj}}$ used in Cases B1–B14 and Cases C1–C13. Unless listed in Table 3, all parameters are the same as the baseline parameters given in Table 2.

**Table 3. Parameters of the various simulation cases, A1–A8, B1–B14, C1–C13, and D1–D3. Further details about the B and C cases are given in Table 4.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1–A8</td>
<td>Constant $P_{\text{inj}}$ from 51 MPa to 58 MPa at 1 MPa increments</td>
</tr>
<tr>
<td>B1–B14</td>
<td>Decreasing $P_{\text{inj}}$ with time from 58 MPa, minimum $P_{\text{inj}} = 51$ MPa</td>
</tr>
<tr>
<td>C1–C13</td>
<td>Decreasing $P_{\text{inj}}$ with time from 58 MPa, minimum $P_{\text{inj}} = 53$ MPa</td>
</tr>
<tr>
<td>D1</td>
<td>Constant $P_{\text{inj}}$ at 58 MPa, $\phi_{\text{Edil}} = 1.0^\circ$</td>
</tr>
<tr>
<td>D2</td>
<td>Constant $P_{\text{inj}}$ at 58 MPa, $d_c = 5$ mm</td>
</tr>
<tr>
<td>D3</td>
<td>Constant $P_{\text{inj}}$ at 58 MPa, production at 3.0 kg/s after injection</td>
</tr>
</tbody>
</table>

**Table 4. The rate of decrease in injection pressure, $P'_{\text{inj}}$, for cases B1–B14 and C1–C13.**

<table>
<thead>
<tr>
<th>Case</th>
<th>$P'_{\text{inj}}$ (MPa/hr)</th>
<th>Case</th>
<th>$P'_{\text{inj}}$ (MPa/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Infinity</td>
<td>C1</td>
<td>Infinity</td>
</tr>
<tr>
<td>B2</td>
<td>-10.1</td>
<td>C2</td>
<td>-7.2</td>
</tr>
<tr>
<td>B3</td>
<td>-5.0</td>
<td>C3</td>
<td>-3.6</td>
</tr>
<tr>
<td>B4</td>
<td>-3.4</td>
<td>C4</td>
<td>-2.4</td>
</tr>
<tr>
<td>B5</td>
<td>-2.5</td>
<td>C5</td>
<td>-1.8</td>
</tr>
<tr>
<td>B6</td>
<td>-2.0</td>
<td>C6</td>
<td>-1.4</td>
</tr>
<tr>
<td>B7</td>
<td>-1.7</td>
<td>C7</td>
<td>-1.2</td>
</tr>
<tr>
<td>B8</td>
<td>-1.4</td>
<td>C8</td>
<td>-1.0</td>
</tr>
<tr>
<td>B9</td>
<td>-1.3</td>
<td>C9</td>
<td>-0.9</td>
</tr>
<tr>
<td>B10</td>
<td>-1.0</td>
<td>C10</td>
<td>-0.7</td>
</tr>
<tr>
<td>B11</td>
<td>-0.8</td>
<td>C11</td>
<td>-0.6</td>
</tr>
<tr>
<td>B12</td>
<td>-0.6</td>
<td>C12</td>
<td>-0.5</td>
</tr>
<tr>
<td>B13</td>
<td>-0.5</td>
<td>C13</td>
<td>-0.4</td>
</tr>
<tr>
<td>B14</td>
<td>-0.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**RESULTS**

Plots of injection rate, injection pressure, and event magnitude versus time for Cases A3, A6, A8, B6, B10, D1, D2, and D3 are shown in Figures 2, 3, 4, 5, 6, 7, 8, and 9. Figures 10, 11, and 12 give summary metrics for Cases A1–A8. Figure 10 shows...
maximum magnitude and number of seismic events magnitude greater than two. Figure 11 shows the average injection rate during the period of time between the first seismic event and shut-in. Figure 12 shows the total fluid injected and total seismic moment released. Figures 13 and 14 show the maximum event magnitude and the number of events magnitude greater than two for Cases

Figure 5. Injection rate (kg/s) and event magnitude for Case B6, decreasing injection rate from 58 MPa to 51 MPa with $p_{\text{inj}}$ equal to 2 MPa/hr.

Figure 6. Injection rate (kg/s) and event magnitude for Case B10, decreasing injection rate from 58 MPa to 51 MPa with $p_{\text{inj}}$ equal to 1 MPa/hr. Note that in this case the injection was stopped before $p_{\text{inj}}$ reached 51 MPa.

Figure 8. Injection rate (kg/s) and event magnitude for Case D2, constant pressure injection of 58 MPa with $d_c$ equal to 5 mm, 100 times larger than the default.

Figure 9. Injection rate (kg/s) and event magnitude for Case D3, constant pressure injection of 58 MPa with fluid production after injection.

Figure 10. Maximum magnitude and the number of events with magnitude greater than 2.0 for Cases A1–A8, constant pressure injection.
B1–B14 and Cases C1–C13. Tables 5 and 6 give summary data for selected cases.

**DISCUSSION**

A number of issues are discussed in the following subsections. First the overall behavior of the model is discussed, focusing on comparison to EGS field observations, the sequential stimulation mechanism that controlled the progression of the stimulation, shut-in events, and changes in injection rate with time. Subsequent subsections discuss estimation of least principal stress, estimation of prestimulation hydraulic diffusivity, the effect of injection pressure for constant pressure injection, the effect of changing injection pressure for constant pressure injection, the effect of producing back fluid to reduce shut-in seismicity, a comparison of rate and state to static/dynamic friction, the effect of slip-induced void aperture dilation, and the effect of the characteristic displacement scale, \( d_c \).

**Similarity and differences compared to EGS field observations**

The behavior of the model was qualitatively consistent with a broad range of observations from EGS projects, with some differences. Similarities were migration of event hypocenters away from the stimulated region (Figure 15; Baisch et al., 2010; Shapiro et al., 1999), shut-in seismicity after injection stopped (Figures 2, 3, 4, 5, 6, 7, 8, and 9; Charléty et al., 2007; Häring et al., 2007; Asanuma et al., 2006; Baisch et al., 2010), and large increases in injection pressure with small changes in injection rate (Figure 11; Cornet et al., 2003). Differences were an underestimation of the number of smaller events (Figures 2, 3, 4, 5, 6, 7, 8, and 9; Baisch et al., 2010), a lack of event hypocenters that were not at the periphery of the stimulated region (Figure 15; Baisch et al., 2010), and shut-in seismicity magnitudes that were lower than magnitudes with time, the effect of producing back fluid to reduce shut-in seismicity, a comparison of rate and state to static/dynamic friction, the effect of slip-induced void aperture dilation, and the effect of the characteristic displacement scale, \( d_c \).

![Figure 11](image1.png)

**Figure 11.** Average injection rate during stimulation for Cases A1–A8, constant pressure injection.

![Figure 12](image2.png)

**Figure 12.** Total fluid injected (kg) and total seismic moment release (N-m) for Cases A1–A8, constant pressure injection.

![Figure 13](image3.png)

**Figure 13.** Maximum magnitude and the number of events with magnitude greater than 2.0 for Cases B1–B14, decreasing injection pressure with time from 58 MPa to a minimum of 51 MPa.

**Table 5. Summary data for Cases A1, A3, A6, A8, B6, B10, and D1–D3. The number of events magnitude greater than 2.0, the maximum magnitude during injection, the maximum magnitude after injection, the total number of events during injection, and the total number of events after injection.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Events &gt;2.0</th>
<th>Max. mag. during injection</th>
<th>Max. mag. after injection</th>
<th>Events during injection</th>
<th>Events after injection</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>2.3</td>
<td>—</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>A3</td>
<td>2</td>
<td>2.2</td>
<td>1.7</td>
<td>26</td>
<td>6</td>
</tr>
<tr>
<td>A6</td>
<td>5</td>
<td>2.2</td>
<td>1.7</td>
<td>41</td>
<td>25</td>
</tr>
<tr>
<td>A8</td>
<td>6</td>
<td>2.2</td>
<td>1.8</td>
<td>51</td>
<td>32</td>
</tr>
<tr>
<td>B6</td>
<td>0</td>
<td>1.7</td>
<td>1.6</td>
<td>35</td>
<td>2</td>
</tr>
<tr>
<td>B10</td>
<td>3</td>
<td>2.2</td>
<td>1.7</td>
<td>61</td>
<td>24</td>
</tr>
<tr>
<td>D1</td>
<td>0</td>
<td>—</td>
<td>—</td>
<td>88</td>
<td>8</td>
</tr>
<tr>
<td>D2</td>
<td>0</td>
<td>—</td>
<td>—</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D3</td>
<td>6</td>
<td>2.2</td>
<td>1.7</td>
<td>51</td>
<td>11</td>
</tr>
</tbody>
</table>
during injection (Figures 2, 3, 4, 5, 6, 7, 8, and 9; Asanuma et al., 2006; Baisch et al., 2010; Majer et al., 2007). Because an unrealistically low value of storativity was used, it was necessary to keep the injection rate low by using a low value of permeability, as discussed in Appendix B.

The differences between the model behavior and reality probably occurred because the model was significantly less heterogeneous than a natural system. The model contained a single, linear fracture with homogeneous properties (other than permeability). Actual EGS stimulations could involve several fracture zones with multiple slip surfaces, nonuniform properties, and nonplanar geometry. All of these factors would encourage heterogeneity in the location of hypocenters and a greater number of smaller events.

As discussed in the section “Effect of shear-induced pore volume dilation,” shear-induced pore volume dilation could also cause hypocenters that are not located at the periphery of the stimulated region, especially because fractures in EGS stimulations are often embedded in high porosity damaged zones. As discussed in the section “Effect of \( d_c \),” \( d_c \) controls the minimum seismic event magnitude (Ruina, 1983; Dieterich, 2007). If a smaller value of \( d_c \) had been used, smaller events could have been simulated. In that case, it would have been necessary to use a finer discretization to avoid numerical instability (Lapusta, 2001). As discussed in the section “Shut-in seismicity,” the shut-in events had lower magnitude likely because the fracture modeled was 1D, not 2D.

In rate and state simulations with appropriately refined discretizations and homogeneous (or mostly homogeneous) properties on a single fracture, frequency-size distributions tend toward a “characteristic” distribution (Rice, 1993). In such models, seismic events, once nucleated, have a tendency to propagate across the entire fracture. In contrast, “inherently discrete” models have coarser discretizations and often use less realistic friction laws and tend to reproduce Gutenberg-Richter frequency-size distributions that have a much greater number of smaller events than larger events (Rice, 1993; Ben-Zion and Rice, 1993; Ben-Zion, 2008). The development of characteristic frequency-size distributions in rate and state simulations suggests that Gutenberg-Richter distributions are not necessarily a consequence of frictional behavior, but rather may arise from heterogeneity in the earth itself. Hillers et al. (2006) replicated a Gutenberg-Richter distribution on a single fault using a rate and state model with spatial heterogeneity in frictional parameters. Aftershock distributions have been modeled successfully with rate and state friction using a distribution of faults (Gomberg et al., 2005).

### Sequential stimulation mechanism

The advance of the stimulated region occurred through a specific mechanism, which we refer to as the sequential stimulation (SS) mechanism. A similar mechanism was described in McClure and Horne (2010) based on results from a simpler shear stimulation model.

Conceptually, it is useful to divide the fracture into two regions. In the stimulated region, significant slip had already occurred. The permeability had increased dramatically, and as a result the fluid pressure had increased significantly. In the unstimulated region, slip had not yet occurred. The permeability was low, and the fluid pressure was near the initial pressure because fluid had not had time to flow beyond the stimulated region. Figures 16, 17, and 18 show the pressure distribution in the fracture at various times during Cases A8, A4, and A1.

Slip events tended to nucleate at the edge of the stimulated region. Once nucleated, slip could propagate easily back across the stimulated region because in the stimulated region the fluid pressure was high and friction was relatively weak. It was more difficult for slip to propagate from the stimulated region into the unstimulated region because in the unstimulated region fluid pressure remained low and friction was relatively strong. Nevertheless, slip events were able to propagate some distance into the unstimulated region before stopping. When that happened, the permeability increased rapidly on the patch of fracture that had slipped for the first time. Fluid was able to rush into the newly slipped patch of fracture, weakening friction and nucleating the next seismic event.

Figures 2, 3, and 4 show that for the base case with constant injection pressure, seismic events were relatively low in magnitude at the beginning of injection and grew larger over time. Magnitude increased over time because magnitude is related directly to the surface area of fracture that slips. At later times, more fracture was available to slip. A similar effect was observed in numerical simulations by McClure and Horne (2010) and by Baisch et al. (2010).

### Table 6. Summary data for Cases A1, A3, A6, A8, B6, B10, and D1–D3. The total seismic moment release both during and after injection, the total duration of injection (including the period prior to the first seismic event), the total amount of fluid injected, and the maximum shear displacement along the fracture.

<table>
<thead>
<tr>
<th>Case</th>
<th>Moment during inj. (N·m)</th>
<th>Moment after inj. (N·m)</th>
<th>Duration of injection (s)</th>
<th>Fluid injected (kg)</th>
<th>Maximum displacement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>5.90E+12</td>
<td>0.00E+00</td>
<td>42,629,200</td>
<td>4073</td>
<td>0.04</td>
</tr>
<tr>
<td>A3</td>
<td>1.06E+13</td>
<td>9.66E+11</td>
<td>235,008</td>
<td>2982</td>
<td>0.07</td>
</tr>
<tr>
<td>A6</td>
<td>1.87E+13</td>
<td>4.44E+12</td>
<td>14,759</td>
<td>3585</td>
<td>0.11</td>
</tr>
<tr>
<td>A8</td>
<td>2.75E+13</td>
<td>8.26E+12</td>
<td>7227</td>
<td>3981</td>
<td>0.14</td>
</tr>
<tr>
<td>B6</td>
<td>5.52E+12</td>
<td>3.15E+11</td>
<td>19,374</td>
<td>2633</td>
<td>0.06</td>
</tr>
<tr>
<td>B10</td>
<td>1.81E+13</td>
<td>4.31E+12</td>
<td>8953</td>
<td>3622</td>
<td>0.11</td>
</tr>
<tr>
<td>D1</td>
<td>9.24E+12</td>
<td>6.08E+11</td>
<td>25,781</td>
<td>20,538</td>
<td>0.12</td>
</tr>
<tr>
<td>D2</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>17,113</td>
<td>4752</td>
<td>0.15</td>
</tr>
<tr>
<td>D3</td>
<td>2.74E+13</td>
<td>2.19E+12</td>
<td>7227</td>
<td>3981</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Shut-in events

Shut-in events occurred because of pressure redistribution. During injection, there was a pressure gradient away from the wellbore, which can be seen in Figures 16, 17, and 18. After injection stopped, the pressure redistributed to become uniform everywhere. The redistribution lowered pressure near the injector and increased pressure away from the injector. The fluid pressure at different times following shut-in during Case A8 is shown in Figure 19.

The idea that pressure diffusion could cause an advance of the pressure front after shut-in has been proposed by other authors. Healy et al. (1968) and Hsieh and Bredehoeft (1981) discussed the possibility that this may have occurred in association with a deep wastewater disposal well outside of Denver. Several numerical models have predicted this effect, including Bruel and Charlety (2007), McClure and Horne (2010), Baisch et al. (2010), and Baisch et al. (2006). Similar mechanisms were investigated in modeling by Hayashi and Abe (1982) and Hayashi and Abe (1983).

The shut-in events were smaller in magnitude than the largest events during injection. In actual EGS stimulation, shut-in events are often larger than events during injection (Majer et al., 2007).
The discrepancy may be partly an artifact of our use of a 1D fracture. For a 2D planar fracture, the periphery would have relatively more surface area than in the 1D case. The shut-in events, which occur at the periphery, would be relatively larger.

In the 1D simulations, peripheral events on either side of the fracture tended to be separated because the center region, which did not slip, prevented slip on one side from triggering slip on the other. For a 2D, planar fracture, the periphery would be a ring-shaped region, and there would be no barrier to prevent slip anywhere in the periphery from inducing slip everywhere else in the periphery. Baisch et al. (2010) simulated induced seismicity in a 2D planar fracture and observed larger events during shut-in than during injection.

Changes in flow rate with time

The flow rate behavior with time for several cases can be seen in Figures 2, 3, 4, 5, 6, 7, 8, and 9. The flow rate tended to spike following a seismic event. Seismic events caused abrupt increases in permeability along the fracture without changing the pressure distribution (except in Case D1, which included the effect of shear-induced pore volume dilation). Because flow rate is proportional to permeability and pressure gradient, increasing permeability while holding pressure gradient constant increased flow rate. Between seismic events, flow rate tended to decrease, as would be expected for constant injection pressure.

During an actual EGS stimulation, it is typical for flow rate to be held constant (except for occasional step changes) and the injection pressure to fluctuate with time. Abrupt changes in injection pressure of at least 0.6 MPa following large slip events have been observed during actual EGS stimulations (Weidler, 2000). The magnitude of the changes in injection pressure observed during EGS stimulations do not appear to be as great as the fluctuations in flow rate observed in Figures 2, 3, 4, 5, 6, 7, 8, and 9. However, as discussed in the following section, “Implications of the sequential stimulation mechanism for estimation of the least principal stress,” small changes in injection pressure can cause large changes in flow rate during shear stimulation. It follows that changes in fracture permeability would cause relatively small changes in injection pressure for constant injection rate and relatively large change in injection rate for constant injection pressure.

Outside of short term fluctuations, the long-term behavior of the injection rate was generally increasing with time for most of the constant pressure injections, but sometimes decreased or remained constant with time. The relationship between flow rate and injection pressure involved competing effects, and can be understood through the framework of the SS mechanism. Increasing permeability across the stimulated region tended to cause an increase in injection rate with time. Growth of the stimulated region tended to cause a decrease in injection rate with time. A more detailed discussion of this relationship can be found in McClure and Horne (2010).

Implications of the sequential stimulation mechanism for estimation of the least principal stress

It was proposed by Cornet and Bérard (2003) and subsequently assumed in Cornet et al. (2007) and Valley and Evans (2007) that during the hydraulic stimulation of the wells GPK1, GPK3, and GPK4 as a part of the EGS projects at Soultz, France, the fluid pressure reached the least principal stress at the top of the openhole section during stimulation. That assumption was used to estimate the magnitude of the least principal stress at Soultz field.

The justification for this assumption was that during the stimulation, large increases in injection rate resulted in relatively small increases in injection pressure. This behavior could be referred to as pressure limiting behavior. When pressure limiting behavior occurs during hydraulic stimulation involving the growth of tensile cracks, it is taken as evidence that tensile fractures are propagating away from the wellbore (Zoback, 2007).

Our model suggests that shear stimulation alone, without the presence of tensile fracturing, could cause pressure limiting behavior. Figure 11 shows the average flow rate for different constant injection pressures during stimulation between the first seismic event and shut-in. It is evident that for the lowest injection pressure case, 51 MPa in Case A1, the average flow rate was extremely low. For an even lower injection pressure, the fluid pressure would not be high enough to propagate slip regardless of the duration of injection. That pressure could be called the shear stimulation threshold pressure.

For injection pressures below the shear stimulation threshold, increasing injection rate in increments would result in relatively large increases in injection pressure because injectivity would be related to the initial, low permeability.

Once shear stimulation began to occur, further increases in injection rate would result in much lower increases in injection pressure. From Figure 11, during Case A1 (51 MPa downhole, corresponding to a \( \Delta P = P_{inj} - P_{init} \) of 11 MPa), the average flow rate (during the period between the first seismic event and shut-in) was 0.02 kg/s. Increasing the injection pressure by one additional mega-Pascal to 52 MPa in Case A2 (\( \Delta P \) of 12 MPa, an increase of 9% from Case A1), doubled the flow rate to 0.04 kg/s. In Case A8, with an injection pressure of 58 MPa, the average flow rate was about
0.7 kg/s. From Case A1 to Case A8, a 64% increase in $\Delta P$ resulted in an increase in injection rate of 3400%.

The large increases in injection rate can be explained by considering the pressure distribution at different times during injection. The difference between the initial fluid pressure and the injection pressure can conceptually be decomposed into two parts. One part is the pressure drop in the stimulated region from the wellbore to the edge of the stimulated region. The second part is the front extension pressure at which the stimulated region was able to advance. The front extension pressure is approximately the fluid pressure that must be reached at the edge of the stimulated region to trigger seismicity that spreads slip into the unstimulated region. At the edge of the stimulated region, the fluid pressure drops rapidly from the front extension pressure to the initial fluid pressure.

Increasing the injection pressure lowered the front extension pressure. In Figure 18, injection at 51 MPa, the extension pressure was around 48 MPa. In Figure 17, injection at 55 MPa, the extension pressure was around 44 MPa. In Figure 16, injection at 58 MPa, the extension pressure apparently had reached nearly 40 MPa, the initial fluid pressure.

The pressure gradient was roughly the difference between the injection pressure and the front extension pressure divided by the distance to the fracture tip. Increasing injection pressure increased pressure gradient in two ways: increasing the pressure at the center of the stimulated region and decreasing it at the edge (by lowering the extension pressure). Further discussion of the relationship between injection pressure and rate can be found in McClure and Horne (2010).

From equation 13, higher fluid pressure led to higher permeability. In addition, greater displacement occurred with higher injection pressure, which also led to greater permeability.

Therefore, when the SS mechanism controlled stimulation, small increases in injection pressure led to large increases in injection rate.

If injection pressure were less than the shear stimulation threshold, significant slip would not occur, and permeability would remain low. With low permeability, the injection rate would remain low, and increases in injection pressure would cause small increases in injection rate.

Therefore, there are different mechanisms controlling the relationship between injection pressure and injection rate for injection pressures above and below the shear stimulation threshold. Injection rates at pressures below the stimulation threshold are related to the initial permeability. Injection rates at pressures above the stimulation threshold are related to the SS mechanism and depend on the much higher stimulated permeability. The change in mechanism at the stimulation threshold causes a sharp change in the relationship between injection pressure and rate.

The shear stimulation threshold pressure could be significantly below the least principal stress. In our model, it was roughly 50 MPa, 15 MPa less than the least principal stress.

Our modeling suggests that pressure limited behavior could occur because of shear stimulation. If pressure limited behavior were incorrectly taken to be evidence of tensile fracturing, the least principal stress could be underestimated.

**Effect of injection pressure for constant pressure injection**

Cases A1–A8 investigated the effect of injection pressure for constant pressure injection. Plots of flow rate and event magnitude with time for Cases A3, A6, and A8 are shown in Figures 2, 3, and 4.

The maximum event magnitude was not affected significantly by the injection pressure. Figure 10 shows that maximum magnitudes were clustered around 2.25 for all cases. The highest magnitudes occurred in Cases A1 and A2, the lowest injection pressures. They were larger because in those cases injection went on for a very long time before the first seismic event occurred (Figure 20). As a result, the pressure in the unstimulated region was significantly elevated by the time seismicity began (Figure 18).

The maximum event magnitude was generally not affected by injection pressure because the stress drop during an earthquake is weakly sensitive to the fluid pressure. Magnitude is related to the area of the product of slip and the displacement. Displacement is related to the stress drop and area of slip. The maximum area of slip was roughly the same in each simulation because it was limited by the size of the fracture. The stress drop is related to the weakening of friction caused by a decrease in the $\theta$ variable in the rate and state friction law. The decrease in $\theta$ during an earthquake depends on the rate and state parameters such as $a$, $b$, and $\eta$, but not on the effective normal stress.

Higher injection pressure led to a greater release of seismic moment, a somewhat greater amount of total fluid injected, and a greater number of relatively large events. These effects can be seen in Figures 10 and 12. The higher injection pressure caused a greater weakening of friction, which allowed more slip to occur, a greater release of seismic moment and more relatively large events.

**Effect of decreasing injection pressure over time**

Cases B1–B14 and Cases C1–C13 tested the effect of decreasing the injection pressure over time. Plots of flow rate and event magnitude with time for Cases B6 and B10 are shown in Figures 5 and 6. In all cases, the initial injection pressure was 58 MPa. The injection pressure was kept constant at 58 MPa until the first seismic event, and then it began to be decreased. In the different cases, the pressure was decreased at different rates with respect to time. In Cases B1–B14, injection pressure was kept constant once it reached 51 MPa. In Cases C1–C13, injection pressure was kept constant once it reached 53 MPa. The cases varied from instantaneous drop to the minimum injection pressure (Cases B1 and C1) to constant injection pressure at 58 MPa (Cases B14 and C13, the same as Case A8). It should be apparent that there was some overlap between Cases B1–B14 and Cases C1–C13.

![Figure 20](image-url)

**Figure 20.** The duration of time until maximum slipping velocity on the fracture reached 10 $\mu$m/s for Cases A1–A8, constant pressure injection.
Figures 10, 13, and 14 show that the cases with decreasing injection pressure over time had reduced seismicity compared to any of the constant injection pressure cases. There was a range of optimal values for $p_{i,j}^\text{inj}$, the pressure derivative, that resulted in significantly smaller maximum magnitudes and fewer events magnitude greater than 2.0 than for either constant injection at 51 MPa, 53 MPa, or 58 MPa. The effect was more pronounced for Cases B1–B14 than Cases C1–C13.

There is a physical explanation for why decreasing injection pressure with time reduced seismicity. Magnitude is related to slip area and shear displacement. The largest events were able to propagate across the entire stimulated region, with the largest events occurring when the stimulated region was largest.

Decreasing the injection pressure over time tended to decrease the fluid pressure across the entire stimulated region over time. Because friction strengthens as pressure decreases, this had the effect of gradually strengthening friction across the fracture over time. Events continued to nucleate at the edge of the stimulated region, but they faced greater frictional resistance in spreading back across the stimulated region. This caused fewer events to spread across the entire stimulated region, and when they did, their total displacements were smaller.

The optimal values of $p_{i,j}^\text{inj}$ corresponded to cases where the minimum injection pressure was reached either some time before or around the time that injection was complete. If injection pressure was lowered too rapidly, the minimum injection pressure was reached early in the simulation, and the simulation was subsequently carried out as a constant pressure injection. If injection pressure was lowered too slowly, the magnitude damping effect of lowering injection pressure over time was limited.

The effect of decreasing injection pressure over time was more pronounced for Cases B1–B14 than C1–C13. In Cases B1–B14, the minimum injection pressure was 51 MPa. Interestingly, in Case A1, 51 MPa injection was almost too low to cause stimulation. Figure 20 shows that for Case A1 it took an exceptionally long time, over 1000 hours of injection, for slip velocity to reach $10^{-5} \text{ m/s}$. Prior to the initiation of significant slip, Case A1 was effectively constant pressure injection into a low permeability fracture. In Cases B1–B14, injection at 51 MPa was clearly able to propagate stimulation across the fracture. It appears that a higher injection pressure was required to initiate seismicity in Case A1, but once seismicity was initiated, it was possible to propagate stimulation with a lower injection pressure.

The exact value of $p_{i,j}^\text{inj}$ that would minimize seismicity is dependent on the details of the model. If decreasing injection pressure with time were to be attempted in practice, the optimal $p_{i,j}^\text{inj}$ would be site specific and need to be estimated in advance with the construction of a full scale stimulation model.

It is possible that with a more complex, site-specific model, the conclusions of this section may not hold or may not be feasible. On the other hand, with a more detailed model perhaps other opportunities to minimize induced seismicity would become apparent. Our results demonstrate in concept that minimizing induced seismicity by manipulating injection strategy could be possible, but to do so would require careful modeling and planning.

Effect of producing fluid back after injection

Case D3 tested the effect of producing fluid back after injection. Prior to the end of injection, Case D3 was identical to Case A8, with constant pressure injection at 58 MPa. The injection rate and event magnitudes for Cases A8 and D3 are shown in Figures 4 and 9 (the vertical scales are different in the two figures). The figures show that producing from the well following injection resulted in reduced postinjection seismicity. The strategy of producing back fluid was also suggested by Baisch et al. (2006).

Producing fluid back mitigated the build-up of fluid pressure at the edge of the stimulated region after shut-in. The postinjection events immediately after the end of injection were not prevented because it took a period of time for the pressure transient caused by the production to reach the periphery of the fracture.

Effect of shear-induced pore volume dilation

Case D1 investigated the potential effect of shear-induced pore volume dilation on seismicity. Laboratory experiments such as Marone et al. (1990) and Morrow and Byerlee (1989) show evidence of fracture pore volume dilation due to sliding. The effects of pore volume dilation have been investigated numerically by several authors, including Segall and Rice (1995), Yamashita (1999), and Segall et al. (2010). We are not aware of field evidence to indicate whether or not shear-induced pore volume dilation happens during EGS stimulation, but it is an interesting phenomenon to investigate.

Pore volume dilation had the effect of damping out seismicity, consistent with results from Yamashita (1999), Segall and Rice (1995), and Segall et al. (2010). This can be seen by comparing Cases D1 and A8 in Figures 7 and 4, which show event magnitude with time. Both show constant pressure injection at 58 MPa. The only difference between the two cases is that there was shear-induced pore volume dilation in Case D1. With pore volume dilation, a larger number of events occurred, but they were of relatively smaller magnitude.

Pore dilation damped out slip events because it caused a decrease in fluid pressure during slip. During rapid slip, fluid flow did not have time to occur, and so the mass of fluid at a given location was nearly fixed. Water density is relatively insensitive to pressure, so to conserve mass, the void aperture at a given location had to be nearly constant during slip. As slip tried to dilate the void aperture, the only way to keep void aperture constant was to decrease fluid pressure, increasing the effective normal traction (equation 13). The higher effective normal traction strengthened friction and tended to inhibit slip from occurring. The same total amount of slip had to occur for the same injection pressure, and so slip was distributed into a larger number of smaller magnitude events.

There were several other differences between Case D1 and the other simulations. Due to pore dilation, the fluid pressure at the injector was decreased following seismic events. Because the well continued to inject at constant pressure, the flow rate spiked as a large pressure gradient was suddenly imposed between the injector and the fracture. Unrealistically high flow rates were possible, and so it was necessary to specify a maximum injection rate of 1 kg/s in the code. Because of the maximum flow rate, the injection rate would sometimes dip below the target of 58 MPa. Because of flow rate spiking following slip events, the flow rate history had a very erratic, unrealistic looking behavior as can be seen in Figure 7.

There were more event hypocenters near the injector in Case D1 than in Case A8. After seismic events, when fluid pressure was lowered near the injector, the fluid pressure increased again rapidly near
the wellbore. Often that triggered another seismic event close to the injector.

A more realistic treatment of fracture zone geometry may have damped out the erratic variations in flow rate. If a fracture were surrounded by porous, permeable material, such as the alteration zones observed at Soultz (see Appendix B), fluid could flow rapidly back into the fracture from the surrounding porosity and raise the fluid pressure up again across the entire fracture. Such a mechanism could lead to complicated behavior that could be interesting to investigate in future work. For example, a rapid decrease in fluid pressure during slip followed by a rapid increase in pressure due to fluid flow from a surrounding damage zone could lead to aftershock sequences with hypocenters distributed throughout the stimulated region as fluid pressure rapidly recovered following slip events.

A final difference between Case D1 and the other cases was that far more fluid was injected in Case D1 (Table 6). The reason was that because of the shear-induced pore volume dilation, a greater amount of fluid was required to increase the fluid pressure across the fracture.

**Effect of \(d_c\)**

In Case D2, a larger value of \(d_c\) resulted in inhibited seismicity. The entire fracture was stimulated, but the slip was slow and aseismic. Figure 8 shows the injection rate and event magnitude with time for Case D2. The injection rate increased continuously during injection because of the shear-induced stimulation, but seismicity did not occur. This result is consistent with the theory of rate and state friction (Ruina, 1983), which predicts that the minimum size of a patch that can slip unstably increases with \(d_c\). If the fracture in Case D2 had been large enough, the region of increased fluid pressure would have eventually grown large enough that it would have slipped unstably in a seismic event.

Differences in the rate and state parameters \(a, b,\) and \(d_c\) in nature help explain why sometimes fractures slip seismically, and sometimes they slip aseismically. Before initiating an injection experiment at a given location, characterization of the parameters \(a, b,\) and \(d_c\) could be useful for predicting seismic hazard.

**Comparison of rate/state friction to static/dynamic friction**

Previously, in McClure and Horne (2010), we modeled injection into a 1D, isolated fracture using static/dynamic friction instead of rate and state friction. See Appendix A for a description of static/dynamic friction. The problem setup was similar, but not identical, to the problem setup used in this paper. An approach similar to that used in McClure and Horne (2010) was used by Baisch et al. (2010), who modeled injection-induced seismicity in a 2D planar fault. Many results from McClure and Horne (2010) and Baisch et al (2010) were consistent with the results from the rate and state modeling described in this paper. In all three investigations, event magnitudes increased with time as the stimulated region grew larger. Hypocenters migrated away from the injector well. Postinjection events occurred because of redistribution of pressure. Investigation of the effect of injection schedule in McClure and Horne (2010) were consistent with the results in this paper. Decreasing injection pressure over time reduced seismicity relative to other strategies. Producing fluid after injection resulted in reduced seismicity.

A difference between the rate and state simulations in this work and the results from Baisch et al. (2010) and McClure and Horne (2010) is that there were fewer low magnitude events in the rate and state simulations. The models in Baisch et al. (2010) and McClure and Horne (2010) are examples of “inherently discrete” models (Ben-Zion and Rice, 1993), which tend to model a greater number of smaller events.

**CONCLUSIONS**

Our modeling suggests that the treatment of friction and stress interaction between elements have a first-order effect on the overall behavior of a shear stimulation model.

The sequential stimulation mechanism was proposed to describe the process by which shear stimulation occurs in fractures and fault zones. The shear stimulation mechanism contrasts with the diffusion controlled mechanism because it involves slip and permeability enhancement advancing ahead of pore pressure perturbation.

If the sequential stimulation mechanism describes shear stimulation realistically, it would undermine assumptions that are sometimes made for the estimation of initial hydraulic diffusivity and the estimation of least principal stress.

Shut-in seismic events occurred because of redistribution of pressure after injection was stopped, a mechanism proposed by several previous investigators. Producing fluid back after injection reduced postinjection seismicity.

Reducing injection strategy over time was identified as a strategy that minimized maximum event magnitude. This result shows in principle that it may be possible to reduce the magnitude of induced events with injection strategy.

The effect of two geological factors, slip-induced pore volume dilation and the characteristic length scale in the rate and state law, were investigated. Pore volume dilation caused more smaller events and a larger characteristic length scale led to aseismic slip.

**ACKNOWLEDGMENTS**

We gratefully acknowledge the Precourt Institute for Energy at Stanford University for funding this research. This work has benefitted greatly from interactions with a number of people at Stanford. Thank you very much to Andrew Bradley for his insight into fast boundary element solutions, as well as the coupling of rate and state and fluid flow simulation. Thank you to Eric Dunham for his help and for lending us a rate and state simulation code which we used to check the accuracy of our simulator. Also thank you to David Pollard, Paul Segall, Mark Zoback, and Hamdi Tchelbi for providing valuable insight.

**APPENDIX A**

**SEISMICITY MODELING IN EGS**

EGS modeling most often has used a treatment of friction in which elements do not slip until their shear traction exceeds their frictional ability to resist slip according to the Coulomb failure criterion

\[
|\tau| = S_0 + \mu(\sigma_n - P)
\]

where \(\tau\) is shear traction, \(S_0\) is a cohesion factor, \(\mu\) is the coefficient of friction, \(\sigma_n\) is the normal traction, and \(P\) is the fluid pressure.
If $\mu$ is assumed constant (Kohl and Mégel, 2007; Ghassemi and Tarasovs, 2006), then slip is gradual and essentially aseismic because the only friction weakening mechanism is pore pressure diffusion.

Some recent modeling has implemented methods for abruptly weakening friction on slipping elements, as occurs in earthquakes. Baisch et al. (2010) imposed an instantaneous drop in stress on slipping elements. McClure and Horne (2010) imposed an instantaneous (but subsequently recovered) drop in $\mu$ on slipping elements. In these approaches, slip occurs instantaneously and so is essentially seismic. We refer to the approach in McClure and Horne (2010) as static/dynamic friction. All of these models are more generally in the class of inherently discrete earthquake models (Ben-Zion and Rice, 1993).

Rate and state friction has several advantages compared to other approaches. Constant friction approaches can model only aseismic slip, and abruptly weakening friction can only model seismic slip. Rate and state friction can model either seismic or aseismic slip. Rate and state friction simulation allows time to be discretized during slip, allowing slip velocity to evolve continuously, although potentially very rapidly.

A variety of approaches to stress transfer have been used in EGS modeling, including the block-spring model (Baisch et al., 2010), the distinct element model (Deng et al., 2011; Raiche and Gentier, 2010), the displacement discontinuity method (Ghassemi and Tarasovs, 2006) and neglecting stress transfer (Bruel, 2007; Kohl and Mégel, 2007). The model in this paper calculated stress transfer using the displacement discontinuity method (Crouch and Starfield, 1983). The displacement discontinuity method assumes linearly elastic deformation in an infinite, isotropic, homogenous medium.

**APPENDIX B**

**RELATIONSHIP OF OUR MODEL TO ACTUAL EGS RESERVOIRS**

Observations made during EGS projects demonstrate the context of this work. The European EGS project at Soultz-sous-Forêts can be used as an example of an EGS project. During the 1990s and 2000s several wells were drilled and stimulated hydraulically in faulted and fractured granite. During each stimulation, thousands of cubic meters of water were injected at high pressure into open wellbore. The injectivity of the wells increased by one to two orders of magnitude following stimulation (Hettkamp et al., 2004; Tischner et al., 2006; Genter et al., 2010).

Spinner and temperature logs of the Soultz wells indicated that during hydraulic stimulation, fluid exited the wellbore at a small number of preexisting fracture zones intersecting the wellbore. Caliper and wellbore imaging logs indicated that the newly permeable fractures had existed prior to stimulation but had been induced to shear, enhancing their permeability. In one example, 70% of flow during injection exited the wellbore GP2K3 at a single location. (Evans et al., 2005a; Evans et al., 2005b; Baria et al., 2006; Tischner et al., 2006; Dezayes et al., 2010).

Wellbore core demonstrated that a typical fracture zone consisted of a fault core surrounded by an alteration zone up to 25 m thick. The fault cores were full of cataclasites, breccia, and secondary precipitation of quartz. The alterations zones had high fracture density and extensive chemical alteration leading to porosities as high as 25% (Genter et al., 2000).

The fault zones observed at Soultz could be considered more or less typical for medium to large-scale faults in granite (Wibberley et al., 2008; Caine et al., 1996; Bruhn et al., 1994; Lockner et al., 2009). However, other fault zones geometries in granite have been described in the literature (Griffith et al., 2009).

In this work, injection into a single, isolated fracture 500 m long was modeled. While there may be a large number of fractures participating in flow at Soultz, they are located primarily in a small number of large scale fracture zones. The larger seismic events require a laterally extensive slip surface (Charlety et al., 2007) and so are likely associated with the fault core. The fracture in our model is intended to represent the fault core. Such a model cannot describe all of the smaller scale seismic events that occur on minor fractures, but it can describe slip on the large scale features.

One challenge for EGS modeling is how to specify the model storativity. Closed fractures in granite (which would be any fracture in frictional contact, and therefore capable of generating an earthquake) have apertures on the order of hundreds of microns (Esaki et al., 1999; Lee and Cho, 2002). A huge number of such fractures would be required to contain the thousands of cubic meters that are injected during EGS stimulation. Because fluid typically exited the wellbore from a small number of fracture zones, a likely source of storativity is the high porosity, heavily fractured alteration zones that surround the fault cores.

In this work, we avoided any complexity associated with the details of fracture zone geometry. We modeled flow only in a single fracture. The storativity of the fracture was supplied by the increase in void aperture caused by increase in fluid pressure, which resulted in a decrease in effective normal stress (equation 13. Void aperture values were on the order of $10^{-3}$ m, and so the fracture had quite limited volume and storativity.

Because the fracture storativity was low, it was necessary to use low flow rates, generally around 1 kg/s. During actual stimulations at Soultz, injection rates reached 50 kg/s or higher (Tischner et al., 2006).

**REFERENCES**


